Linear Growth

If a quantity starts at size P_0 and grows by d every time period, then the quantity after n time periods can be determined using either of these relations:

Recursive form:
$P_n = P_{n-1} + d$

Explicit form:
$P_n = P_0 + d\,n$

In this equation, d represents the **common difference** – the amount that the population changes each time n increases by 1

Exponential Growth

If a quantity starts at size P_0 and grows by $R\%$ (written as a decimal, r) every time period, then the quantity after n time periods can be determined using either of these relations:

Recursive form:
$P_n = (1+r)\,P_{n-1}$

Explicit form:
$P_n = (1+r)^n\,P_0$ or equivalently, $P_n = P_0\,(1+r)^n$

We call r the **growth rate**.
The term $(1+r)$ is called the **growth multiplier**, or common ratio.

Logistic Growth

If a population is growing in a constrained environment with carrying capacity K, and absent constraint would grow exponentially with growth rate r, then the population behavior can be described by the logistic growth model:

$$P_n = P_{n-1} + r\left(1 - \frac{P_{n-1}}{K}\right)P_{n-1}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$y = mx + b$$

$$y - y_1 = m(x - x_1)$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y = a(x - h)^2 + k$$

$$h = \frac{-b}{2a} \qquad k = f(h)$$

$$(x - h)^2 + (y - k)^2 = r^2$$

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

$$\log_b x = y \Leftrightarrow b^y = x$$

$$\log_b xy = \log_b x + \log_b y$$

$$\log_b \frac{x}{y} = \log_b x - \log_b y$$

$$\log_b x^r = r\log_b x$$

$$\log_b x = \frac{\log x}{\log b} \quad \text{or} \quad \log_b x = \frac{\ln x}{\ln b}$$

$$A = lw$$

$$P = 2l + 2w$$

$$V = lwh$$

$$a^2 + b^2 = c^2$$

$$d = rt$$

$$A = P\left(1 + \frac{r}{m}\right)^{tm}$$

$$I = Prt$$

$$(f \circ g)(x) = f[g(x)]$$

$$i = \sqrt{-1}$$

$$i^2 = -1$$

$$\frac{1}{a}(x) + \frac{1}{b}(x) = 1$$

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{x}$$

$$A = Pe^{rt}$$

Difference Quotient=

$$\frac{f(x + h) - f(x)}{h}$$

Basic Probability

Given that all outcomes are equally likely, we can compute the probability of an event *E* using this formula:

$$P(E) = \frac{\text{Number of outcomes corresponding to the event E}}{\text{Total number of equally - likely outcomes}}$$

Odds in favor = (favorable outcomes / unfavorable outcomes)
Odds against= (unfavorable outcomes / favorable outcomes)

Complement of an Event

The **complement** of an event is the event "*E* doesn't happen"

The notation $\overline{E}$ is used for the complement of event *E*.

We can compute the probability of the complement using $P\left(\overline{E}\right) = 1 - P(E)$

Notice also that $P(E) = 1 - P\left(\overline{E}\right)$

Independent Events

Events A and B are **independent events** if the probability of Event B occurring is the same whether or not Event A occurs.

***P*(*A* and *B*) for independent events**

If events *A* and *B* are independent, then the probability of both *A* and *B* occurring is

$P(A \text{ and } B) = P(A) \cdot P(B)$

where $P(A \text{ and } B)$ is the probability of events *A* and *B* both occurring, $P(A)$ is the probability of event *A* occurring, and $P(B)$ is the probability of event *B* occurring

***P*(*A* or *B*)**

The probability of either *A* or *B* occurring (or both) is

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Conditional Probability Formula

If Events *A* and *B* are not independent, then
$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$

Bayes' Theorem

$$P(A \mid B) = \frac{P(A)P(B \mid A)}{P(A)P(B \mid A) + P(\overline{A})P(B \mid \overline{A})}$$

Basic Counting Rule

If we are asked to choose one item from each of two separate categories where there are m items in the first category and n items in the second category, then the total number of available choices is $m \cdot n$.

This is sometimes called the multiplication rule for probabilities.

Factorial

$n! = n \cdot (n - 1) \cdot (n - 2) \cdots 3 \cdot 2 \cdot 1$

Permutations

${}_nP_r = n \cdot (n - 1) \cdot (n - 2) \cdots (n - r + 1)$

We say that there are ${}_nP_r$ **permutations** of size r that may be selected from among n choices *without replacement* when *order matters*.

It turns out that we can express this result more simply using factorials.

$$ {}_nP_r = \frac{n!}{(n-r)!} $$

Combinations

$$ {}_nC_r = \frac{{}_nP_r}{{}_rP_r} $$

We say that there are ${}_nC_r$ **combinations** of size r that may be selected from among n choices *without replacement* where *order doesn't matter*.

We can also write the combinations formula in terms of factorials:

$$ {}_nC_r = \frac{n!}{(n-r)!r!} $$

Expected Value

Expected Value is the average gain or loss of an event if the procedure is repeated many times.

We can compute the expected value by multiplying each outcome by the probability of that outcome, then adding up the products.