Section 1.1 Linear Equations

Equation : A statement that two expressions are equal.

Example: $x + 2 = 9$

Solve: Find <u>all</u> numbers that makes the equation a true statement.

Solution: <u>a</u> number that makes the equation a true statement.

Solution Set: <u>all</u> the numbers that make the equation a true statement.

Identity equation : An equation that is true for <u>every</u> real number in the domain of the variable in the equation.

$$\cancel{2x}+7 = \cancel{2x}+7$$
$$-2x \qquad\qquad -2x$$
$$\boxed{7 = 7}$$

☆ **Conditional equation :** an equation that is satisfied (A Solution) by some numbers but not by others.

Contradiction: An equation which is false for every value of the variable in the equation.

$$\cancel{2x}+5 = \cancel{2x}+1$$
$$-2x \qquad\qquad -2x$$
$$\boxed{5 \neq 1}$$

Steps for Solving a Linear Equation

To solve a linear equation, follow these steps:

Step1: List any restrictions on the variable.

Step 2: If necessary, clear the equation of fractions by multiplying both sides (of the equation) by the least common multiple (LCM) of the denominators of **ALL** the fractions in the equation.

Step 3: Remove all parentheses and simplify.

Step 4: Collect all terms containing the variable on one side and all remaining terms on the other side.

Step 5: Simplify and solve.

Step 6: Check your solution(s).

Linear equation in one variable:

> an equation that can be written in the form:
> **$ax + b = 0$,** where $a \neq 0$.

Example 1: $3x - 5 = 4$

$+5 \quad +5$

$3x = 9$

$\frac{\cancel{3}x}{\cancel{3}} = \frac{9}{3}$

$\boxed{x = 3}$

check:

$3(3) - 5 = 4$

$9 - 5 = 4$

$4 = 4$ ✓

Example 2: $6(x - 1) + 4 = 3(7x + 1)$

$6x - 6 + 4 = 21x + 3$

$6x - 2 = \cancel{21x} + 3$

$-21x \qquad -\cancel{21x}$

$-15x \cancel{-2} = 3$ $\nearrow$

$+\cancel{2} \qquad +2$

$\frac{-15x}{-15} = \frac{5}{-15}$

$\boxed{x = \frac{-1}{3}}$

Solving equations with fractions in them?

Clear the equation of all fractions. This is done by multiplying both members of the equation by the Least Common Denominator of all the fractions in the equation .

Example: $\frac{1}{2}(x+5) - 4 = \frac{1}{3}(2x - 1)$

LCD = 6

$$\overset{3(\cancel{6})}{\frac{1}{\cancel{2}}}x + \overset{3(\cancel{6})}{\frac{5}{\cancel{2}}} - \overset{(6)}{\frac{4}{1}} = \overset{2(\cancel{6})}{\frac{2}{\cancel{3}}}x - \overset{2(\cancel{6})}{\frac{1}{\cancel{3}}}$$

$3x + 15 - 24 = 4x - 2$

$3x - 9 = \cancel{4x} - 2$ $\nearrow$

$-4x \qquad -\cancel{4x}$

$-x \cancel{-9} = -2$

$+\cancel{9} \qquad +9$

$\frac{-1x}{-1} = \frac{7}{-1}$

$\boxed{x = -7}$

$\cancel{(x-2)}(x-1) \qquad (x-2)\cancel{(x-1)} \qquad \cancel{(x-2)(x-1)}$

Example: $\frac{3}{\cancel{x-2}} = \frac{1}{\cancel{x-1}} + \frac{7}{\cancel{(x-1)(x-2)}}$

$x \neq 2$

$x \neq 1$

LCD = $(x-2)(x-1)$

$$3(x-1) = 1(x-2) + 7$$

$$3x-3 = x-2+7$$

$$3x-3 = \cancel{x}+5$$

$$-x \qquad -\cancel{x}$$

$$2x-\cancel{3} = 5$$

$$+\cancel{3} \qquad +3$$

$$\frac{2x}{2} = \frac{8}{2} \qquad \boxed{x=4}$$

check:

$$\frac{3}{4-2} = \frac{1}{4-1} + \frac{7}{(4-1)(4-2)}$$

$$\frac{3}{2} = \frac{1}{3}\cdot\frac{2}{2} + \frac{7}{6}$$

$$\frac{3}{2} = \frac{2}{6} + \frac{7}{6}$$

$$\frac{3}{2} = \frac{9}{6}$$

$$\frac{3}{2} = \frac{3}{2} \checkmark$$

An equation with no solution:

Example: $\frac{3x}{x-1} + \frac{2}{1} = \frac{3}{(x-1)}$

$x \neq 1$

$$\cancel{(x-1)}\ \frac{3x}{\cancel{x-1}} + (x-1)\frac{2}{1} = \cancel{(x-1)}\ \frac{3}{\cancel{x-1}}$$

$$3x + 2(x-1) = 3$$

$$3x + 2x - 2 = 3$$

$$5x - 2 = 3$$

$$+2 \qquad +2$$

$$\frac{5x}{5} = \frac{5}{5}$$

no solution

$$\cancel{\boxed{x=1}}$$

Steps for Solving Applied Problems

Step1: Read the problem carefully, perhaps two or three times. Pay particular attention to the question being asked in order to identify what you are looking for. If you can, determine realistic possibilities for the answer.

Step 2: Assign a letter (variable) to represent what you are looking for, and, if necessary, express any remaining unknown quantities in terms of this variable. Let x = number

Step 3: Make a list of all the known facts, and translate them into mathematical expressions. These may take the form of an equation (or later, an inequality) involving the variable. If possible, draw an appropriately labeled diagram to assist you. Sometimes a table or chart helps.

Step 4: Solve the equation for the variable, and then answer the question, usually using a complete sentence.

Step 5: Check the answer with the facts in the problem. If it agrees, congratulations! If it does not agree, try again.

EXAMPLE:

ANDY GROSSED $435 ONE WEEK BY WORKING 52 HOURS. HIS EMPLOYER PAYS TIME-AND-A-HALF FOR ALL HOURS WORKED IN EXCESS OF 40 HOURS. WHAT IS ANDY'S HOURLY WAGE?

	hours worked	hourly wage	salary
Regular	40	x	$40x$
Overtime	12	$1.5x$	$12(1.5x) = 18x$

$$40x + 18x = 435$$

$$\frac{58x}{58} = \frac{435}{58}$$

$$\boxed{x = \$7.50}$$

Step 4:

$$
\begin{aligned}
18{,}000 - x &= \frac{1}{2}x \\
18{,}000 &= x + \frac{1}{2}x \\
18{,}000 &= \frac{3}{2}x \\
\left(\frac{2}{3}\right)18{,}000 &= \left(\frac{2}{3}\right)\left(\frac{3}{2}x\right) \\
12{,}000 &= x
\end{aligned}
$$

So, \$12,000 is invested in stocks and \$18,000 − \$12,000 = \$6000 is invested in bonds.

Step 5: The total invested is \$12,000 + \$6000 = \$18,000, and the amount in bonds, \$6000, is half that in stocks, \$12,000.

Now Work PROBLEM 83

EXAMPLE 9 Determining an Hourly Wage

Shannon grossed \$435 one week by working 52 hours. Her employer pays time-and-a-half for all hours worked in excess of 40 hours. With this information, can you determine Shannon's regular hourly wage?

Solution

Step 1: We are looking for an hourly wage. Our answer will be in dollars per hour.

Step 2: Let x represent the regular hourly wage; x is measured in dollars per hour.

Step 3: We set up a table:

	Hours Worked	Hourly Wage	Salary
Regular	40	x	$40x$
Overtime	12	$1.5x$	$12(1.5x) = 18x$

The sum of regular salary plus overtime salary will equal \$435. From the table, $40x + 18x = 435$.

Step 4:

$$
\begin{aligned}
40x + 18x &= 435 \\
58x &= 435 \\
x &= 7.50
\end{aligned}
$$

Shannon's regular hourly wage is \$7.50 per hour.

Step 5: Forty hours yields a salary of 40(7.50) = \$300, and 12 hours of overtime yields a salary of 12(1.5)(7.50) = \$135, for a total of \$435.

Now Work PROBLEM 87

SUMMARY Steps for Solving a Linear Equation

To solve a linear equation, follow these steps:

Step 1: List any restrictions on the variable.

Step 2: If necessary, clear the equation of fractions by multiplying both sides by the least common multiple (LCM) of the denominators of all the fractions.

Step 3: Remove all parentheses and simplify.

Step 4: Collect all terms containing the variable on one side and all remaining terms on the other side.

Step 5: Simplify and solve.

Step 6: Check your solution(s).

§1.2 Quadratic Equations

Quadratic Equation: an equation which can be written in the form $ax^2 + bx + c = 0$ where a, b, c are real numbers and $a \neq 0$. This is also called standard form.

1) Solving by Factoring

Zero-Factor Property - If a and b are complex numbers with $ab = 0$, then $a = 0$ or $b = 0$ or both.

Square Root Property - If $x^2 = k$, then $x = \pm\sqrt{k}$.

example: Solve by factoring

a) $x^2 + 6x = 0$

$x(x+6) = 0$

$\boxed{x = 0}$ | $x+6 = 0$

$-6 \quad -6$

$\boxed{x = -6}$

b) $2x^2 = x + 3$

$2x^2 - x - 3 = 0$

$(2x-3)(1x+1) = 0$

$2x-3 = 0$ | $x+1 = 0$

$\frac{2x}{2} = \frac{3}{2}$ | $\boxed{x = -1}$

$\boxed{x = \frac{3}{2}}$

c) $x^2 = 5$

$\sqrt{x^2} = \sqrt{5}$

$\boxed{x = \pm\sqrt{5}}$

d) $(x-2)^2 = 16$

$\sqrt{(x-2)^2} = \sqrt{16}$

$x-2 = \pm 4$

$x = 2 \pm 4$

$\boxed{x = 6}$ $\boxed{x = -2}$

2) Solving by Completing the Square

Start	Add	Result
$x^2 + 4x$	4	$x^2 + 4x + 4 = (x + 2)^2$
$x^2 + 12x$	36	$x^2 + 12x + 36 = (x + 6)^2$
$x^2 - 6x$	9	$x^2 - 6x + 9 = (x - 3)^2$
$x^2 + x$	$\frac{1}{4}$	$x^2 + x + \frac{1}{4} = \left(x + \frac{1}{2}\right)^2$

Start	Add	Result
$x^2 + mx$	$\left(\frac{m}{2}\right)^2$	$x^2 + mx + \left(\frac{m}{2}\right)^2 = \left(x + \frac{m}{2}\right)^2$

EXAMPLE Completing the Square

Start	Add	Result	Factored Form
$y^2 + 8y$	$\left(\frac{1}{2}\cdot 8\right)^2 = 16$	$y^2 + 8y + 16$	$(y + 4)^2$
$x^2 + 12x$	$\left(\frac{1}{2}\cdot 12\right)^2 = 36$	$x^2 + 12x + 36$	$(x + 6)^2$
$a^2 - 20a$	$\left(\frac{1}{2}\cdot(-20)\right)^2 = 100$	$a^2 - 20a + 100$	$(a - 10)^2$
$p^2 - 5p$	$\left(\frac{1}{2}\cdot(-5)\right)^2 = \frac{25}{4}$	$p^2 - 5p + \frac{25}{4}$	$\left(p - \frac{5}{2}\right)^2$

example: Solve(by completing the square)

$x^2 + 5x + 4 = 0$

$\left(\frac{5}{2}\right)^2 = \frac{25}{4}$

$\boxed{x^2 = 16}$

$x^2 + 5x + \frac{25}{4} = \frac{-4}{1} + \frac{25}{4}$

$\left(x + \frac{5}{2}\right)^2 = \frac{-4}{1} \cdot \frac{4}{4} + \frac{25}{4}$

$\left(x + \frac{5}{2}\right)^2 = \frac{-16}{4} + \frac{25}{4}$

↗

$\left(x + \frac{5}{2}\right)^2 = \frac{9}{4}$

$x + \frac{5}{2} = \pm\sqrt{\frac{9}{4}}$

$x + \frac{5}{2} = \pm\frac{3}{2}$

↗

$x = \frac{-5}{2} \pm \frac{3}{2}$

$\boxed{x = \frac{-2}{2} = -1}$

$\boxed{x = \frac{-8}{2} = -4}$

3) Solving Using the Quadratic Formula

The solutions of $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Note: The fraction bar in the quadratic formula extends under the –b term in the numerator.

example: Solve(by quadratic formula)

$3x^2 - 5x + 1 = 0$

$a = 3$
$b = -5$
$c = +1$

PEMDAS

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(1)}}{2(3)}$$

EMS

$$= \frac{5 \pm \sqrt{25 - 12}}{6}$$

$$= \frac{5 \pm \sqrt{13}}{6} = \boxed{\frac{5}{6} \pm \frac{\sqrt{13}}{6}}$$

ex.:

$x = \frac{4 \pm \sqrt{7}}{2} \neq 2 \pm \sqrt{7}$

$x = \frac{4}{2} \pm \frac{\sqrt{7}}{2}$

$\boxed{x = 2 \pm \frac{\sqrt{7}}{2}}$

Discriminant of a Quadratic Equation

For a quadratic equation $ax^2 + bx + c = 0$:

1. If $b^2 - 4ac > 0$, there are two unequal real solutions.
2. If $b^2 - 4ac = 0$, there is a repeated real solution, a root of multiplicity 2.
3. If $b^2 - 4ac < 0$, there is no real solution. complex solution.

FOIL

$$(x+2)(x+3)$$

$$= x^2 + 3x + 2x + 6$$

$$0 = x^2 + 5x + 6$$

§1.3 Complex Numbers

Complex number: a number of the form a + bi, where a and b are real numbers. a is called the <u>REAL</u> part of the complex number a + bi, bi is called the <u>IMAGINARY</u> part of the complex number a + bi.

Imaginary number: a complex number of the form a + bi, where b is nonzero.

Standard Form of a complex number:
a + bi or a + ib (Discuss $i\sqrt{5}$ & $\sqrt{5i}$)

Definition of i: $\boxed{i=\sqrt{-1}}$ or $\boxed{i^2=-1}$

$=\sqrt{-1}\cdot\sqrt{a}$

Definition of $\sqrt{-a}$ If a > 0, then $\sqrt{-a} = i\sqrt{a}$

Example: $\sqrt{-16}$

$\sqrt{+4} = 2$ b/c $(2)(2)=4$
$\sqrt{+9} = 3$ b/c $(3)(3)=9$
$\sqrt{25} = 5$ b/c $(5)(5)=25$

$\sqrt{-16} = \boxed{4i}$ b/c $(4i)(4i) = -16$
$\sqrt{-16} = \sqrt{-1}\,\sqrt{16} = i\sqrt{16} = 4i$

Simplify

example 4: a) $\sqrt{-4}$
$= \sqrt{-1}\,\sqrt{4} = \boxed{2i}$

b) $\sqrt{-8} = \sqrt{-1}\cdot\sqrt{8}$
$= i\sqrt{4}\cdot\sqrt{2} = \boxed{2i\sqrt{2}}$

OPERATIONS WITH COMPLEX NUMBERS

Addition or Subtraction of Complex Numbers:

1. Combine the real parts.
2. Combine the imaginary parts.
3. Leave the result in the form a + bi.

Note: Add (or subtract) the real numbers then add the imaginary numbers.

example 1:

a) $(3 + 5i) + (-2 + 3i)$

$= \boxed{1 + 8i}$

a + bi

b) $(6 + 4i) - (3 + 6i)$

$= 6 + 4i - 3 - 6i$

$= \boxed{3 - 2i}$ a + bi

Multiplication of Complex Numbers:

1. Multiply the numbers as if they are two binomials (FOIL METHOD).
2. Substitute -1 for i^2
3. Combine the like terms and leave the result in the form a + bi.

example 2: a) $(5 + 3i)(2 + 7i)$

$= 10 + 35i + 6i + 21i^2$

$= 10 + 35i + 6i + 21(-1)$

$= 10 + 41i - 21$

$= \boxed{-11 + 41i}$ a + bi

b) $(4 + 3i)^2$

$= (4 + 3i)(4 + 3i)$

$= 16 + 12i + 12i + 9i^2$

$= 16 + 24i - 9$

$= \boxed{7 + 24i}$ a + bi

Properties of Complex Conjugates:

If $z = a + bi$ then the conjugate $\bar{z} = a - bi$:

$$z \cdot \bar{z} = (a + bi)(a - bi) = a^2 + b^2$$

Division of Complex Numbers:

1. Write the division as a fraction.
2. Multiply the numerator and denominator by the conjugate of the denominator:

$$\frac{a+bi}{c+di} \cdot \frac{c-di}{c-di}.$$

3. Multiply and simplify in the numerator (by FOIL). Multiply and simplify in the denominator to a real number (by FOIL).
4. Write the result in the form a + bi.

example 3 a) $\dfrac{1+4i}{5-12i}$ b) $\dfrac{2-3i}{4-3i}$

a) $$\frac{1+4i}{5-12i} \cdot \frac{5+12i}{5+12i} = \frac{5+12i+20i+48i^2}{25\cancel{+60i}\cancel{-60i}-144i^2} = \frac{5+32i-48}{25+144}$$

$$= \frac{-43+32i}{169} = \boxed{\frac{-43}{169} + \frac{32i}{169}}$$ a + bi

b) $$\frac{2-3i}{4-3i} \cdot \frac{4+3i}{4+3i} = \frac{8+6i-12i-9i^2}{16\cancel{+12i}\cancel{-12i}-9i^2} = \frac{17-6i}{25}$$

$$= \boxed{\frac{17}{25} - \frac{6}{25}i}$$ a + bi

Powers of i: $i^1 = i$ $\quad i^2 = -1$ $\quad i^3 = -i$ $\quad \boxed{i^4 = 1}$

example 4: a) i^{27}

$$= (i^4)^6 \cdot i^3$$

$$\begin{array}{r} 6 \\ 4\overline{)27} \\ \underline{24} \\ \boxed{3} \end{array}$$

$$= (1)^6 \cdot i^3$$

$$= \boxed{-i}$$

b) i^{101} $= (i^4)^{25} \cdot i^1$

$$= \boxed{i}$$

c) $i^{36} = (i^4)^9 = (1)^9$

$$= \boxed{1}$$

example 5: Solve $x^2 - 4x + 8 = 0$

$a = 1$
$b = -4$
$c = 8$

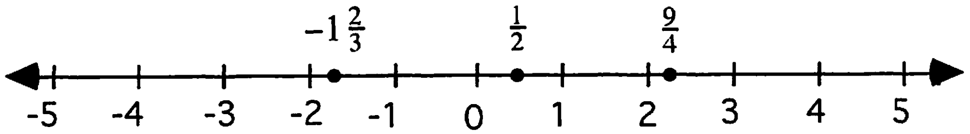
~~$(\quad)(\quad) = 0$~~

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(8)}}{2(1)}$$

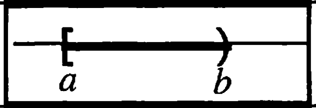
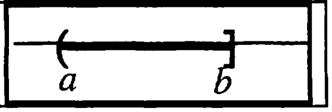
$$= \frac{+4 \pm \sqrt{16 - 32}}{2} = \frac{4 \pm \sqrt{-16}}{2}$$

$$= \frac{4 \pm 4i}{2} = \frac{4}{2} \pm \frac{4}{2}i = \boxed{2 \pm 2i}$$

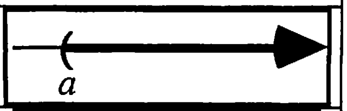
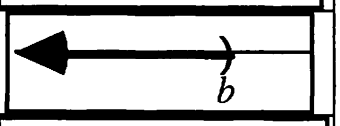
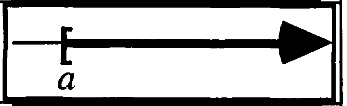
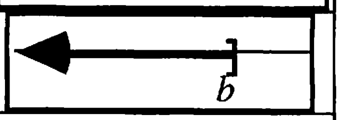
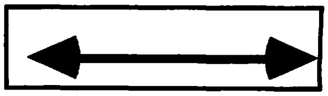


Interval Notation

Bounded Intervals on the Real Number Line

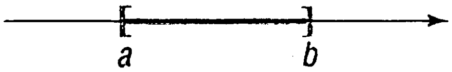
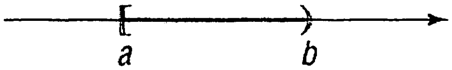
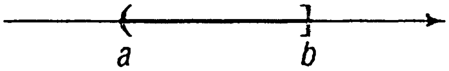
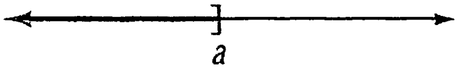
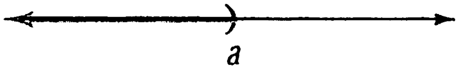
Type of Interval	Inequality	Interval Notation	Graph
Closed	$a \leq x \leq b$	$[a,b]$	
Open	$a < x < b$	(a,b)	
Half - Open	$a \leq x < b$	$[a,b)$	
Half - Open	$a < x \leq b$	$(a,b]$	

Unbounded Intervals on the Real Number Line

Type of Interval	Inequality	Interval Notation	Graph
Open	$x > a$	(a,∞)	
Open	$x < b$	$(-\infty,b)$	
Half - Open	$x \geq a$	$[a,\infty)$	
Half - Open	$x \leq b$	$(-\infty,b]$	
All Real Numbers	ℝ	$(-\infty,\infty)$	

§1.5 Solving Inequalities

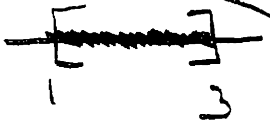
Interval Notation

Interval	**Inequality**	**Graph**
The open interval (a, b)	$a < x < b$	
The closed interval $[a, b]$	$a \leq x \leq b$	
The half-open interval $[a, b)$	$a \leq x < b$	
The half-open interval $(a, b]$	$a < x \leq b$	
The interval $[a, \infty)$	$x \geq a$	
The interval (a, ∞)	$x > a$	
The interval $(-\infty, a]$	$x \leq a$	
The interval $(-\infty, a)$	$x < a$	
The interval $(-\infty, \infty)$	All real numbers	

Examples 1 Write in interval notation and graph.

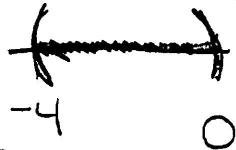
a.) $1 \leq x \leq 3$

I.N $[1,3]$

G 

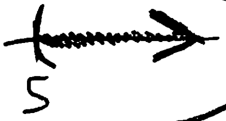
b.) $-4 < x < 0$

I.N. $(-4, 0)$

G 

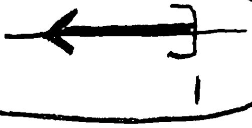
c.) $x > 5$

I.N. $(5, \infty)$

G 

d.) $x \leq 1$

I.N $(-\infty, 1]$

G 

Properties of Inequalities - Let a, b and c be real numbers.

1.) $a < b$ and $a + c < b + c$ are equivalent.
(addition property)
2.) If $c > 0$, then $a < b$ and $ac < bc$ are equivalent.
(multiplication property)
3.) If $c < 0$, then $a < b$ and $ac > bc$ are equivalent.
(multiplication property) *

Note: Replacing < with >, ≤ or ≥ results in equivalent properties.

* **Note**: When multiplying or dividing both sides of the inequality by a negative number, we must reverse the direction of the inequality symbol.

Linear Inequalities: an inequality that can be written in the form $\mathbf{ax + b > 0}$ where $a \neq 0$. (Note: Any inequality symbol may be used<, >, ≤, ≥.)

Use the properties of inequalities to solve linear inequalities by isolating the variable.

Example 2 Solve. write answers in interval notation.

a.) $3 - 2x < 5$

$-3 \qquad -3$

$\frac{-2x}{-2} < \frac{2}{-2}$

$x > -1$

$\boxed{(-1, \infty)}$

b.) $4x + 7 \geq \cancel{2}x - 3$

$-2x \qquad \cancel{-2x}$

$2x \cancel{+7} \geq -3$

$\cancel{-7} \qquad -7$

$2x \geq -10$

$\frac{2x}{2} \geq \frac{-10}{2}$

$x \geq -5$

$\boxed{[-5, \infty)}$

Double Inequalities: Isolate the variable in the middle. Perform operations on each part of the inequality.

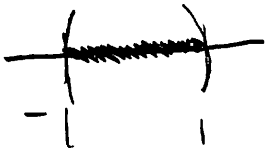
c.) $-5 < 3x \cancel{-} 2 < 1$

$+2 \quad\quad +\cancel{2} \quad +2$

$\frac{-3}{3} < \frac{\cancel{3}x}{\cancel{3}} < \frac{3}{3}$

$-1 < x < 1$

$\boxed{(-1, 1)}$



d.) $$\overset{(2)}{-1} \le \overset{\cancel{(2)}}{\frac{3-5x}{\cancel{2}}} \le \overset{(2)}{9}$$

$-2 \le \cancel{3}-5x \le 18$

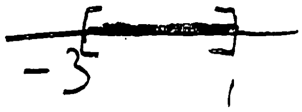
$-3 \quad\quad -\cancel{3} \quad\quad\quad -3$

$\frac{-5}{-5} \le \frac{-5x}{-5} \le \frac{15}{-5}$

$1 \ge x \ge -3$

$-3 \le x \le 1$

$[-3, 1]$



§1.6 Equations and Inequalities with Absolute Value

Solving Equations Involving Absolute value: Theorem

if a is a positive real number and if u is any algebraic expression, then

$|u| = a$ is equivalent to

$$u = a \quad \text{or} \quad u = -a$$

Example Solve. $|5| = 5 \quad |-3| = 3$

a.) $|x+4| = 13$

$x + 4 = 13$ or $x + 4 = -13$

$\quad -4 \quad -4 \qquad\qquad -4 \quad -4$

$\boxed{x = 9}$ ✓ $\qquad \boxed{x = -17}$ ✓

b.) $|2x-3| \cancel{+2} = 7$

$\quad \cancel{-2} \quad -2$

$|2x-3| = 5$

$2x - 3 = 5$ or $2x - 3 = -5$

$\quad +3 \quad +3 \qquad\qquad +3 \quad +3$

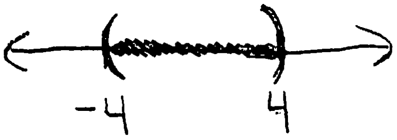
$\frac{2x}{2} = \frac{8}{2} \qquad \frac{2x}{2} = \frac{-2}{2}$

$\boxed{x = 4}$ ✓ $\qquad \boxed{x = -1}$ ✓

Solving Inequalities Involving Absolute value: Theorem

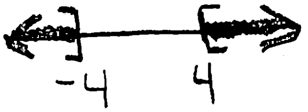
Examples Solve.

a.) $|x| < 4$



$\boxed{(-4,4)}$

b.) $|x| \geq 4$



if a is a positive real number and if u is any algebraic expression, then

(1) $|u| < a$ is equivalent to $-a < u < a$

(2) $|u| > a$ is equivalent to $u < -a$ or $u > a$

Note: You may see < or ≤. You may see > or ≥ .

Examples Solve.

a.) $|2x + 4| \leq 3$

$-3 \leq 2x+4 \leq 3$

$-4 \qquad -4 \qquad -4$

$\frac{-7}{2} \leq \frac{2x}{2} \leq \frac{-1}{2}$

$\frac{-7}{2} \leq x \leq \frac{-1}{2}$

I.N.

$\boxed{\left[\frac{-7}{2}, \frac{-1}{2}\right]}$

b.) $|1 - 4x| < 5$

$-5 < 1-4x < 5$

$-1 \qquad -1 \qquad -1$

☆ $\frac{-6}{-4} < \frac{-4x}{-4} < \frac{4}{-4}$

$\frac{3}{2} > x > -1$

$-1 < x < \frac{3}{2}$

$\boxed{\left(-1, \frac{3}{2}\right)}$

$|x| = 3$

$x = 3$ or $x = -3$

c.) $|2x-5|>3$

$|u|>a$

$u<-a$ or $u>a$

$2x\cancel{-5}<-3$ or

$\cancel{+5} \quad +5$

$\frac{2x}{2}<\frac{2}{2}$

$x<1$

$\boxed{(-\infty,1)}$

$2x\cancel{-5}>3$

$\cancel{+5} \quad +5$

$\frac{2x}{2}>\frac{8}{2}$

$x>4$

$\boxed{(4,\infty)}$

$\boxed{(-\infty,1)\cup(4,\infty)}$

d) $|x-4|\cancel{+2}\geq 5$

$\cancel{-2} \quad -2$

☆ $|x-4|\geq 3$

$x\cancel{-4}\leq-3$ or $x-4\geq 3$

$\cancel{+4} \quad +4 \qquad +4 \quad +4$

$x\leq 1 \qquad x\geq 7$

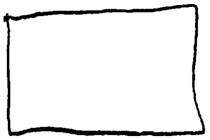
$\boxed{(-\infty,1]\cup[7,\infty)}$

§ 1.7 Problem Solving: Interest, Mixture, Constant Rate Applications

Translating Verbal Descriptions into Mathematical Expressions

Example 1: Write an algebraic expression for the verbal description.

a) The length of a rectangle is four times its width.



w = width

4w = length

b) Five times a number, decreased by 3.

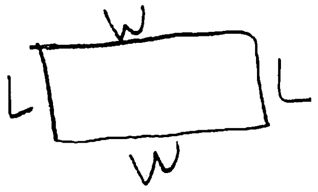
let x = number $\quad$ $5x - 3$

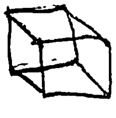
c) The product of 3 and a number increased by 5.

let x = number $\quad$ $3x + 5$

Example 2: Solve for the indicated variable.

a) P = 2l + 2w (for l). $\quad$ b) V = l · w · h (for w)





$$\frac{P-2w}{2} = \frac{2L}{2}$$

$$\boxed{L = \frac{P-2W}{2}} = \frac{P}{2} - \frac{\cancel{2}w}{\cancel{2}} = \boxed{\frac{P}{2} - w}$$

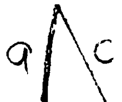
$$\frac{V}{lh} = \frac{\cancel{l} \cdot w \cdot \cancel{h}}{\cancel{l}\,\cancel{h}}$$

$$\boxed{w = \frac{V}{lh}}$$

c) $P = \cancel{a} + b + \cancel{c}$

$\quad -a - c \quad -a \quad -c$

$$\boxed{b = P - a - c}$$



(b)

Solving Applied Problems

Step 1: Read the problem thoroughly.
Step 2: Give one unknown quantity a variable name and write it down.
Step 3: Draw a picture or make a chart to show the information. (If applicable).
Step 4: Write all other unknowns in terms of the variable. (step 2)
Step 5: Write an equation in one variable.
Step 6: Solve the equation.
Step 7: Check the solution in the words of the problem to be sure it makes sense.

Simple Interest Problems

If a principal of **P** dollars is borrowed for a period of **t** years at a interest rate **r**, the interest **I** charged is

$$\boxed{I = Prt}$$

Example 3: Suppose that Juanita borrows $500 for 6 months at the simple interest rate of 9% per year. What is the interest that she will be charged on the loan? How much does Juanita owe after 6 months?

$$I = Prt$$

$$I = (500)(.09)\left(\frac{6}{12}\right) = (500)(.09)\left(\frac{1}{2}\right)$$

$$\boxed{I = \$22.50} \quad \$500 + 22.50 = \boxed{\$522.50}$$

Solve Mixture Problems

These problems combine two or more quantities to form a mixture. $(\%)(\text{amt}) + (\%)(\text{amt}) = (\%)(\text{amt})$

1 mixture   2nd mixture   final mixture

Example 4:

The manager of a Starbucks store decides to experiment with a new blend of coffee. She will mix some B grade Colombian coffee that sells for \$5 per pound with some A grade Arabica coffee that sells for \$10 per pound to get 100 pounds of the new blend. The selling price of the new blend is to be \$7 per pound, and there is to be no difference in revenue from selling the new blend versus selling the other types. How many pounds of the B grade Colombian and A grade Arabica coffees are required?

(B Grade) + (A Grade) = final mixture

$(5)(\quad) + (10)(\quad) = (7)(100)$

let x = lbs of B Grade coffee

$(5)(x) + 10(100-x) = (7)(100)$

$5x + 1000 - 10x = 700$

$-5x + 1000 = 700$

$\quad -1000 \quad -1000$

$\frac{-5x}{-5} = \frac{-300}{-5}$

$\boxed{x = 60 \text{ lbs B Grade}}$

$100 - x$

$100 - 60$

$\boxed{40 \text{ lbs A Grade}}$

B Grade \$300

A Grade \$400

700 ✓

Solve Constant Rate Job Problems

If a "machine" can perform a task in 5 hours, then it completes $\frac{1}{5}$ of the task each hour. This is the machine's **work rate**. The combined work rate of two or more "machines" is the sum of their individual work rates.

Example 5:

At 10 AM Danny is asked by his father to weed the garden. From past experience, Danny knows that this will take him 4 hours, working alone. His older brother, Mike, when it is his turn to do the job, requires 6 hours. Since Mike wants to go golfing with Danny and has a reservation for 1 PM, he agrees to help Danny. Assuming no gain or loss of efficiency, when will they finish if they work together? Can they make the golf date?

	Job(hr)	work rate
Danny's work rate	(4)	$\frac{1}{4}$
Mike's work rate	(6)	$\frac{1}{6}$

LCD = 12t

$$\overset{3}{\cancel{12}t}\,\frac{1}{\cancel{4}} + \overset{2}{\cancel{12}t}\,\frac{1}{\cancel{6}} = 12\cancel{t}\,\frac{1}{\cancel{t}}$$

Let t = time it takes work together.

$$3t + 2t = 12$$

$$\frac{5t}{5} = \frac{12}{5}$$

$$\boxed{t = \frac{12}{5} = 2\frac{2}{5} \text{ hrs}}$$

2 hrs 24 min

finish 12:24 pm

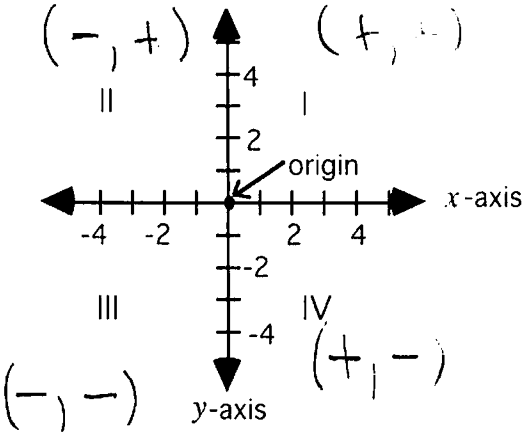
§ 2.1 The Distance and Midpoint Formulas

- an **ordered pair** consists of two numbers where order (or sequence) is important.

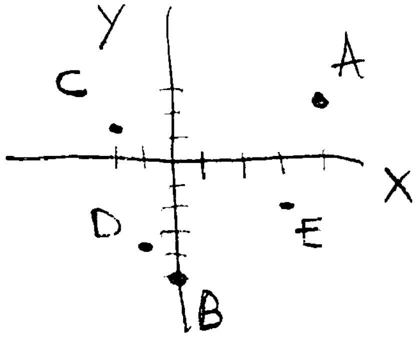
Example (1,2) and (2,1) are different ordered pairs.

The Rectangular Coordinate System (or Cartesian coordinate system)

- the rectangular coordinate system has four **quadrants** (I, II, III, IV).
- the point (0,0) is called the **origin.**
- points in the rectangular coordinate system are ordered pairs (x, y) where x and y are the **coordinates** of the point.



Example Plot the points A(4, 3), B(0,-5), C(-2, 1), D(−1, − 4) and E(3, -2).

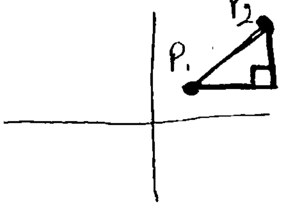


The Distance Formula:

- the distance between two points $P_1(x_1,y_1)$ and $P_2(x_2,y_2)$ is : $d(P_1,P_2)=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Example Find the distance $d(P_1, P_2)$ given $P_1(-4, 5)$ and $P_2(3, 2)$.

x_1 y_1 x_2 y_2

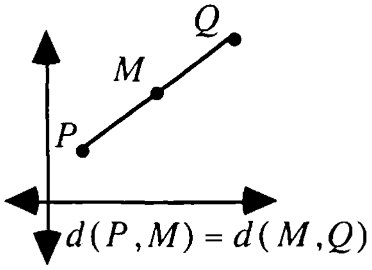


$$d(P_1,P_2) = \sqrt{(3 - -4)^2 + (2 - 5)^2}$$

$$= \sqrt{(7)^2 + (-3)^2}$$

$$= \sqrt{49 + 9} = \boxed{\sqrt{58}} = 7. \ldots$$

Midpoint Formula:



- the midpoint of the line segment PQ with endpoints

$P(x_1,y_1)$ and $Q(x_2,y_2)$ is

$$M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

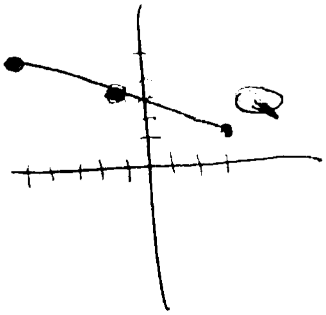
Note:	the midpoint formula gives the coordinates of the midpoint not the distance (length) of it.

Example Find the midpoint of the line segment PQ given P(–5, 5) and Q(3, 1).

x_1 y_1

x_2 y_2

$$M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$



$$= \left(\frac{-5+3}{2}, \frac{5+1}{2}\right)$$

$$= \left(\frac{-2}{2}, \frac{6}{2}\right)$$

$$= \boxed{(-1, 3)}$$

§ 2.2 Graphs of Equations in Two Variables; Intercepts; Symmetry

Graphing by Plotting Points

Are the following points on the graph of 2x – y = 6?

a) (2, 3)

$x = 2$

$y = 3$

$2x - y = 6$

$2(2) - (3) = 6$

$4 - 3 = 6$

$1 \neq 6$ no

b) (2, –2)

$x = 2$

$y = -2$

$2(2) - (-2) = 6$

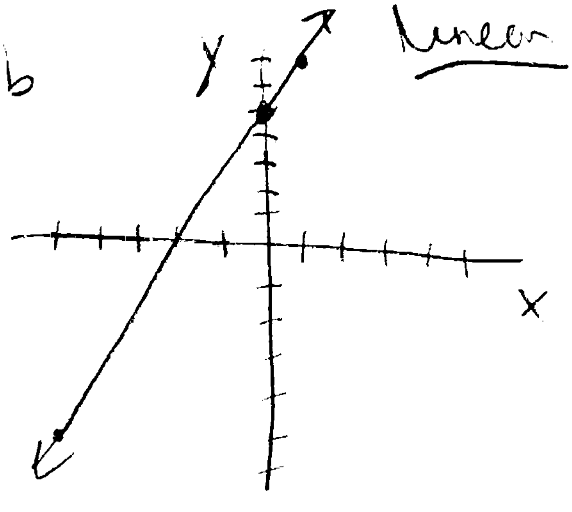
$4 - (-2) = 6$

$6 = 6$ ✓ yes

Example: Sketch the graph of the line $y = 2x + 5$ by completing the table and then plotting the points.

x	y
0	$2(0)+5 = 5$
1	$2(1)+5 = 7$
–5	$2(-5)+5 = -5$

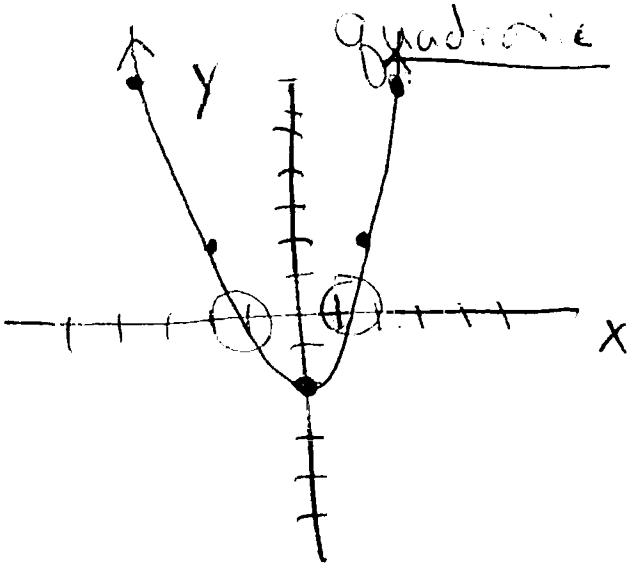
$y = mx + b$



linear

Example: Sketch the graph of $y = x^2 - 2$ by completing a table and then plotting the points.

X	Y
0	$(0)^2-2 = -2$
2	$(2)^2-2 = +2$
-2	$(-2)^2-2 = +2$
3	$(3)^2-2 = 7$
-3	$(-3)^2-2 = 7$



Intercepts of a Graph

x-intercepts- where the graph crosses the x-axis.
Also called **roots** or **zeros**.
To find x-intercepts, let y = 0 and solve for x.

y-intercepts- where the graph crosses the y-axis.
To find the y-intercept, let x = 0 and solve for y.

Example: Find the x- and y- intercepts of $y = x^2 - 4$

X int (y=0)

$y = x^2-4$

$0 = x^2-4$

$+4 \qquad +4$

$x^2 = 4$

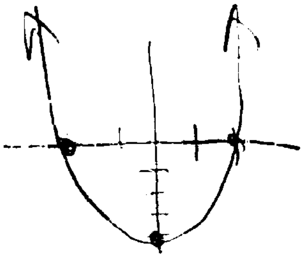
$x = \pm 2$

Y int (x = 0)

$y = x^2-4$

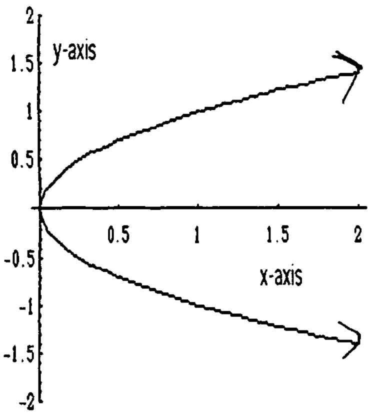
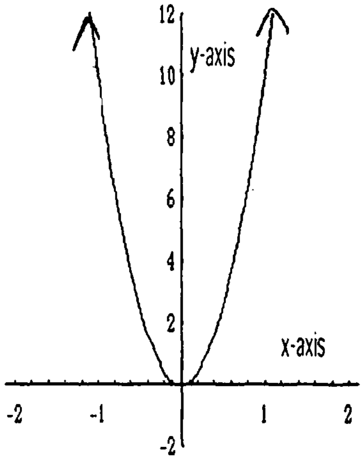
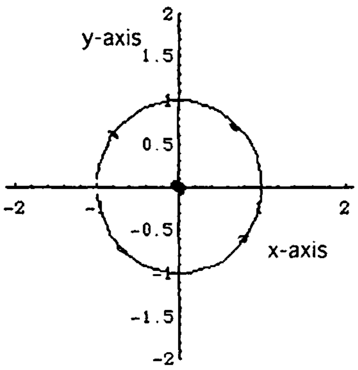
$y = 0^2-4$

$y = -4$



Tests for Symmetry

Symmetric with respect to:

	x – Axis	**y – Axis**	**Origin**
TEST	Replace y with – y (same equation should result)	Replace x with – x (same equation should result)	Replace y with – y **and** replace x with – x (same equation should result)
Example			

Example: Test for symmetry with respect to the x-axis, y-axis, and origin.

$$y = \frac{4x^2}{x^2+1}$$

x-axis (y→-y)

$y = \frac{4x^2}{x^2+1}$ (orig)

$(-y) = \frac{4x^2}{x^2+1}$ (new)

no

y-axis (x→-x)

$y = \frac{4x^2}{x^2+1}$ (orig)

$y = \frac{4(-x)^2}{(-x)^2+1}$ yes

$y = \frac{4x^2+1}{x^2+1}$ (new)

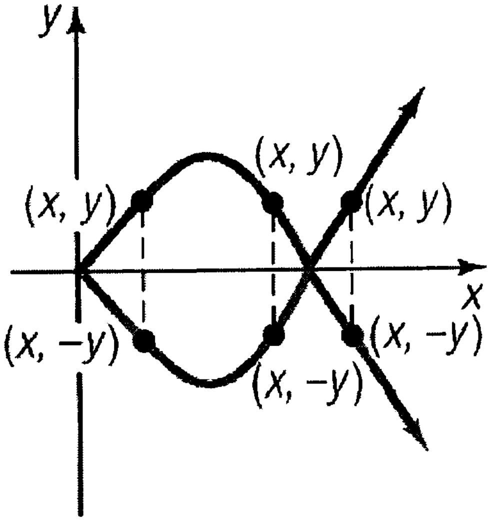
Origin

(x→-x, y→-y) no

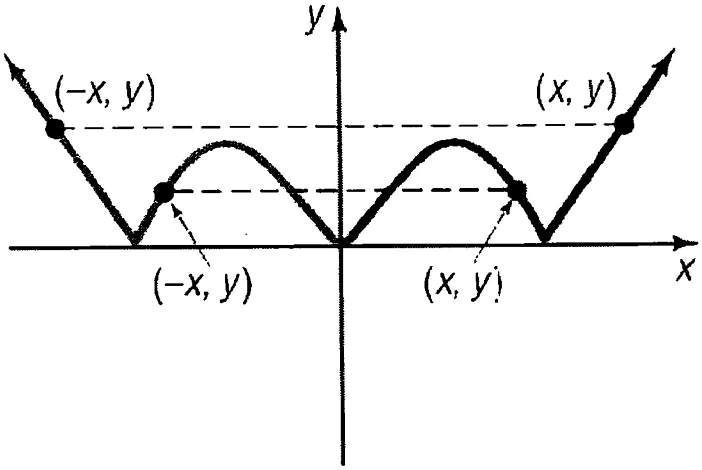
$y = \frac{4x^2}{x^2+1}$

$(-y) = \frac{4(-x)^2}{(-x)^2 [illegible]}$

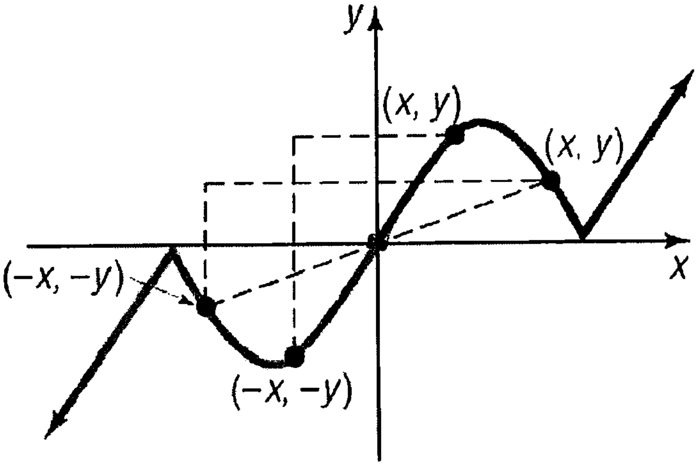
$-y = 4x$ [illegible]



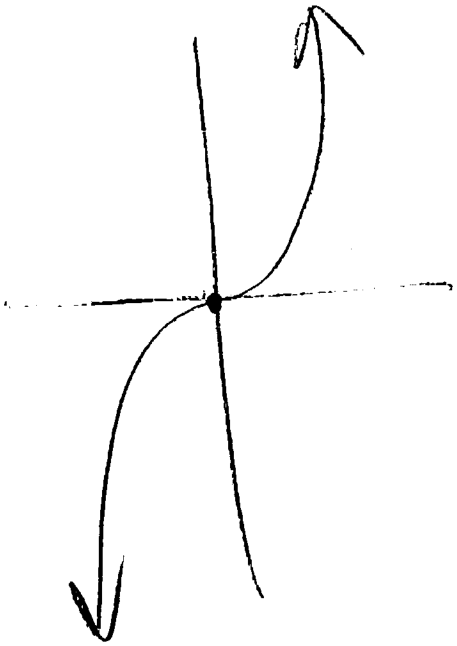
Symmetry with respect
to the *x*-axis



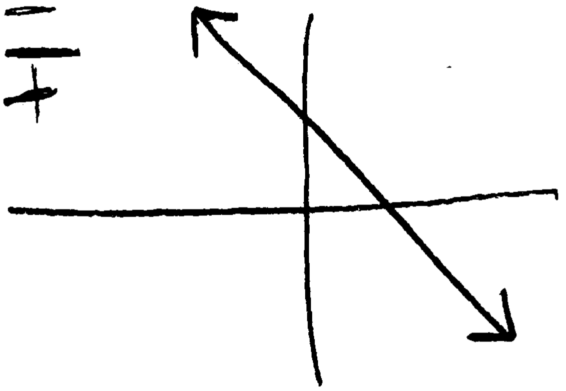
Symmetry with respect
to the *y*-axis



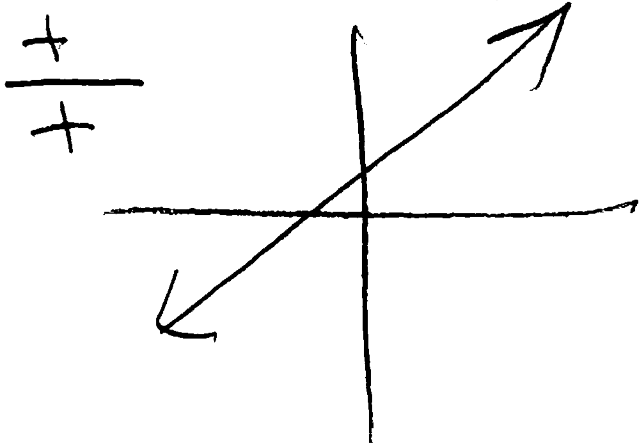
Symmetry with respect
to the origin



neg slope



pos slope



§ 2.3 Lines

Slope of a Line

the **slope** m of the line through the points (x_1, y_1) and (x_2, y_2) is :

$$\boxed{m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}}}$$

The slope of a **horizontal line is 0** and the slope of a **vertical line is undefined**.

Example Find the slope of the line through (1, 2) and (5, –3).

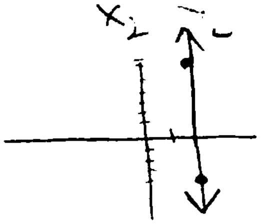
x_1 y_1

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 2}{5 - 1} = \boxed{\frac{-5}{4}}$$

Example Find the slope of the line through (2, 7) and (2, -4).

x_1 y_1

x_2 y_2



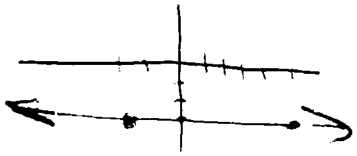
$$m = \frac{-4 - 7}{2 - 2} = \frac{-11}{0} = \boxed{\text{undefined}}$$

Example Find the slope of the line through (5, -3) and (-2, -3).

x_1 y_1

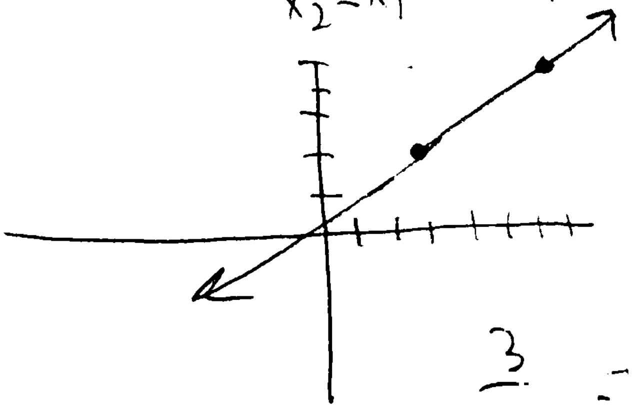
x_2 y_2

$$m = \frac{-3 - -3}{-2 - 5} = \frac{0}{-7} = \boxed{0}$$

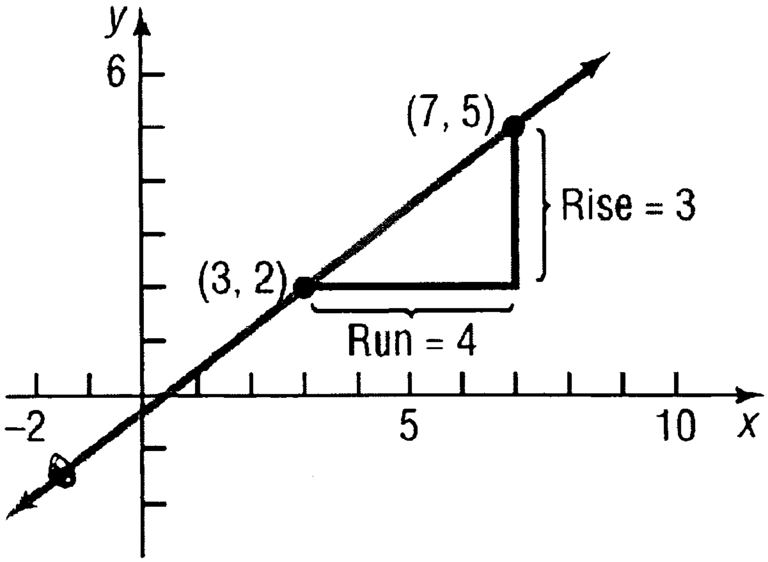


Example Graph the line through (3, 2) having slope $m = \frac{+3}{+4}$.

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}}$



$\frac{3}{4} = \frac{-3}{-4}$



Point-Slope Form of the Equation of a Line

The line with slope m passing through the point (x_1, y_1) has equation :

$$\boxed{y - y_1 = m(x - x_1)}$$

Example Write the equation of the line in standard form.

a.) through $(\overset{x_1}{1}, \overset{y_1}{2})$ and m = 4

$y - y_1 = m(x - x_1)$

$y - 2 = 4(x - 1)$

$y - 2 = 4x - 4$

$+2 \qquad +2$

$\boxed{y = 4x - 2}$

$\boxed{y = mx + b}$

LCC = 3

$\frac{2}{3} + \frac{3}{1} \cdot \frac{3}{3}$

$\frac{2}{3} + \frac{9}{3}$

$\boxed{\frac{-1}{3} \cdot \frac{-2}{1} = +\frac{2}{3}}$

b.) through $(\overset{x_1}{2}, \overset{y_1}{3})$ and $(\overset{x_2}{-4}, \overset{y_2}{5})$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 3}{-4 - 2} = \frac{2}{-6} = \frac{-1}{3}$

$y - y_1 = m(x - x_1)$

$y - 3 = \frac{-1}{3}(x - 2)$

$y - 3 = \frac{-1}{3}x + \frac{2}{3}$

$y = \frac{-1}{3}x + \frac{2}{3} + \frac{3}{1}$

$\boxed{y = \frac{-1}{3}x + \frac{11}{3}}$

Slope-Intercept Form of the Equation of a Line

The line with slope m and y-intercept (0, b) has equation

$$\boxed{y = mx + b}$$

Example Find the slope and y-intercept of 2x + 4y = 8

$-2x \qquad -2x$

$\frac{4y}{4} = \frac{-2x}{4} + \frac{8}{4}$

$\boxed{y = \frac{-1}{2}x + 2}$

$\boxed{m = \frac{-1}{2}}$ y int (0, 2)

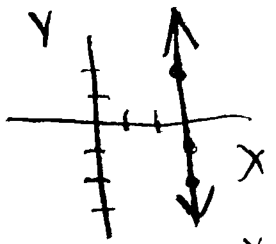
Equation of a vertical line through the point (a, b) is:

$$\boxed{x = a}$$

m = undefined

(3, 7)

$\boxed{x = 3}$



x	y
3	2
3	-1
3	-2

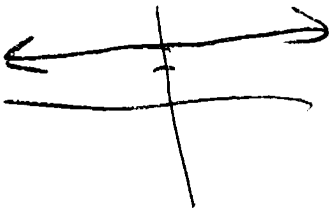
Equation of a horizontal line through the point (a, b) is:

$$\boxed{y = b}$$

m = 0

(5, 2)

$\boxed{y = 2}$



Parallel and Perpendicular Lines

- parallel lines have the same slope. \\
- the slopes of perpendicular lines are negative reciprocals $m_1 = -\dfrac{1}{m_2}$

m_1	m_2
$\frac{2}{3}$	$-\frac{3}{2}$
$-\frac{4}{5}$	$+\frac{5}{4}$

Example Show that two lines are parallel.

L_1: $2x + 3y = 6$ L_2: $4x + 6y = 0$

$$\frac{3y}{3} = \frac{-2x}{3} + \frac{6}{3}$$

$$y = \boxed{\frac{-2}{3}}x + 2$$

$$\frac{6y}{6} = \frac{-4x}{6} + \frac{0}{6}$$

$$y = \boxed{-\frac{2}{3}}x + 0$$

yes

Example Write the equation of the line in standard form.

a) through (2, –3) and parallel to 2x + y = 6

$$y - y_1 = m(x - x_1)$$

$$y + 3 = m(x - 2)$$

$$y + 3 = -2(x - 2)$$

$$y + 3 = -2x + 4$$

$$-3 \qquad -3$$

$$y = -2x + 6$$

$$\underline{m = -2}$$

$$\boxed{y = -2x + 1}$$

b) through (2, –3) and perpendicular to 2x + y = 6

$$y - y_1 = m(x - x_1)$$

$$y + 3 = \frac{1}{2}(x - 2)$$

$$y + 3 = \frac{1}{2}x - 1$$

$$-3 \qquad -3$$

$$\boxed{y = \frac{1}{2}x - 4}$$

$$y = -2x + 6$$

$$m = \frac{-2}{1}$$

$$\boxed{+\frac{1}{2}}$$

$$\frac{1}{2} \cdot \frac{-2}{1} = \frac{-2}{2} = -1$$

§ 3.1 Functions

Relation: a set of ordered pairs. example:

$\{(4,5),(7,2),(8,11)\}$

Domain: - the x-values.

Function: a relation in which each element (number) in the domain corresponds to <u>exactly</u> one element (number) in the range. (Note: The elements in the Domain CANNOT repeat !)

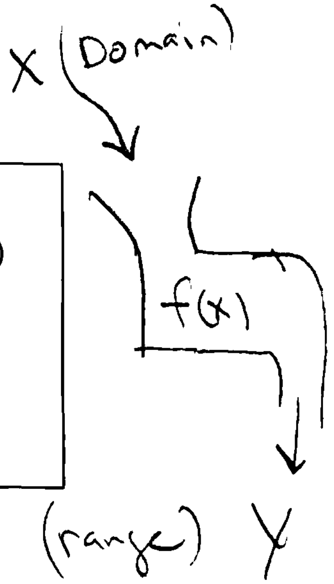
Example: $\{(1,2),(3,4),(5,4)\}$ <u>IS THIS A FUNCTION</u> ?

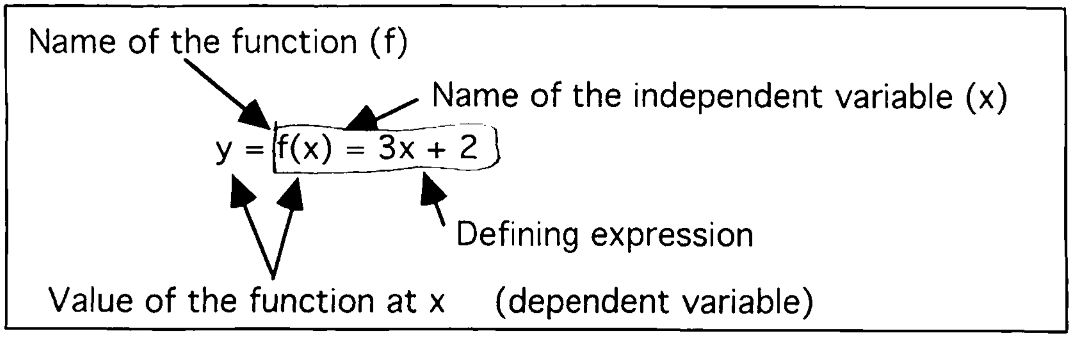
Yes

Example: $\{(1,2),(1,3),(5,4)\}$ <u>IS THIS A FUNCTION</u> ?

No 1's repeat

Function Notation: f(x) is read "f of x " or "the function f evaluated at x".





Independent variable: x is called the independent variable because it determines f(x), which is the y - coordinate.

Dependent variable: y is called the dependent variable because it is determined by x. (it depends on x)

Example:1 Let $g(x)=3\sqrt{x}$, $h(x)=1+4x$, $k(x)=x^2+3$.

PEMDAS

Find a) g(16)

$g(x) = 3\sqrt{x}$

$g(16) = 3\sqrt{16}$

$= 3(4)$

$= \boxed{12}$

$\boxed{(16, 12)}$

b) h(3)

$h(x) = 1+4x$

$h(3) = 1+4(3)$

$= \boxed{13}$

$\boxed{(3, 13)}$

c) k(b)

$k(x) = x^2+3$

$k(b) = (b)^2+3$

$= \boxed{b^2+3}$

$\boxed{(b, b^2+3)}$

Domain & Range: (of a function)

1) If the function is in the form $y=\dfrac{P(x)}{Q(x)}$, solve $Q(x)=0$. (This gives the restrictions on the value(s) of the variable (x)).

2) If the function is in the form $y=\sqrt{P(x)}$, solve the inequality $P(x)\geq 0$.

Example 2: State the Domain for each of the following:

a) $k(x)=\dfrac{3x}{x-5}$

$x-5=0$

$+5 \quad 5$

$D \Rightarrow x \neq 5$ or

$(-\infty,5)\cup(5,\infty)$

b) $h(x)=x^2+5x$

$D=(-\infty,\infty)$

$\mathbb{R}$

c) $g(x)=\sqrt{x-2}$

$x-2\geq 0$

$+2 \quad +2$

$x\geq 2$

$D=[2,\infty)$

Operations on Functions

If f and g are functions, then for all values of x for which both f(x) and g(x) exist,

the SUM of f and g is defined by:
$(f + g)(x) = f(x) + g(x)$

the DIFFERENCE of f and g is defined by:
$(f - g)(x) = f(x) - g(x)$

the PRODUCT of f and g is defined by:
$(f \bullet g)(x) = f(x) \bullet g(x)$

the QUOTIENT of f and g is defined by:
$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, where $g(x) \neq 0$.

Example 1: Let $f(x) = 3x - 5$, and $g(x) = x + 1$
Find:

a) $(f + g)(x)$

$= f(x) + g(x)$

$= (3x-5) + (x+1)$

$= \boxed{4x - 4}$

b) $(f - g)(x)$

$= f(x) - g(x)$

$= (3x-5) - (x+1)$

$= 3x - 5 - x - 1 = \boxed{2x - 6}$

c) $(f \bullet g)(x)$

$= f(x) \cdot g(x)$

$= (3x-5)(x+1)$

$= 3x^2 + 3x - 5x - 5$

$= \boxed{3x^2 - 2x - 5}$

d) $(f/g)(x)$

$= \frac{f(x)}{g(x)} = \boxed{\frac{(3x-5)}{(x+1)}}$

$=$

e) $(f - g)(10)$.

$= 2(10) - 6$

$= \boxed{14}$

$f(x) - g(x) = \boxed{14}$

$f(10) - g(10) = 25 - 11$

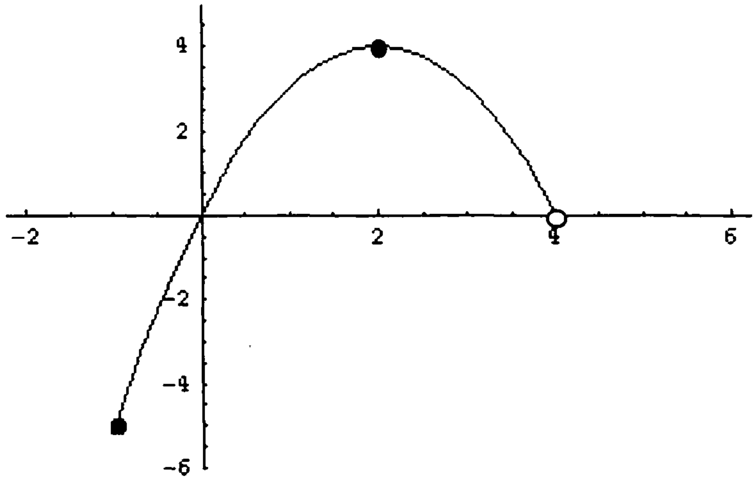
§ 3.2 The Graph of a Function

The **graph of a function f** is the collection of ordered pairs (x, f(x)) such that x is in the domain of f.

To find **domain** from graph look at the x - values (left to right)

To find **range** from graph look at the y - values (up and down)

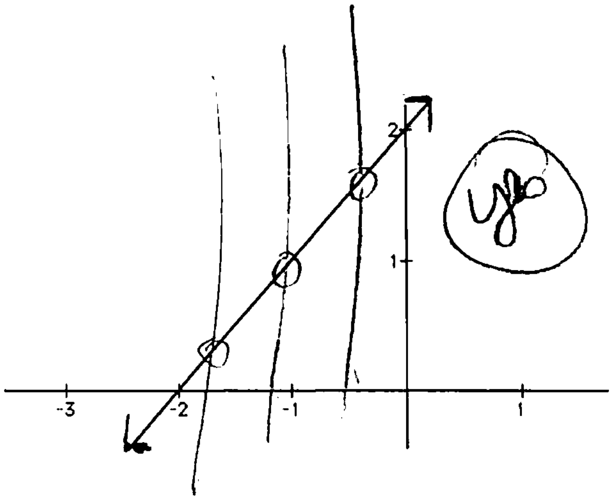
Example:

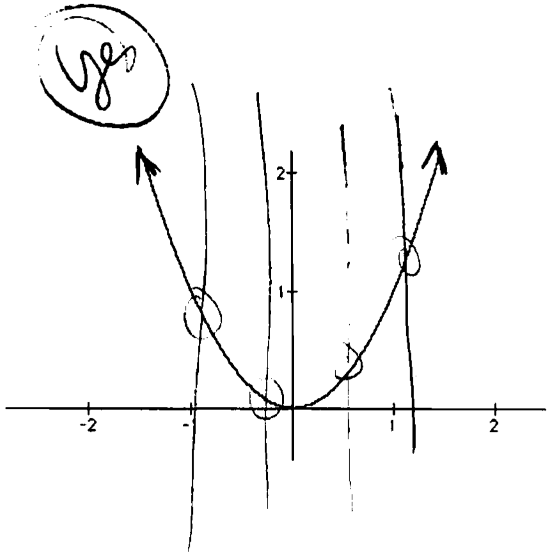
	Domain = Range =

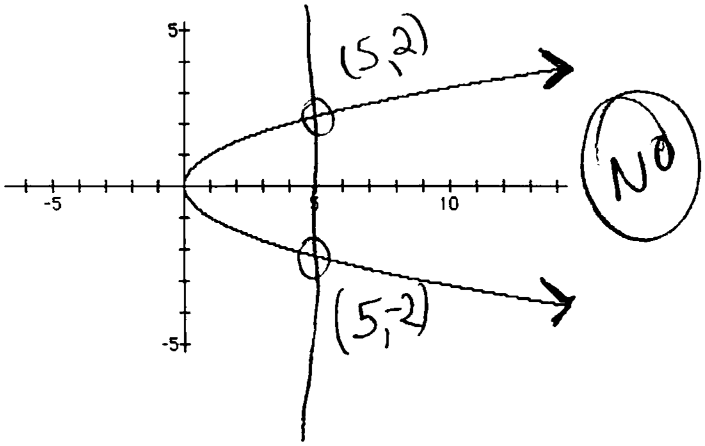
Vertical Line Test:

If every vertical line drawn intersects a graph in no more than one point, the graph is the graph of a function.

Example: Are the following graphs functions ?







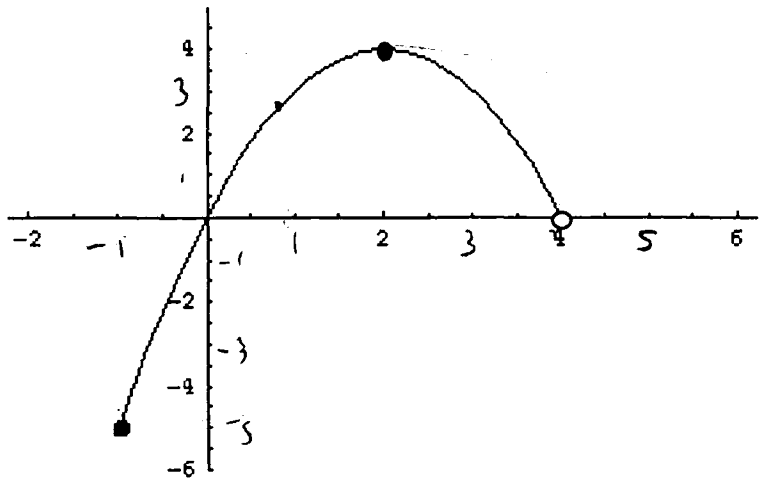
§ 3.2 The Graph of a Function

The **graph of a function f** is the collection of ordered pairs (x, f(x)) such that x is in the domain of f. (x, y)

To find **domain** from graph look at the x - values (left to right)

To find **range** from graph look at the y - values (up and down)

Example:



Domain =

$[-1, 4)$

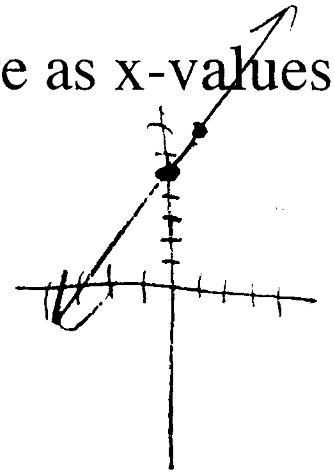
Range =

$[-5, 4]$

Increasing Function: A function where as x-values increase so do the y-values.
(Note: graph will rise up to the right)

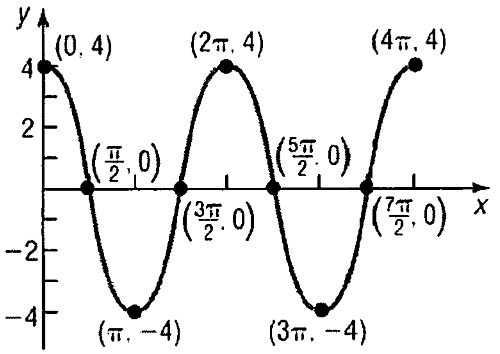
Example: Graph $y = 2x + 5$

x	y
1	7
2	9
3	11



EXAMPLE

Obtaining Information from the Graph of a Function



(a) What are $f(0), f\left(\frac{3\pi}{2}\right)$, and $f(3\pi)$?

(b) What is the domain of f?

(c) What is the range of f?

(d) List the intercepts.

(e) How often does the line $y = 2$ intersect the graph?

(f) For what values of x does $f(x) = -4$?

(g) For what values of x is $f(x) > 0$?

EXAMPLE

Obtaining Information from the Graph of a Function

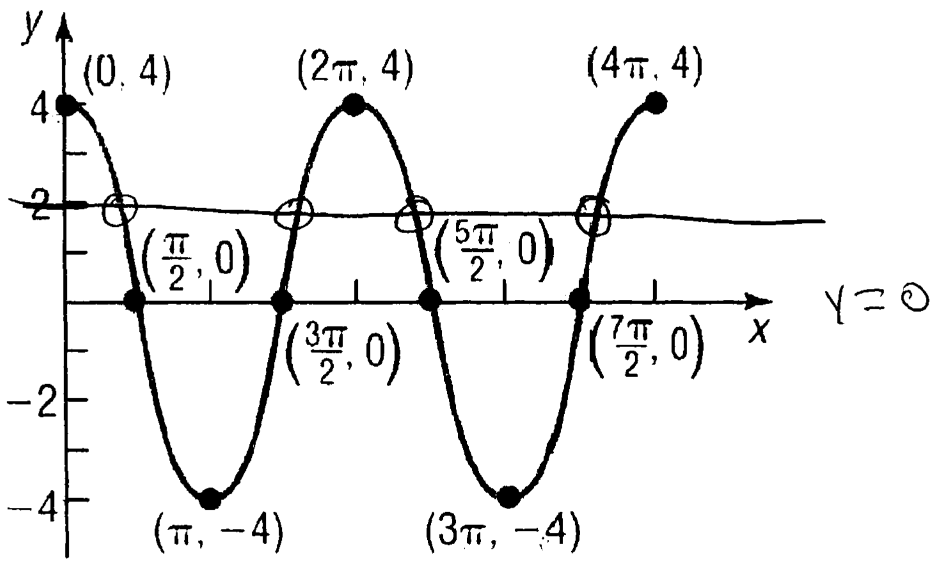
Consider the function $f(x) = \frac{x}{x+1}$

(a) Is the point $\left(1, \frac{1}{2}\right)$ on the graph of f?

(b) If $x = 2$, what is $f(x)$? What point is on the graph of f?

(c) If $f(x) = 2$, what is x? What point is on the graph of f?

Example: Obtaining Information from a graph



X | Y

(a) What are $f(0)$, $f\left(\frac{3\pi}{2}\right)$, and $f(3\pi)$?

4   0   −4

(b) What is the domain of f?

$D = [0, 4\pi]$

(c) What is the range of f?

$R = [-4, +4]$

(d) List the intercepts.

X int $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$ Y int = (0, 4)

(e) How often does the line $y = 2$ intersect the graph?

4 times $y = -4$

(f) For what values of x does $f(x) = -4$?

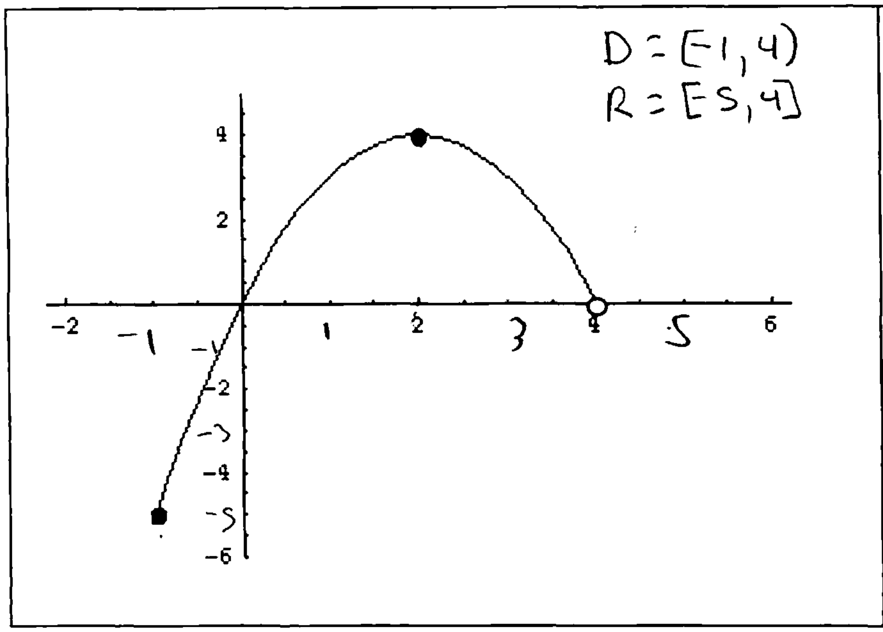
$\pi, 3\pi$

(g) For what values of x is $f(x) > 0$?

$Y > 0$

$[0, \frac{\pi}{2}) \cup (\frac{3\pi}{2}, \frac{5\pi}{2}) \cup (\frac{7\pi}{2}, 4\pi]$

§ 3.3 Properties of Functions



x-values

Increasing =

[-1, 2]

Decreasing =

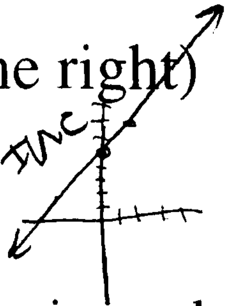
[2, 4)

Constant =

none

Increasing Function: A function where as x-values increase so do the y-values.
(Note: graph will rise up to the right)

Example: Graph $y = \frac{2}{1}x + 5$



x	y
1	7
2	9
3	11

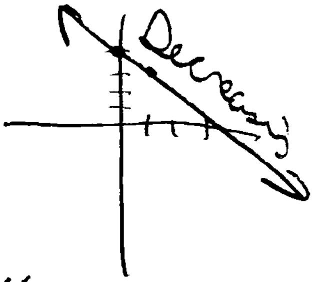
↓ inc ↓ inc

Decreasing Function: A function where as x-values increase y-values decrease.
(Note: graph will fall down to the right)

Example: Graph $y = -\frac{1}{1}x + 4$

x	y
1	3
2	2
3	1

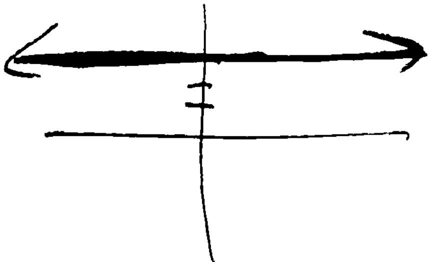
↓ dec

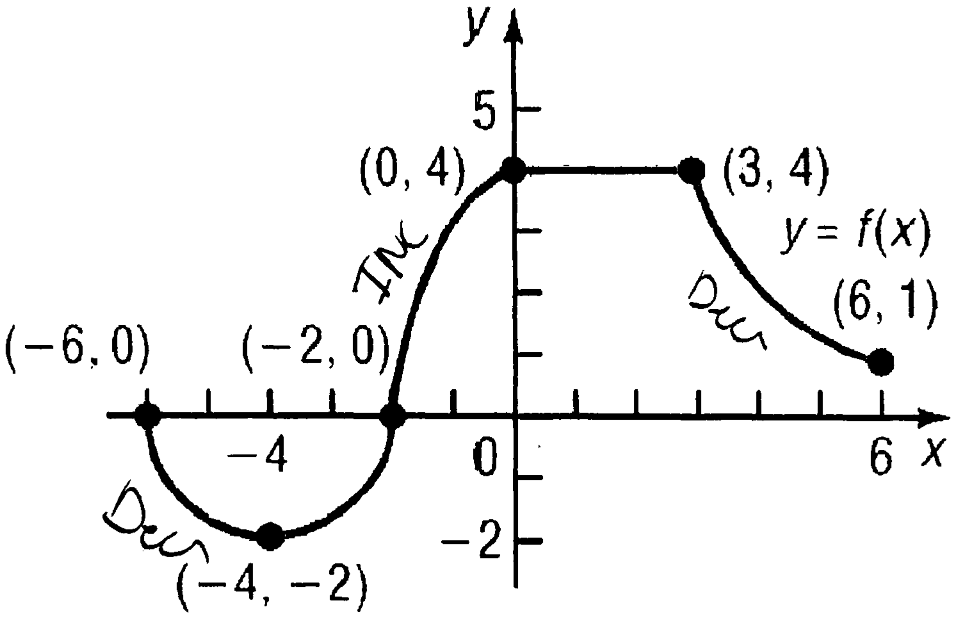


Constant function: The graph is a flat horizontal line.

Example: Graph y = 3

x	y
1	3
2	3
3	3
4	3





X-VALUES

Where is the function increasing?

[-4 0]

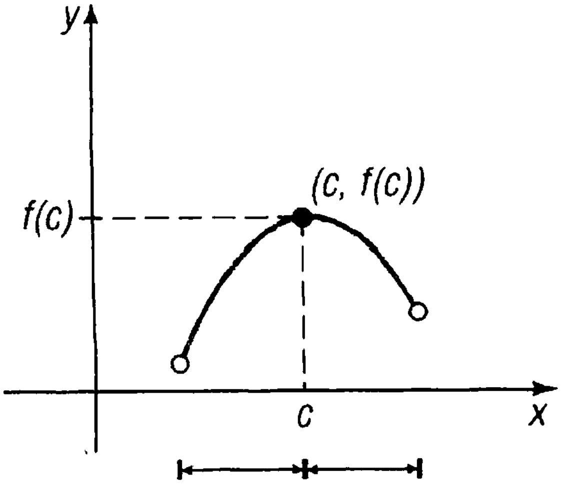
Where is it decreasing?

[-6,-4]U[3,6]

Where is it constant?

[0,3]

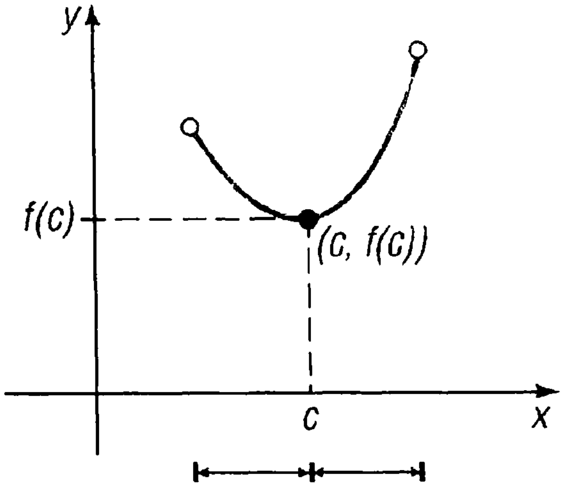
LOCAL MAXIMA



increasing decreasing

The local maximum
is *f(c)* and occurs
at $x = c$.

LOCAL MINIMA

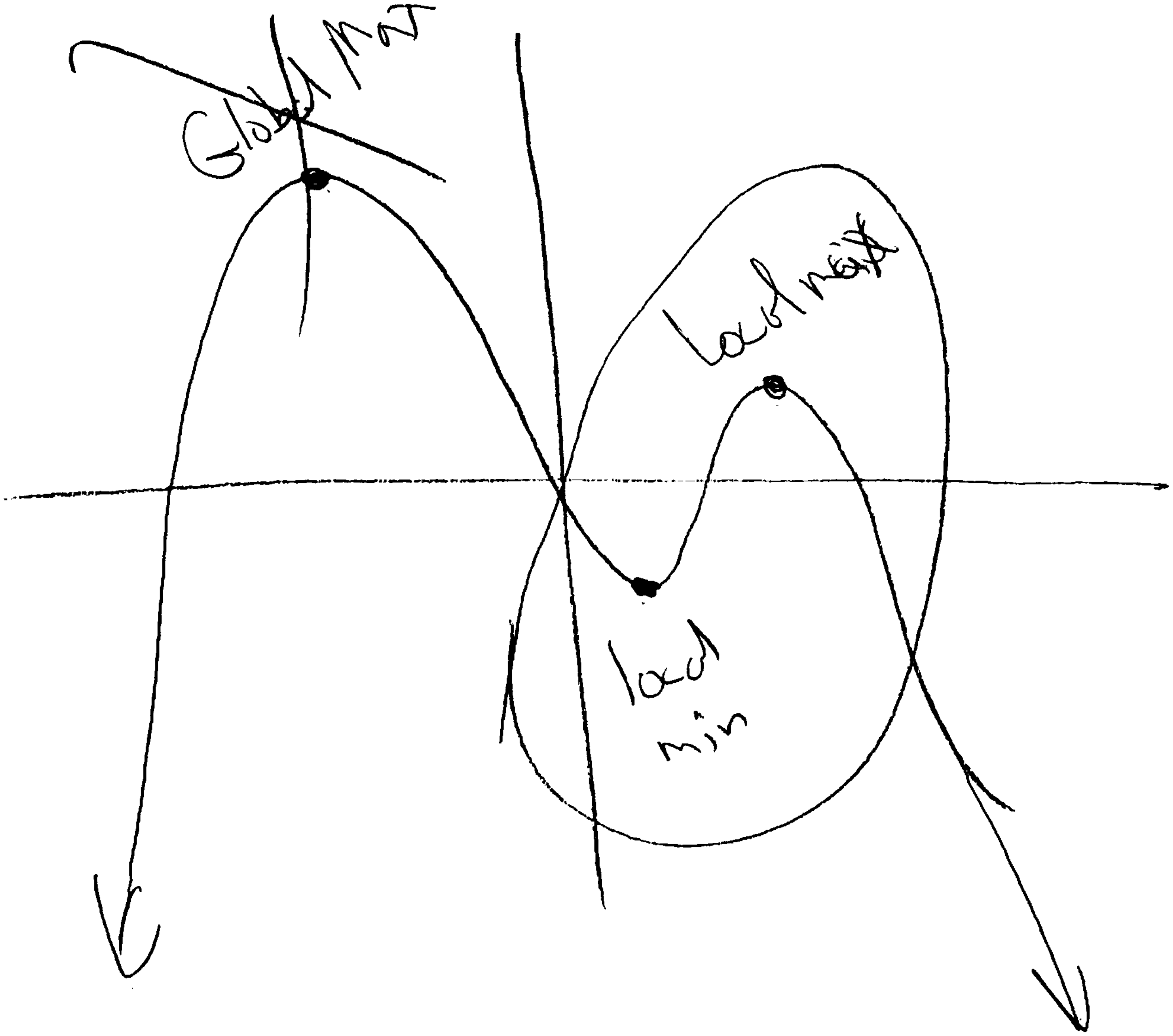


decreasing increasing

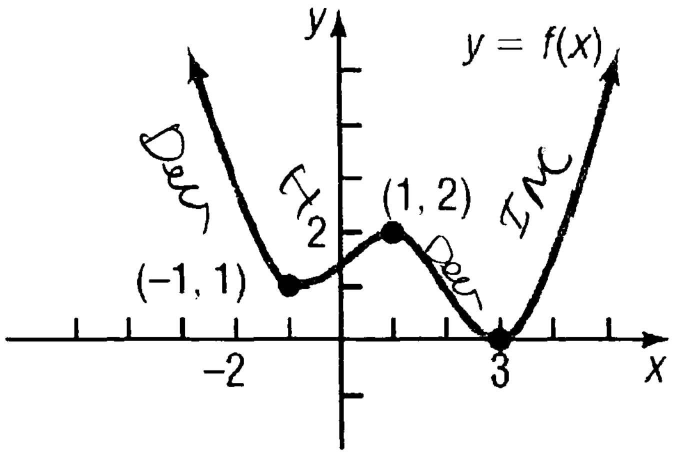
The local minimum
is *f(c)* and occurs
at $x = c$.

A function f has a **local maximum** at c if there is an open interval I containing c so that, for all $x \neq c$ in I, $f(x) \leq f(c)$. We call $f(c)$ a **local maximum of** f.

A function f has a **local minimum** at c if there is an open interval I containing c so that, for all $x \neq c$ in I, $f(x) \geq f(c)$. We call $f(c)$ a **local minimum of** f.


Global max
Local max
local min

-∞ ∈



a) At what number(s), if any, does f have a local maximum?

X=1

b) What are the local maxima?

(1,2)

c) At what number(s), if any, does f have a local minimum?

X=-1 X=3

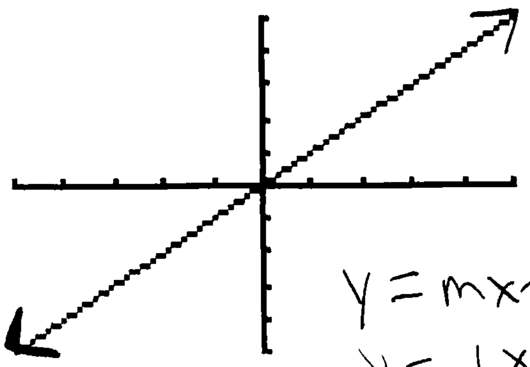
d) What are the local minima?

(-1,1) (3,0)

e) List the intervals on which f is increasing. List the intervals on which f is decreasing.

Inc [-1,1]∪[3,∞) Dec =(-∞,-1]∪(1,3]

3.4 Library of Functions- MEMORIZE THESE !



y = mx+b

y = 1x+0

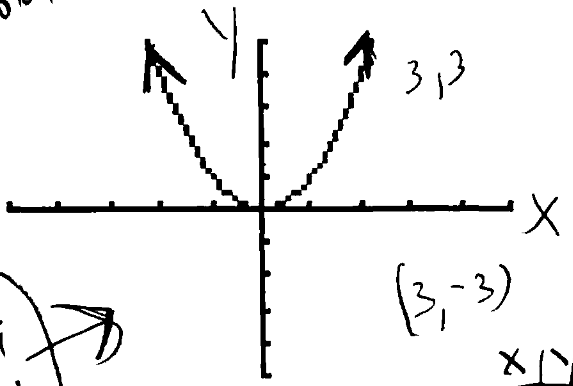
X	Y
3	3
2	2
-1	-1

Identity Function

$\boxed{y = x}$

D = (–∞, ∞) R = (–∞, ∞)

parabola



3,3

(3,-3)

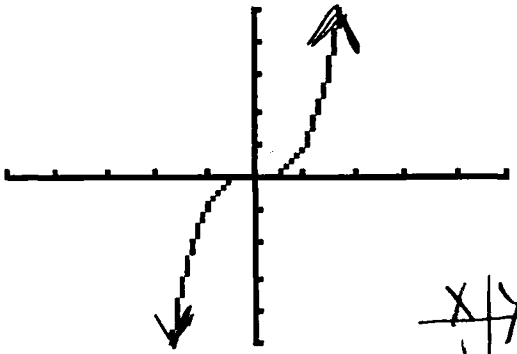
X	Y
-1	+1
1	+1
-2	4
2	4

x	y
1	1
2	4
3	9
5	

Squaring Function

$\boxed{f(x) = x^2}$

D = (–∞, ∞) R = [0,∞)

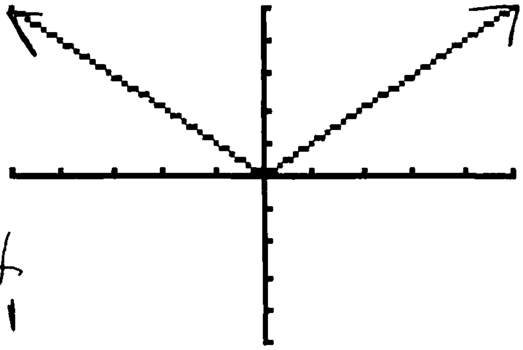


X	Y
1	1
2	8
3	27

Cubing Function

$\boxed{f(x) = x^3}$

D = (–∞, ∞) R = (–∞, ∞)

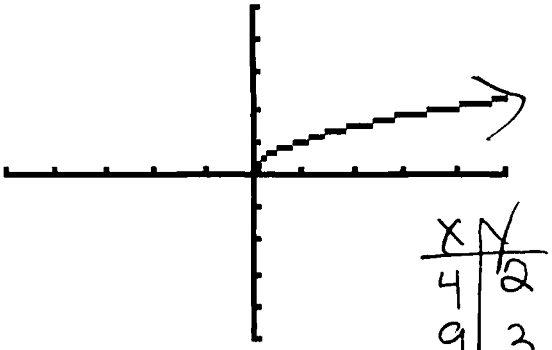


X	Y
-1	1
-2	2
3	3

Absolute Value Function

$\boxed{f(x) = |x|}$

D = (–∞, ∞) R = [0,∞)

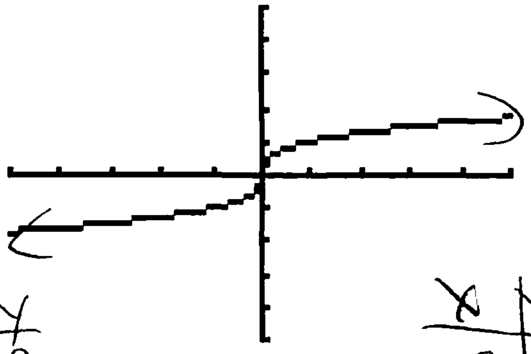


X	Y
4	2
9	3
1	1

Square Root Function

$\boxed{f(x) = \sqrt{x}}$

D = [0, ∞) R = [0,∞)



X	Y
8	2
27	3
0	0
-1	-1

Cube Root Function

$\boxed{f(x) = \sqrt[3]{x}}$

X	Y
-8	-2
-27	-3

D = (–∞, ∞) R = (–∞, ∞)

§ 3.5 Graphing Techniques: Transformations (Graphing Rules)

Shifting Graphs (Rigid Translations) - given a function y = f(x) and c > 0

Basic

(1) the graph of y = f(x) + c is the graph of y = f(x) shifted up c units. ↖ outside

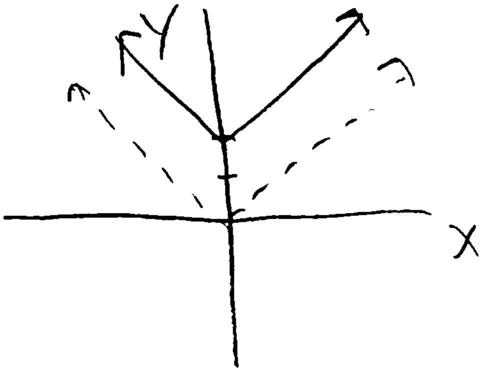
(2) the graph of y = f(x) - c is the graph of y = f(x) shifted down c units. ↖ Outside

(3) the graph of y = f(x + c) is the graph of y = f(x) shifted left c units. ↖ inside

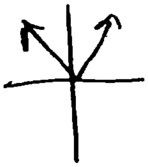
(4) the graph of y = f(x - c) is the graph of y = f(x) shifted right c units. ↖ inside

Example 1: Graph.

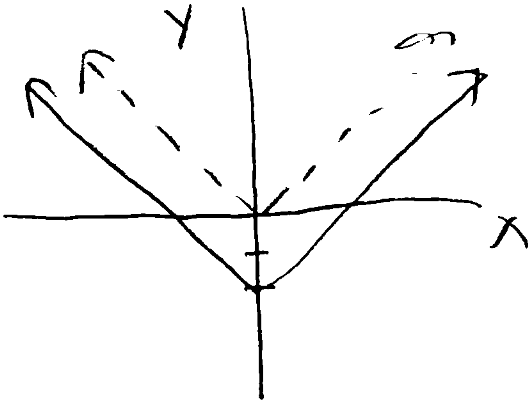
a.) $y = |x| + 2$



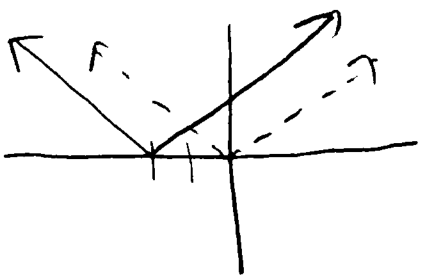
y=|x|



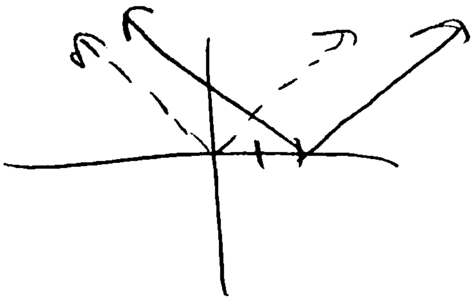
b.) $y = |x| - 2$



c.) $y = |x + 2|$



d.) $y = |x - 2|$



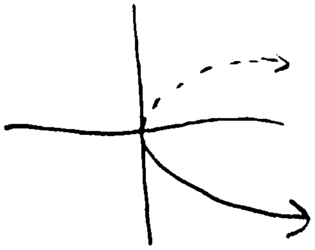
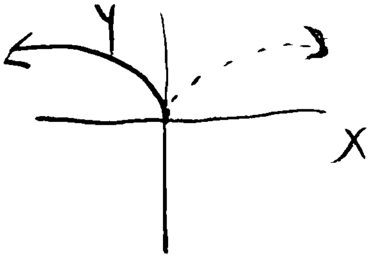
Reflecting Graphs - given a function y = f(x)

X	Y
1	-1
2	-2

(1) the graph of y = −f(x) is the graph of y = f(x) reflected over the x-axis.
inside
(2) the graph of y = f(−x) is the graph of y = f(x) reflected over the y-axis.

Example 2: Graph. a.) $y = -\sqrt{x}$ b.) $y = \sqrt{-x}$

y=√x

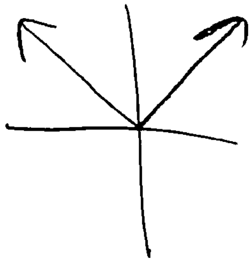
Narrowing and Broading (Non-Rigid Translations):

widening

The graph of g(x) = c * f(x) has the same general shape as the graph of f(x).

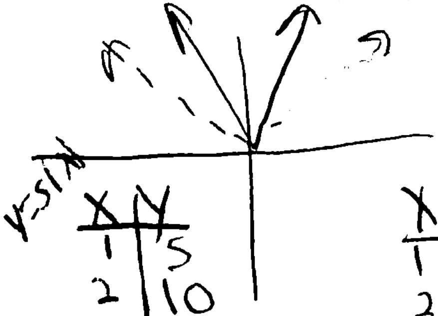
1) It is narrowed vertically compared to the graph of f(x) if $c \overset{+}{>} 1$.
2) It is broadened vertically compared to the graph of f(x) if 0 < c < 1.

Example 3: Graph. a.) $y = 5|x|$ b.) $y = \frac{1}{2}|x|$



y=|x|

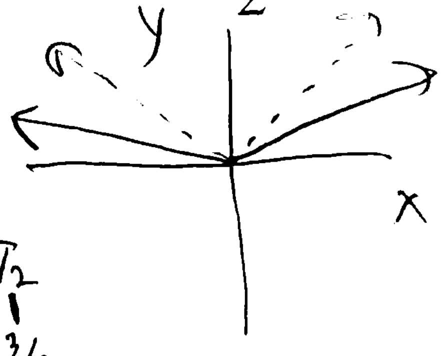
X	Y
1	1
2	2
-3	3



y=5|x|

X	Y
1	5
2	10
-3	15

X	Y
1	1/2
2	1
-3	3/2

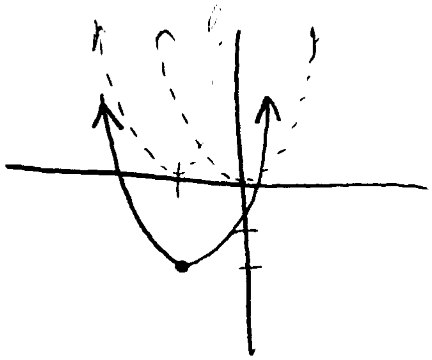


(Note: when an equation contains more than one shifting or reflecting rule, use steps (one rule at a time) and work from the inside of the function to the outside.)

$y = x^2$

basic graph?

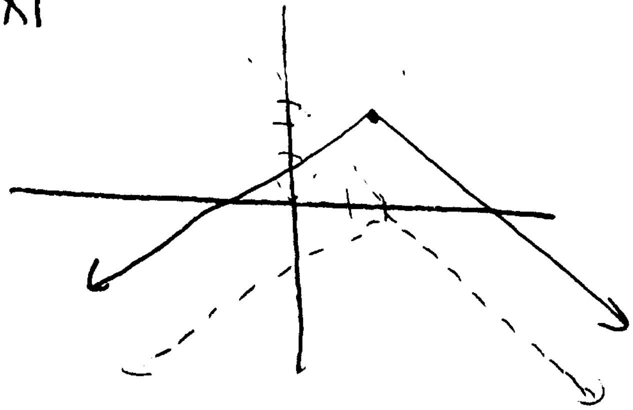
1) $y = (x+1)^2 - 2$



inside

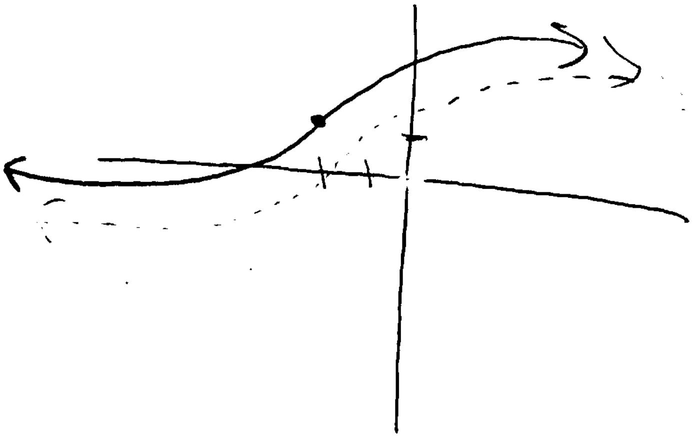
2) $y = -\frac{1}{3}|x-2| + 3$

$y = |x|$



$y = \sqrt[3]{x}$

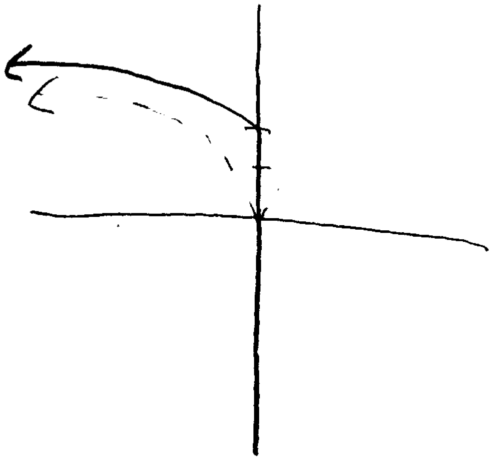
3) $y = \sqrt[3]{x+2} + 1$



4) $y = 3\sqrt{-x} + 2$

basic

$y = \sqrt{x}$



§5.1 Polynomial Functions and Models

A polynomial function is a function of the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0$$

where $a_n, a_{n-1}, \ldots, a_1, a_0$ are real numbers and n is a nonnegative integer.

Determine which of the following are polynomial functions. For those that are, state the degree; for those that are not, tell why not.

(a) $f(x) = 2 - 3x^4$

yes $-3x^4 + 2$

(b) $g(x) = \sqrt{x} = x^{\frac{1}{2}}$

no no radicals with variable

(c) $h(x) = \dfrac{x^2 - 2}{x^3 - 1}$

no no division of variables

(d) $F(x) = 0x^0$

yes $y = 0$

(e) $G(x) = 8$

yes $8x^0$

(f) $H(x) = -2x^3(x - 1)^2$

$= -2x^3(x-1)(x-1)$

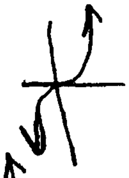
yes

$= -2x^3(x^2 - 2x + 1)$

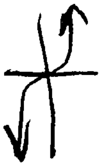
$= \boxed{-2x^5 + 4x^4 - 2x^3}$

$f(x) = ax^{\textcircled{1}} + b$ <u>line</u> 

$f(x) = ax^{\textcircled{2}} + bx^{\textcircled{1}} + c$ <u>parabola</u> 

$f(x) = ax^{3} + bx^{2} + cx^{1} + d$ <u>cubic</u> 

$f(x) = ax^{4} + bx^{3} + cx^{2} + dx + e$ 

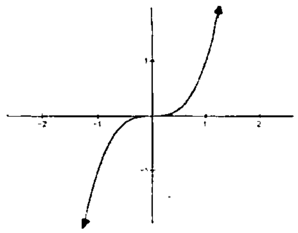
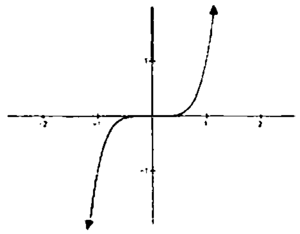
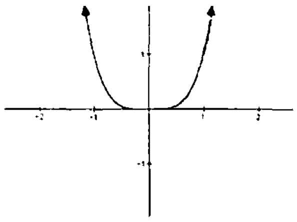
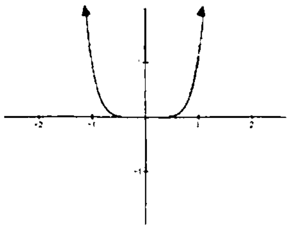
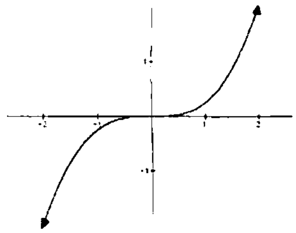
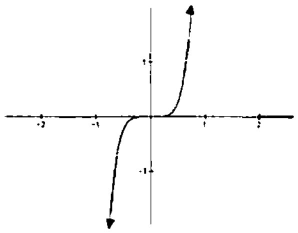
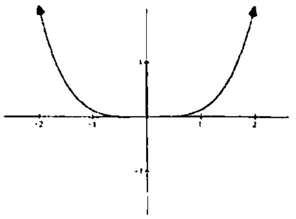
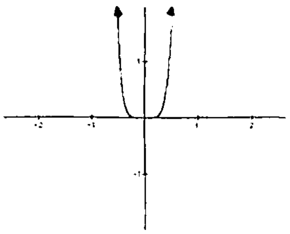
$f(x) = ax^{5} + bx^{4} + cx^{3} + dx^{2} + ex + f$ 

⋮

$f(x) = 4x^{1} + 6$

Polynomials are **<u>continuous</u>** (no breaks in the graph) and **<u>smooth</u>** (no sharp angles, only rounded curves)

Graphing Functions of the Form: $P(x) = ax^n$

$P(x) = x^3$ 	$P(x) = x^5$ 	$P(x) = x^4$ 	$P(x) = x^6$ 
 $P(x) = \frac{1}{3}x^3$	 $P(x) = 8x^5$	 $P(x) = \frac{1}{8}x^4$	 $P(x) = 9x^6$

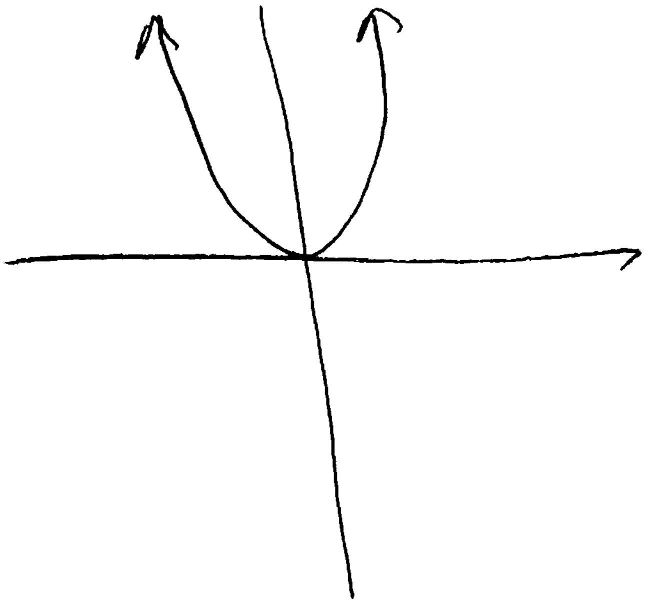
<u>Note</u>: The graph of $y = x^n$ is similar to the graph of

$\begin{cases} y = x^2 \text{ if n is even} \\ y = x^3 \text{ if n is odd} \end{cases}$, except that the greater n is, the flatter

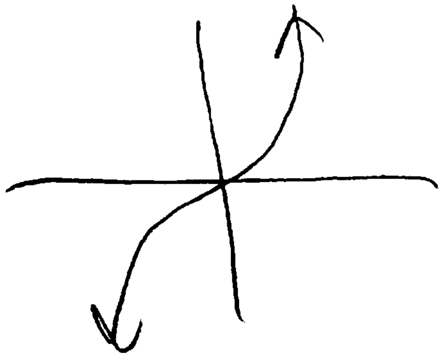
the graph is on [-1, 1] and the steeper it is on

$(-\infty, -1) \cup (1, \infty)$.

$f(x) = x^{100}$

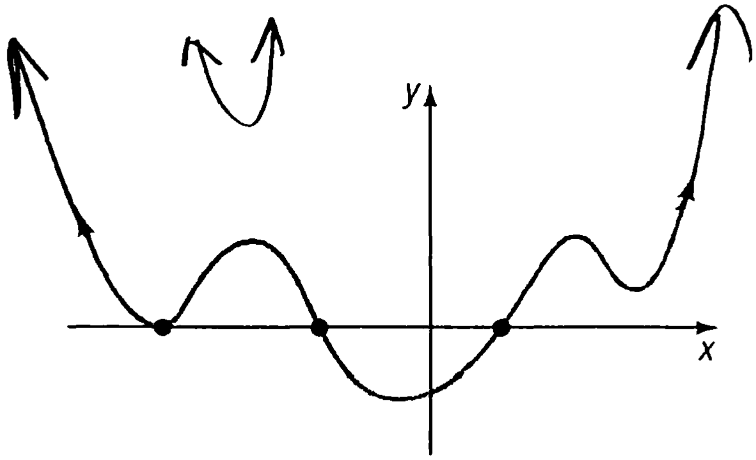


$f(x) = x^{\textcircled{59}}$

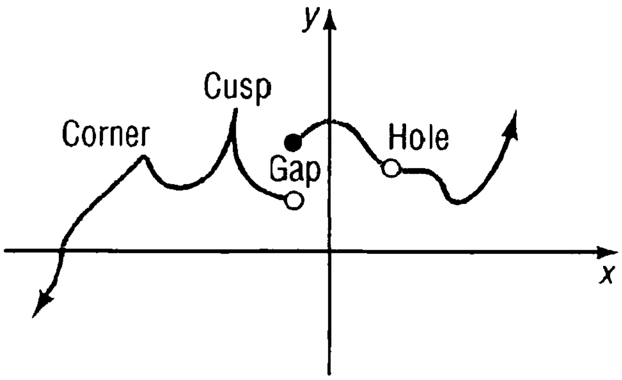


Summary of the Properties of the Graphs of Polynomial Functions

Degree	Form	Name	Graph
No degree	$f(x) = 0$	Zero function	The x-axis
0	$f(x) = a_0, \quad a_0 \neq 0$	Constant function	Horizontal line with y-intercept a_0
1	$f(x) = a_1x + a_0, \quad a_1 \neq 0$	Linear function	Nonvertical, nonhorizontal line with slope a_1 and y-intercept a_0
2	$f(x) = a_2x^2 + a_1x + a_0, \quad a_2 \neq 0$	Quadratic function	Parabola: Graph opens up if $a_2 > 0$; graph opens down if $a_2 < 0$



(a) Graph of a polynomial function: smooth, continuous



(b) Cannot be the graph of a polynomial function

If f is a polynomial function and r is a real number for which $f(r) = 0$, then r is called a (real) **zero of *f***, or **root of *f***. If r is a (real) zero of f, then

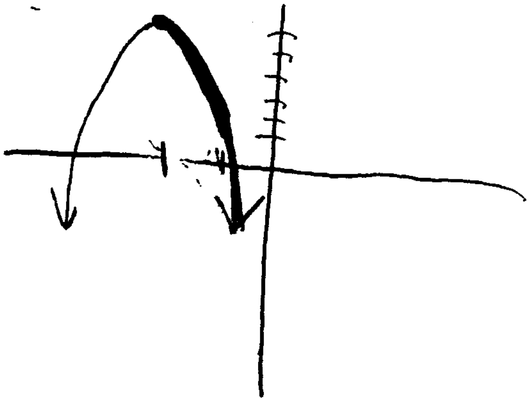
(a) r is an x-intercept of the graph of f.

(b) $(x - r)$ is a factor of f.

Examining Vertical and Horizontal Translations (Shifts):

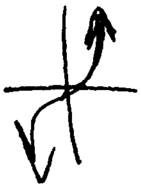
Example 1: Graph basic $y = x^4$

a.) $y = -(x + 2)^4 + 6$

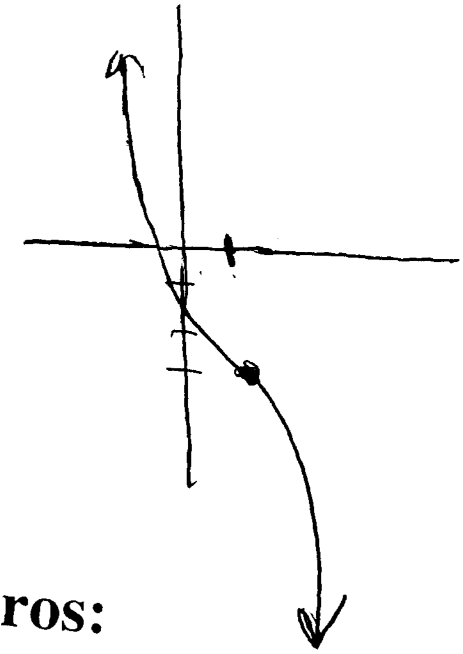


b.) $y = -3 - (x - 1)^3$

basic



$Y = -(x-1)^3 - 3$

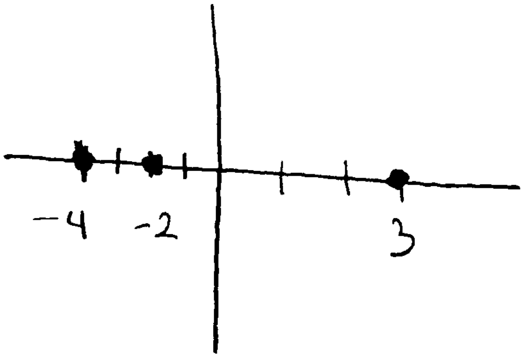


Finding a polynomial from its Zeros:

Example Find a polynomial of degree 3 whose zeros are -4, -2, and 3.

$\boxed{\text{x-int} = \text{zero of the polynomial}}$

basic



$X = -4 \qquad X = -2 \qquad X = 3$

$\boxed{(X+4) \quad (X+2) \quad (X-3)}$

$(X^2+6x+8)(X-3)$

$$\begin{array}{r} x^2+6x+8 \\ x-3 \\ \hline x^3+6x^2+8x \\ -3x^2-18x-24 \\ \hline x^3+3x^2-10x-24 \end{array} = \boxed{(x+4)(x+2)(x-3)}$$

General Form

Factored Form

Identifying Zeros and Their Multiplicities

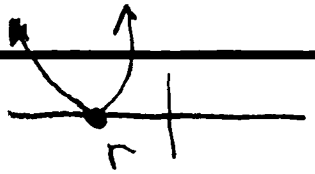
For the polynomial, list all zeros and their multiplicities.

$$f(x) = -2(x-2)^1(x+1)^3(x-3)^4$$

$x - 2 = 0$
$+2 \quad +2$
$x = 2$

$\boxed{x = 2}$ m of 1 | $\boxed{x = -1}$ m of 3 | $\boxed{x = 3}$ m of 4

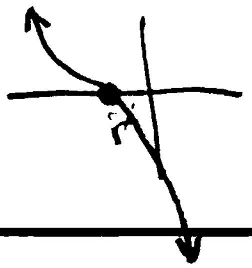
If r Is a Zero of Even Multiplicity



Sign of $f(x)$ does not change from one side of r to the other side of r.

Graph **touches** x-axis at r.

If r Is a Zero of Odd Multiplicity



Sign of $f(x)$ changes from one side of r to the other side of r.

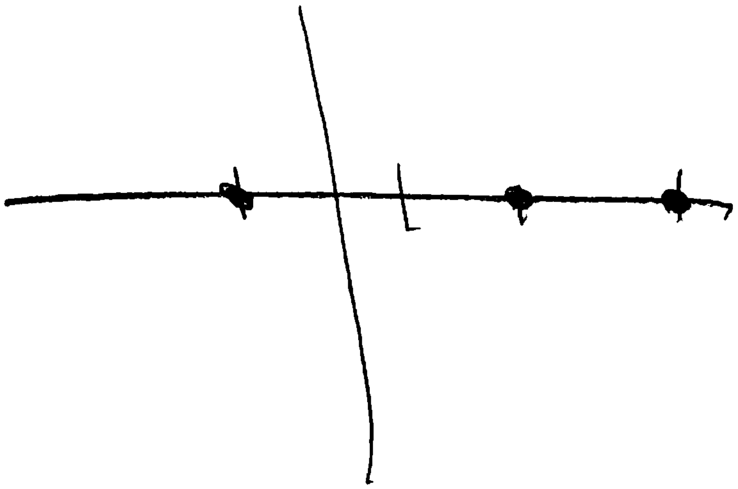
Graph **crosses** x-axis at r.

Theorem

Turning Points

If f is a polynomial function of degree n, then f has at most $n - 1$ turning points.

If the graph of a polynomial function f has $n - 1$ turning points, the degree of f is at least n.



Example Graphing a Polynomial using x-intercepts

zeros? $\boxed{x=2}$ $\boxed{x=0}$

For the polynomial: $f(x) = x^2(x-2)^1$ $= \boxed{x^3-2x^2}$

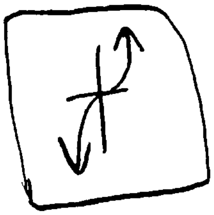
$n=3$

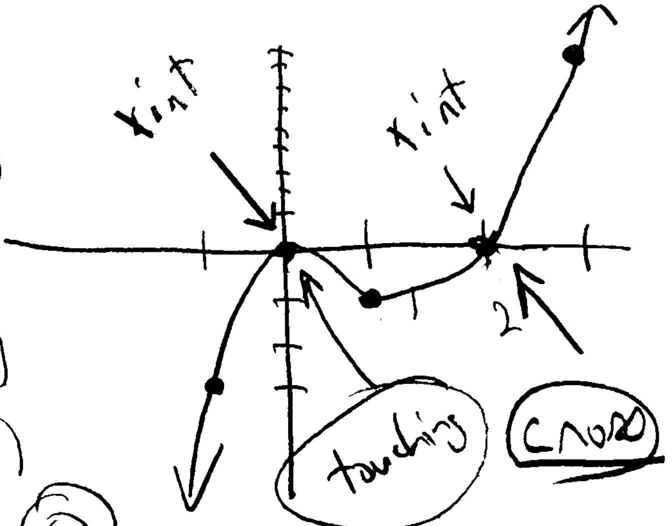
$n-1=\textcircled{2}$

(a) Find the x- and y-intercepts of the graph of f.

(b) Use the $\boxed{x\text{-intercepts}}$ to find the intervals on which the graph of f is above the x-axis and the intervals on which the graph of f is below the x-axis.

(c) Locate other points on the graph and connect all the points plotted with a smooth, continuous curve.





a) $\underline{x \text{ int}}$

$y=0$

$x^2(x-2)=0$

$x^2=0$ | $x-2=0$

$x=0$ | $x=2$

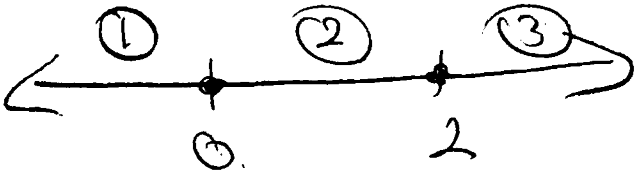
$\underline{y \text{ int}}$

$x=0$

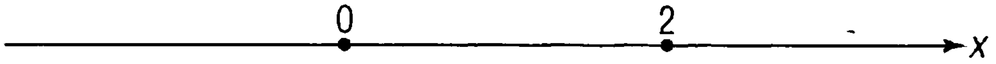
$x^2(x-2)$

$0^2(0-2)$

$0(-2)=\textcircled{0}$



$x=-1$	$x=1$	$x=3$
$y=-3$	$y=-1$	$y=9$



Interval	$(-\infty, 0)$	$(0, 2)$	$(2, \infty)$
Number Chosen	-1	1	3
Value of *f*	$f(-1) = -3$	$f(1) = -1$	$f(3) \overset{+}{=} 9$
Location of Graph	Below x-axis	Below x-axis	Above x-axis
Point on Graph	$\boxed{(-1, -3)}$	$\boxed{(1, -1)}$	$\boxed{(3, 9)}$

$x + 5 = 0$?

$\quad -5 \quad -5$

$\boxed{x \neq -5}$

$x^2 - 4 = 0$

$\sqrt{x^2} = \sqrt{4}$

$\boxed{x \neq \pm 2}$

$x^2 + 1 = 0$?

$\sqrt{x^2} = \sqrt{-1}$

$\cancel{\boxed{x = \pm i}}$

$\boxed{1 \neq X}$

§5.2 Properties of Rational Functions

Rational Function - a function of the form $f(x)=\dfrac{p(x)}{q(x)}$ where p(x) and q(x) are polynomials with $q(x)\neq 0$.

Find the Domain of a Rational Function

The domain is the set of all real numbers where the denominator $\neq 0$. Rule Set Denom $=0$

(a) The domain of $R(x)=\dfrac{2x^2-4}{x+5}$ is the set of all real numbers x except -5; that is, $\{x \mid x\neq -5\}$.

(b) The domain of $R(x)=\dfrac{1}{x^2-4}$ is the set of all real numbers x except -2 and 2, that is, $\{x \mid x\neq -2, x\neq 2\}$.

(c) The domain of $R(x)=\dfrac{x^3}{x^2+1}$ is the set of all real numbers.

(d) The domain of $R(x)=\dfrac{-x^2+2}{3}$ is the set of all real numbers.

(e) The domain of $R(x)=\dfrac{x^2-1}{x-1}$ is the set of all real numbers x except 1, that is, $\{x \mid x\neq 1\}$. ◀

It is important to observe that the functions

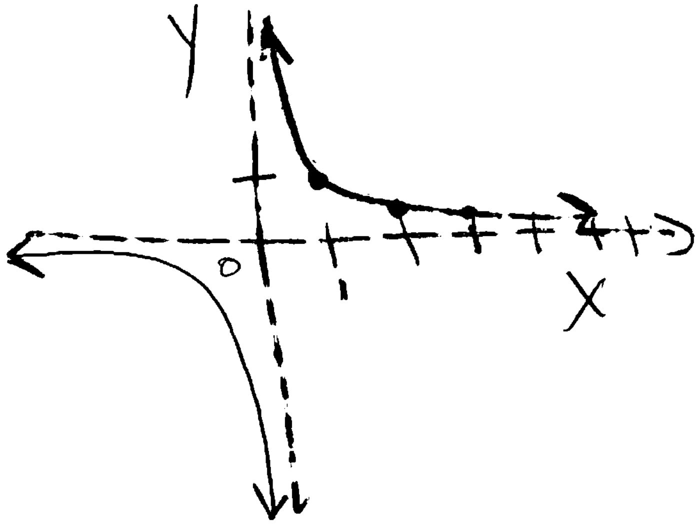
$$R(x)=\frac{x^2-1}{x-1} \quad \text{and} \quad f(x)=x+1$$

are not equal, since the domain of R is $\{x \mid x\neq 1\}$ and the domain of f is the set of all real numbers.

$f(x) = \frac{1}{x}$

Domain?

$x \neq 0$



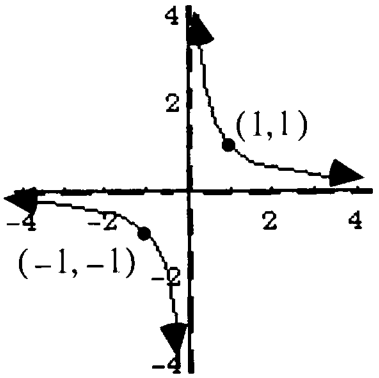
x	y
0	undefined
1	$1/1 = 1$
2	$1/2 = .5$
3	$1/3 = .\bar{3}$
5	$1/5 = .2$
10	$1/10 = .1$
?	0
$1/2$	$\frac{1}{1/2} = 2$
$1/5$	$\frac{1}{1/5} = 5$
$1/10$	$\frac{1}{1/10} = 10$

$\frac{1}{x} = 0$?

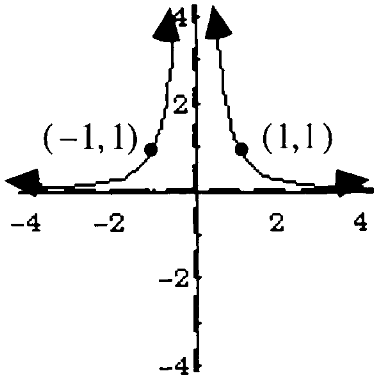
$\frac{1}{1\,000\,000} = .000001$

The graphs of rational functions approach (get closer and closer to) lines called **asymptotes**.

Basic Graphs (memorize these)



$y = \frac{1}{x}$

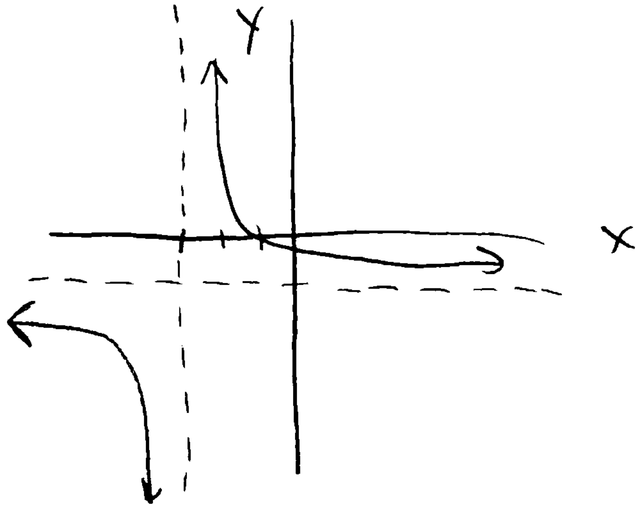


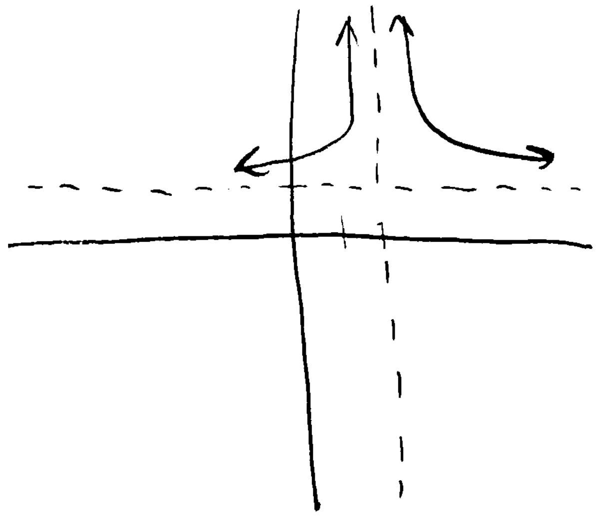
$y = \frac{1}{x^2}$

Example 1 Use stretching/shrinking, reflecting and shifting rules to graph the following.

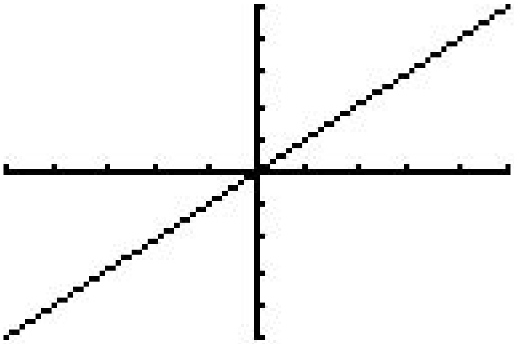
a.) $f(x) = \frac{1}{(x+3)} - 1$

b.) $f(x) = \frac{1}{(x-2)^2} + 1$





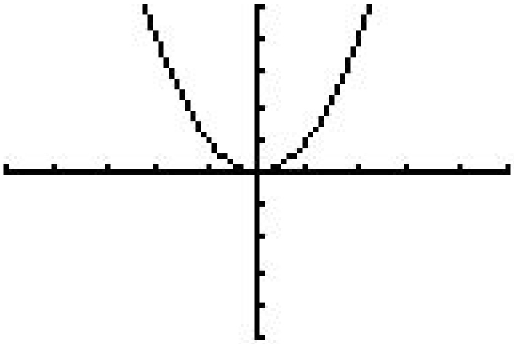
3.4 Library of Functions- MEMORIZE THESE !



Identity Function

$\boxed{y = x}$

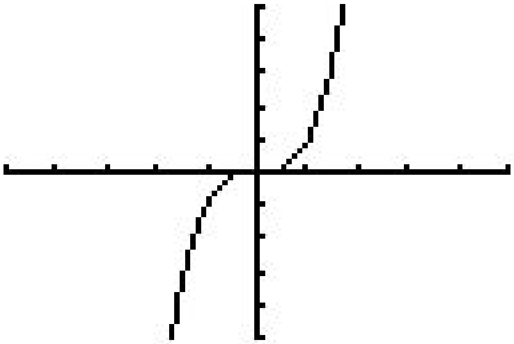
D = (–∞, ∞) R = (–∞, ∞)



Squaring Function

$\boxed{f(x) = x^2}$

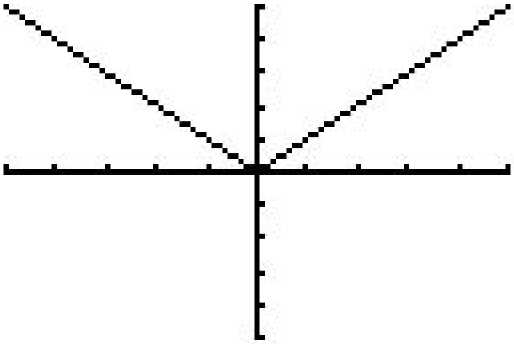
D = (–∞, ∞) R = [0,∞)



Cubing Function

$\boxed{f(x) = x^3}$

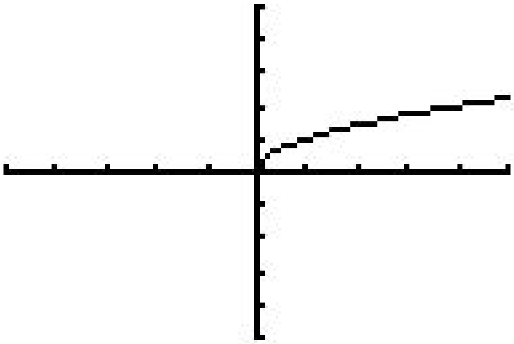
D = (–∞, ∞) R = (–∞, ∞)



Absolute Value Function

$\boxed{f(x) = |x|}$

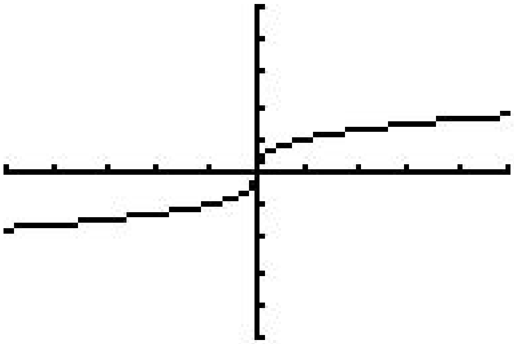
D = (–∞, ∞) R = [0,∞)



Square Root Function

$\boxed{f(x) = \sqrt{x}}$

D = [0, ∞) R = [0,∞)



Cube Root Function

$\boxed{f(x) = \sqrt[3]{x}}$

D = (–∞, ∞) R = (–∞, ∞)

$$f(x) = \frac{x^{\square}}{x^{\square}}$$

$$= \frac{x^2}{x^5}$$

$$= \boxed{\frac{4x^{\textcircled{3}}}{7x^{\textcircled{3}}}} \quad f(x) = \frac{4}{7} \qquad \enclose{circle}{y = \frac{4}{7}}$$

$$= \frac{x^4}{x^2}$$

$$y = \frac{x^5}{x^3}$$ no SA

VA -When in lowest terms !

To Find the Asymptotes of a Rational Function:

(1) Vertical Asymptotes - Find any vertical asymptotes by setting the denominator equal to 0 and solving for x to get the equation $\boxed{\mathbf{x = a}}$. ☐

(2) Horizontal Asymptotes

bottom heavy

Rule 1: If the numerator has lower degree than the denominator, the horizontal asymptote is $\boxed{\mathbf{y = 0}}$.

Rule 2: If the numerator and denominator have the same degree and a_n is the leading coefficient of the numerator and b_n is the leading coefficient of the denominator, the horizontal asymptote is $\boxed{y = \frac{a_n}{b_n}}$.

Rule 3: If the numerator has higher degree than the denominator, there is **no horizontal asymptote**.

Top heavy

(3) Slant Asymptotes - If the numerator is of degree exactly one more than the denominator, there is an slant asymptote. To find it, divide the numerator by the denominator and disregard any remainder. The equation of the slant asymptote is the result of setting y = to the quotient. RARE $y = mx + b$

Example 2 Give the equations of the vertical, horizontal and/or slant asymptotes of the rational function.

a.) $f(x) = \dfrac{3x}{(x+1)(x-2)}$

V.A. set D = 0 $(x+1)(x-2) = 0$

$x+1=0$ | $x-2=0$

$\boxed{x=-1}$ | $\boxed{x=2}$

H.A.

N → 1 bottom

D → 2 heavy

$\boxed{y=0}$

S.A. none

b.) $f(x) = \dfrac{(x-5)(x-2)}{x^2+9}$

V.A.

Set D = 0

$x^2+9=0$

none

$\sqrt{x^2} = \sqrt{-9}$

~~$x = 3i$~~

H.A.

N = ② same $y = \frac{a_n}{b_n} = \frac{1}{1}$

D = ②

$\boxed{y=1}$

S.A. none

c.) $f(x) = \dfrac{2x^2+3}{x-4}$

V.A.

$x - 4 = 0$

$x = 4$

H.A. top heavy

N = ②

D = ①

none

S.A.

$$\begin{array}{r} 2x+8 \\ x-4\,\overline{)\,2x^2+0x+3} \\ \ominus\, 2x^2 \overset{\oplus}{-} 8x \\ \hline 8x+3 \\ \ominus\, 8x \overset{\oplus}{-} 32 \\ \hline \cancel{35} \end{array}$$

$\boxed{y=2x+8}$

d.) $f(x) = \dfrac{2(3x-1)(x+4)}{(x+2)(5x-3)}$

V.A.

$(x+2)(5x-3) = 0$

$x+2=0$ | $5x-3=0$

$x=-2$ | $x=\frac{3}{5}$

H.A.

N → ② $y = \frac{a_n}{b_n} = \frac{6}{5}$

D → ②

$\boxed{y=\frac{6}{5}}$

S.A. none

Example 3 Find the x-intercepts and y-intercept of the rational function.

a.) $f(x) = \dfrac{3x}{(x+1)(x-2)}$

x int (y=0)

Set N = 0

$\dfrac{3x}{\cancel{(x+1)(x-2)}} = 0$

$\dfrac{3x}{3} = \dfrac{0}{3}$

$\boxed{x = 0}$

y int (x=0)

f(0)

$= \dfrac{0}{-2}$

$= 0$

b.) $f(x) = \dfrac{(x-5)(x-2)}{x^2+9}$

x int (y=0)

Set N = 0 $(x-5)(x-2) = 0$

$x=5$ $x=2$

y int (x=0)

$f(0) = \dfrac{(0-5)(0-2)}{0^2+9} = \boxed{\dfrac{10}{9}}$

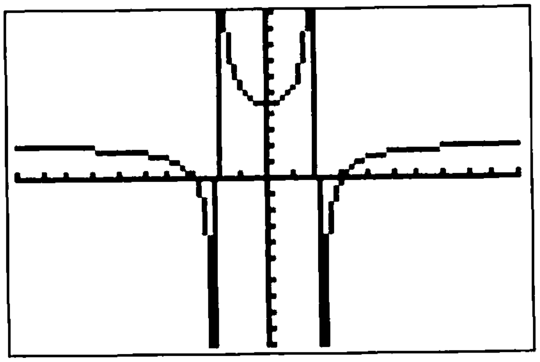
Examples Sketch the graph and provide information about intercepts and asymptotes.

a.) $f(x)=\dfrac{2(x^2-9)}{x^2-4}$

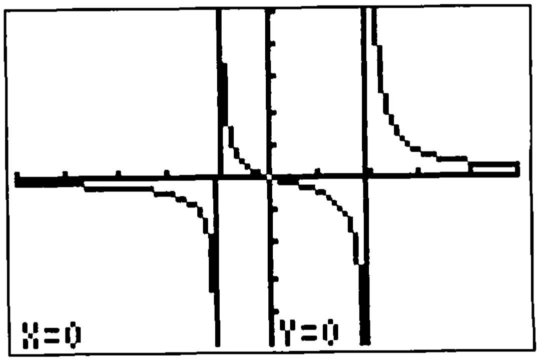
b.) $f(x)=\dfrac{x}{x^2-x-2}$

c.) $f(x)=\dfrac{x^2-x-2}{x-1}$

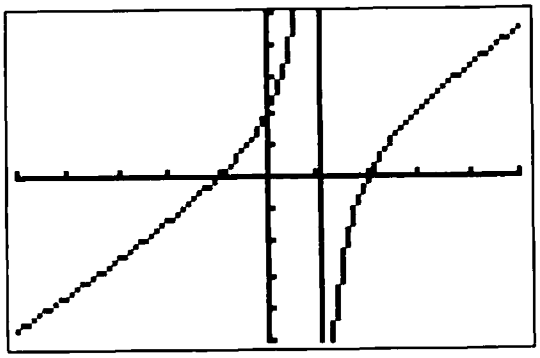
a)

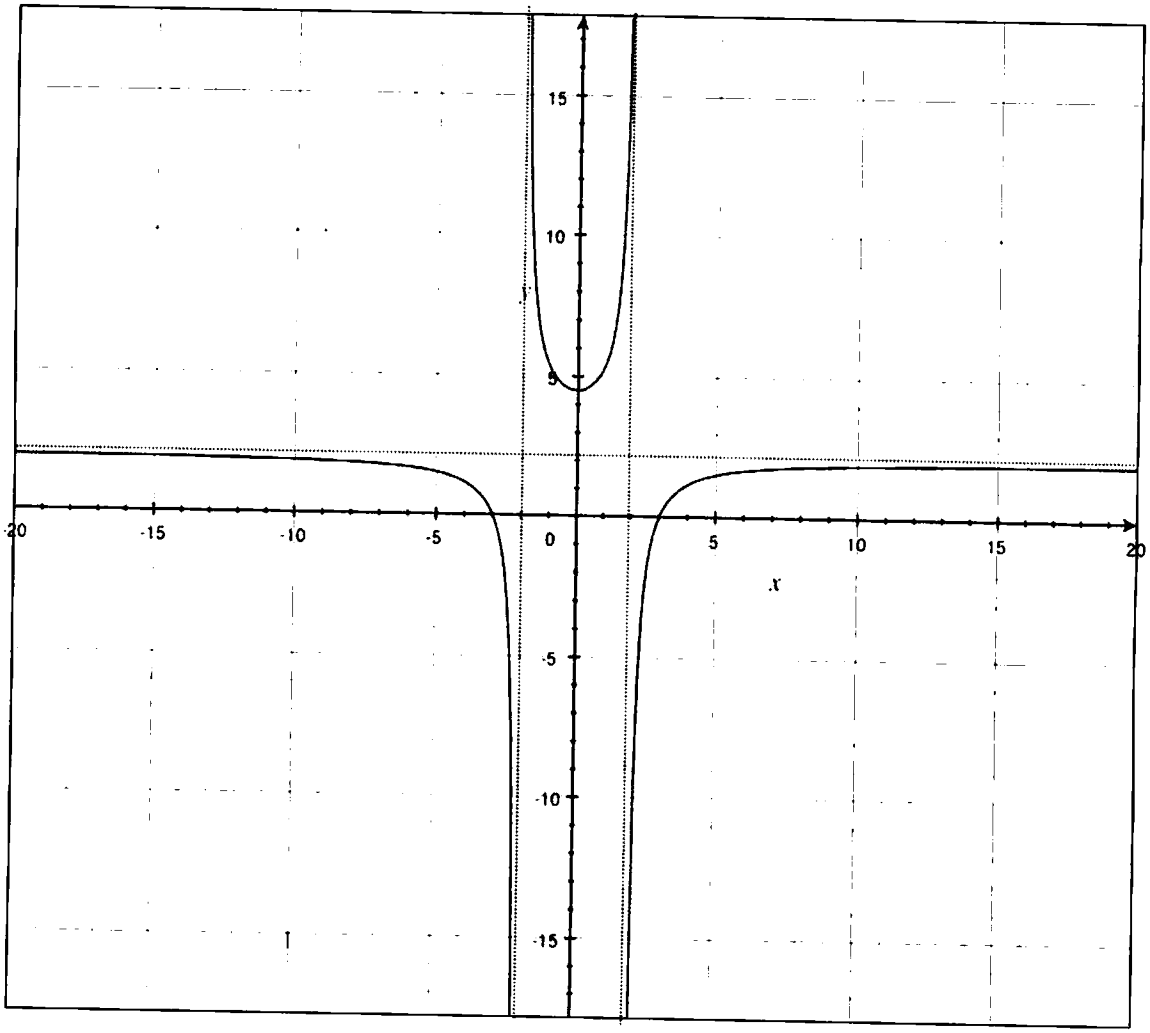


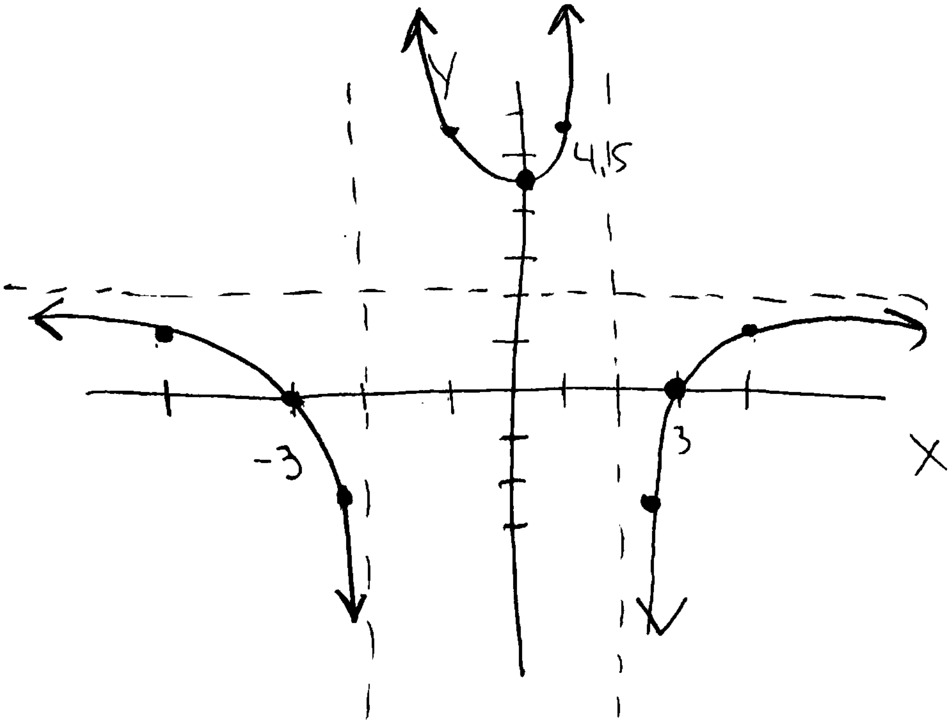
b)



c)



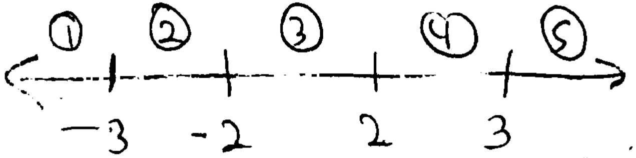

15
10
5
-5
-10
-15
-20
-15
-10
-5
0
5
10
15
20
x


Y
4.15
-3
3
X

§5.3 Graphs of Rational Functions

Guidelines for Graphing Rational Functions

1. Find and plot the x-intercepts.
   (Set numerator = 0 and solve for x)
2. Find and plot the y-intercepts.
   (Let x = 0 and solve for y)
3. Find and plot the Vertical Asymptotes.
   (Set denominator = 0 and solve for x)
4. Find and plot the Horizontal Asymptotes.
   (Top heavy, Bottom heavy or Same)
5. Find and plot the Slant Asymptotes.
   (Divide numerator by denominator.)
6. Find where the graph will intersect its nonvertical asymptote by solving f(x) = k, where k is the y-value of the horizontal asymptote, or f(x) = mx + b, where y = mx + b is the equation of the oblique asymptote.
7. Plot at least one point between and beyond each x-intercept and vertical asymptotes.



Use smooth curves to complete the graph between and beyond the vertical asymptotes.

Example Sketch the graph and provide information about intercepts and asymptotes.

$$f(x) = \frac{x}{x^2 - x - 2}$$

1. Find and plot the x-intercepts. (Set numerator = 0 and solve for x)

$x = 0$

---

2. Find and plot the y-intercepts. (Let x = 0 and solve for y)

$$f(0) = \frac{0}{0^2 - 0 - 2} = 0$$

---

3. Find and plot the Vertical Asymptotes. (Set denominator = 0 and solve for x)

$x^2 - x - 2 = 0$
$(x + 1)(x - 2) = 0$
$x = -1$ and $x = 2$

---

4. Find and plot the Horizontal Asymptotes. (Top heavy, [✓ Bottom heavy] or Same)

(Rule 1) $\boxed{y = 0}$ ↑ none ↑ $\boxed{Y = 0}$ $Y = \frac{a_n}{b_n}$

---

5. Find and plot the Slant Asymptotes. (Divide numerator by denominator.)

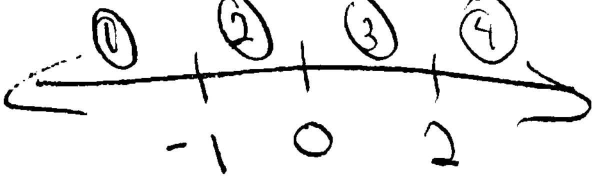
None

---

6. Plot at least one point between and beyond each x-intercept and vertical asymptotes.

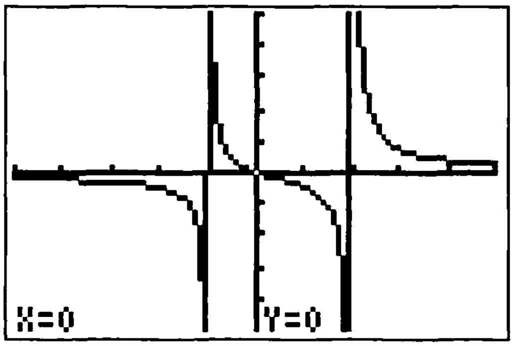
choose:

①	②	③	4
x = -2 y = -.5	x = -.5 y = .4	x = 1 y = -.5	x = 3 y = .75



Note: YOU MAY WANT TO PICK MORE POINTS TO GET A BETTER GRAPH !

ANSWER:



Guidelines for Graphing Rational Functions

example a.) $f(x) = \frac{2(x^2 - 9)}{x^2 - 4}$

1. Find and plot the x-intercepts. (Set numerator = 0 and solve for x).

$2(x^2 - 9) = 0$

$x^2 - 9 = 0$

$x^2 = 9$

$x = \pm 3$

---

2. Find and plot the y-intercepts. (Let x = 0 and solve for y)

$$f(0) = \frac{2(0^2 - 9)}{0^2 - 4} = \boxed{\frac{9}{2}} = \frac{-18}{-4}$$

---

3. Find and plot the Vertical Asymptotes. (Set denominator = 0 and solve for x)

$x^2 - 4 = 0$

$x^2 = 4$

$x = \pm 2$

---

4. Find and plot the Horizontal Asymptotes. (Top heavy, Bottom heavy or [✓ Same])

$f(x) = \frac{2(x^2 - 9)}{x^2 - 4}$ Rule 2 Numerator and denominator have the <u>same degree.</u>

$$y = \frac{2}{1} = \boxed{y = 2 \text{ H.A.}}$$

---

5. Find and plot the Slant Asymptotes. (Divide numerator by denominator.)

RARE

None ! Only have these if Numerator is exactly 1 degree higher than denominator!

---

6. Find where the graph will intersect its nonvertical asymptote by solving f(x) = k. where k is the y-value of the horizontal asymptote, or f(x) = mx + b, where y = mx + b is the equation of the oblique asymptote.

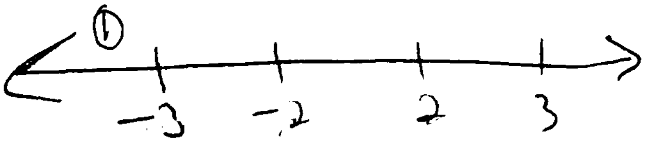
Solve $2 = \frac{2(x^2 - 9)}{x^2 - 4}$ (No solution!) No oblique asymptotes.

---

7. Plot at least one point between and beyond each x-intercept and vertical asymptotes.

Remember Test Points ?

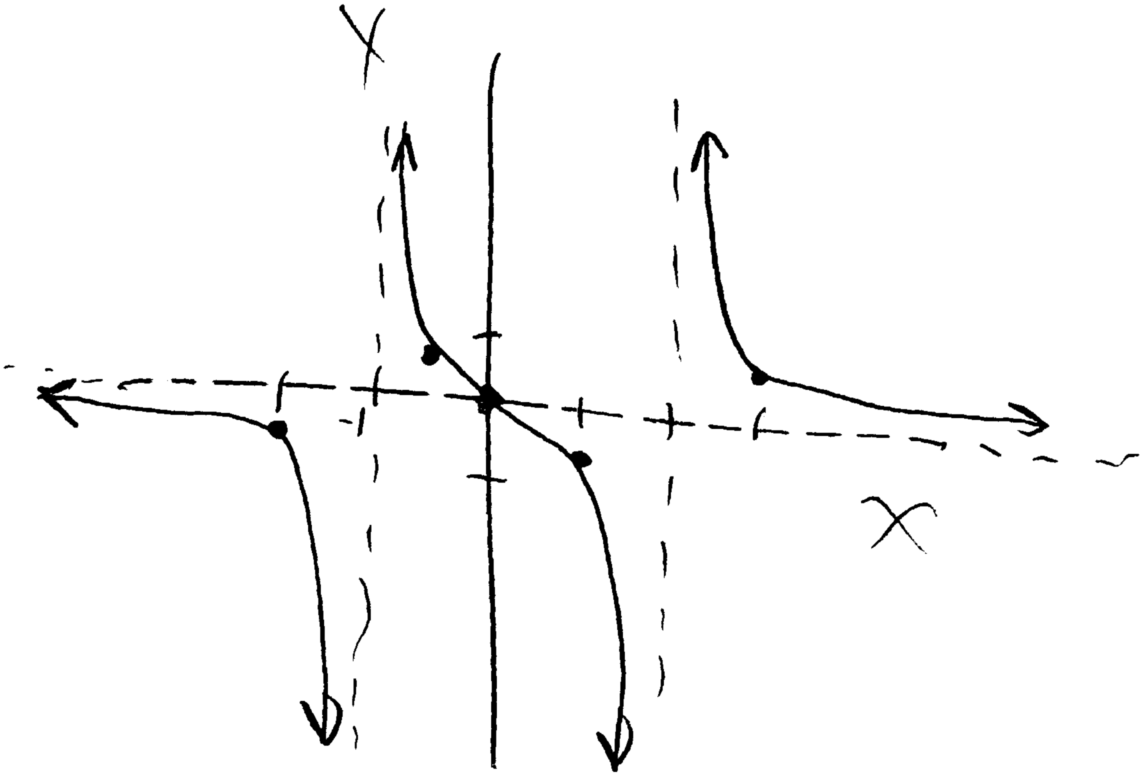
Choose test points carefully!



x = -4	x = -2.5	x = 0	x = 2.5	x = 4			x = -1	x = 1	
y = 1.16	y = -2.4	y = 4.5	y = -2.4	y = 1.16			y = 5.3	y = 5.3	

Note: YOU STILL MAY HAVE TO PLOT ADDITIONAL POINTS !

Use smooth curves to complete the graph between and beyond the vertical asymptotes.


Y
X

Example Sketch the graph and provide information about intercepts and asymptotes.

$$f(x) = \frac{x^2 - x - 2}{x - 1}$$

1. Find and plot the x-intercepts. (Set numerator = 0 and solve for x)

$x^2 - x - 2 = 0$
$(x + 1)(x - 2) = 0$
$x = -1$ and $x = 2$

---

2. Find and plot the y-intercepts. (Let x = 0 and solve for y)

$$f(0) = \frac{0^2 - 0 - 2}{0 - 1} = \frac{-2}{-1} = 2$$

---

3. Find and plot the Vertical Asymptotes. (Set denominator = 0 and solve for x)

$(x - 1) = 0$ $x = 1$

---

4. Find and plot the Horizontal Asymptotes. (Top heavy, Bottom heavy or Same)

(Rule 3) Top Heavy none !

---

5. Find and plot the Slant Asymptotes. (Divide numerator by denominator.)

$$x - 1\overline{)x^2 - x - 2} \qquad y = x$$

$$\underline{-x^2 + x}$$

$$0$$

---

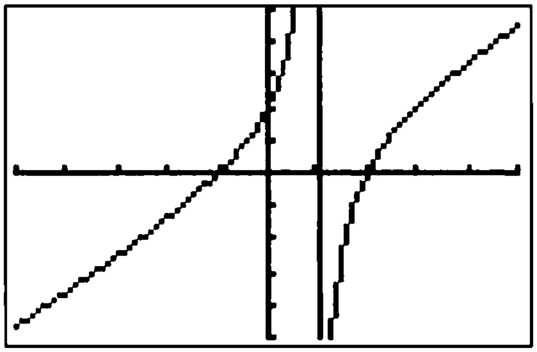
6. Plot at least one point between and beyond each x-intercept and vertical asymptotes.

choose:

x = -2	x = 0	x = 1.5	x = 3
y = -1.3	y = 2	y = -2.5	y = 2

Note: YOU MAY WANT TO PICK MORE POINTS TO GET A BETTER GRAPH !

ANSWER:



§ 6.1 Composite Functions

Composition of Functions:

If f and g are functions, then the <u>composite</u> function or <u>composition</u>, of g and f is:

$(g \circ f)(x) = g[f(x)]$ (Note: this is read "g of f of x".)

for all x in the domain of f such that f(x) is in the domain of g.

Example 1: Let $f(x) = x^2 + 3x - 1$ & $g(x) = 2x + 3$

Find $(f \circ g)(x)$ and $(g \circ f)(x)$.

$(2x+3)(2x+3) = 4x^2+6x+6x+9$

$(f \circ g)(x) = f[\underline{g(x)}] = (2x+3)^2 + 3(2x+3) - 1$

$4x^2+12x+9+6x+9-1 = \boxed{4x^2+18x+17}$

$(g \circ f)(x) = g[f(x)]$

$= 2(x^2+3x-1) + 3$

$= 2x^2+6x-2+3 = \boxed{2x^2+6x+1}$

Example 2: Let $f(x) = 2x^2 - 3$ and $g(x) = 4x$

Find $(f \circ g)(1)$ and $(g \circ f)(1)$ and $(f \circ f)(-2)$.

$(f \circ g)(x) = f[g(x)] = 2(4x)^2 - 3 = 2(16x^2) - 3 = \boxed{32x^2-3}$

$(f \circ g)(1) = 32x^2 - 3 = 32(1)^2 - 3 = \boxed{29}$

$(f \circ g)(1) = f[g(1)] = f[4(1)] = f[4] = 2(4)^2 - 3 = \boxed{29}$

$(g \circ f)(1) = g[f(1)] = g[2(1)^2-3] = g[-1] = 4(-1) = \boxed{-4}$

$$
\begin{aligned}
(f \circ f)(-2) &= f[f(-2)] = 2(\overset{\overset{x}{\downarrow}}{-2})^2-3 \\
&= f[5] = 2(5)^2-3 \\
&= 50-3 = \boxed{47}
\end{aligned}
$$

Show That Two Composite Functions Are Equal

Example 3:

If $f(x) = 3x - 4$ and $g(x) = \frac{1}{3}(x+4)$

Show that $(f \circ g)(x)$ and $(g \circ f)(x) = x$

$$(f \circ g)(x) = f[\underline{g(x)}] = 3\left(\frac{1}{3}(x+4)\right) - 4$$

$$= x + 4 - 4 = \boxed{x}$$

$f(x) = 2x + 3$

$f(5) = 2(5) + 3 = \boxed{13}$

(X, Y)

$(5, 13)$

$f(x) = 2x + 3 \qquad g(x) = 3x + 1$

$(f+g)(x) = ?$

$f(x) + g(x) = (2x+3) + (3x+1)$

$= \boxed{5x+4}$

$(f+g)(1) = 5(1) + 4 = \boxed{9}$

$f(x) + g(x) = f(1) + g(1) =$

$= (5) + (4) = \boxed{9}$

$f(x) = 8 \cdot x$ $\quad$ $g(x) = \frac{x}{8}$

$f(12) = 8(12)$

$= 96$

$(12, 96)$

$g(96) = \frac{96}{8} = 12$

$(96, 12)$

§6.2 One-to-One Functions; Inverse Functions

Example: Let $f(x) = 8x$ and $g(x) = \frac{1}{8}x = \frac{x}{8}$

Find f(12) and g(96) ? What do you notice about these results?

$f(12) = 96$ $g(96) = 12$

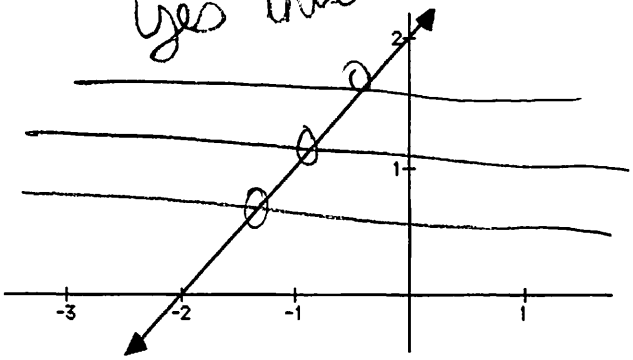
VLT ⇒ ?

Horizontal Line Test:

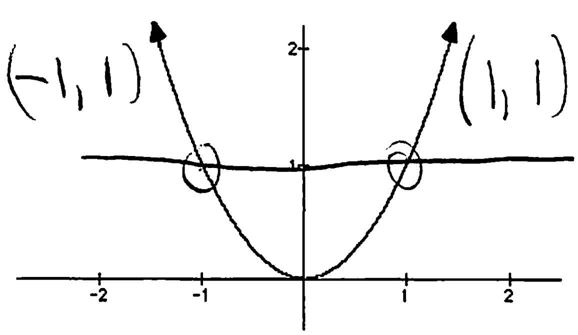
A function f has a inverse function if and only if no horizontal line intersects the graph of f at more than one point.

Example: Do the following graphs of functions have inverses ?

yes inverse.



no inverse



Inverse Function (Verifying)

Let f and g be two functions such that:

$(f \circ g)(x) = x$ for every x in the domain of g,

and $(g \circ f)(x) = x$ for every x in the domain of f.

The function g is the **inverse** of the function f and is denoted by $\boxed{f^{-1}(x)}$ where

$f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.

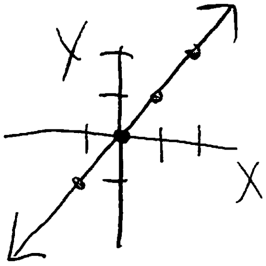
Example: Let $f(x) = x^3 - 1$, and let $g(x) = \sqrt[3]{x+1}$.
Is g the inverse of f ?

$$(f \circ g)(x) = f[g(x)] = \left(\sqrt[3]{x+1}\right)^3 - 1 = (x+1) - 1 = x$$

$$(g \circ f)(x) = g[f(x)] = \sqrt[3]{(x^3-1)+1} = \sqrt[3]{x^3} = x$$

What's the inverse of a function defined by a set of ordered pairs?

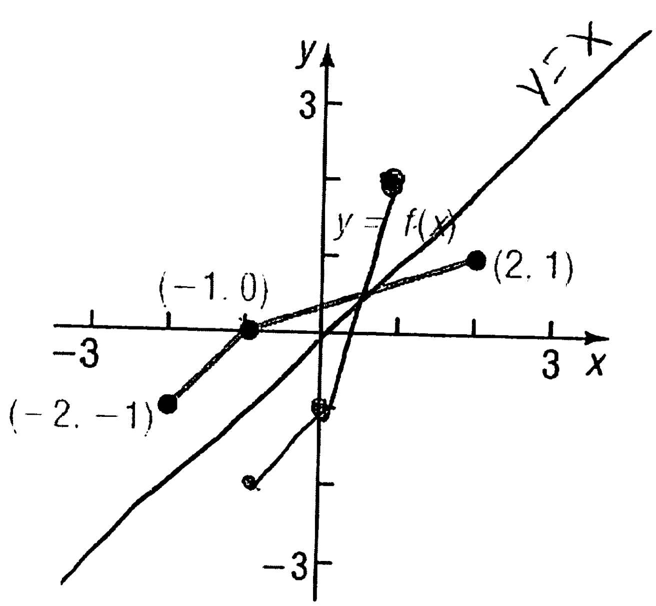
Find the inverse of:
{(–3, –27), (–2, –8), (–1, –1), (2, 8), (3, 27)}

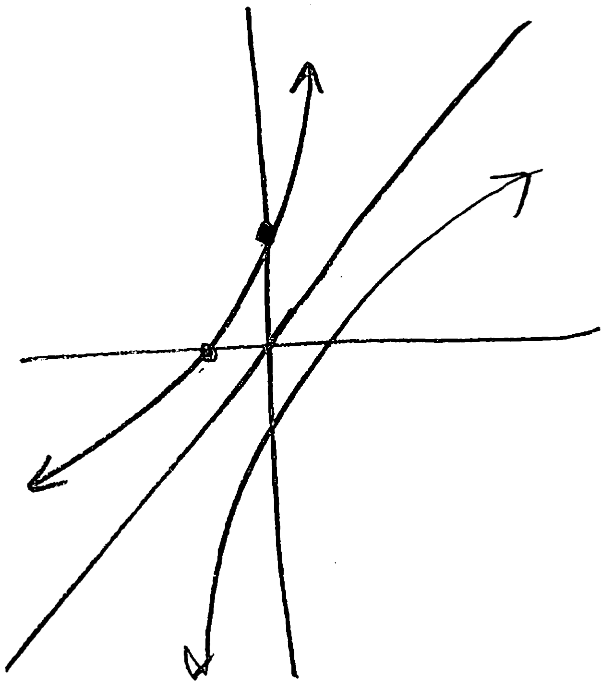


inverse: {(−27, −3), (−8, −2), (−1, −1), (8, 2)(27, 3)}

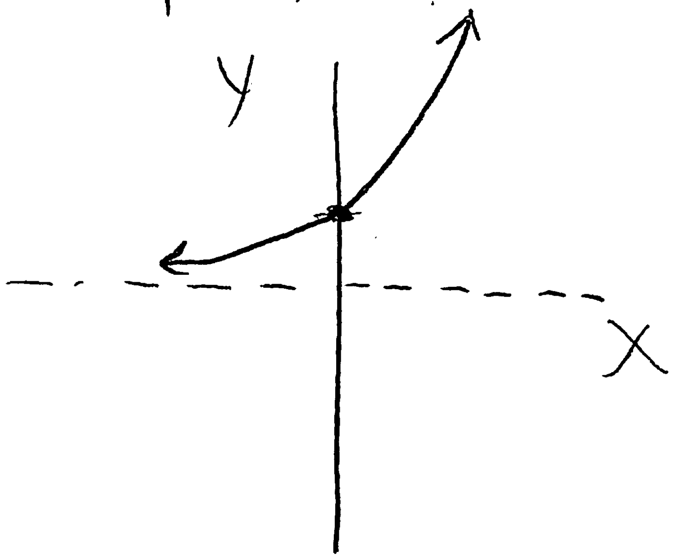
Graphs of Inverses:

(A graph and it's inverse are symmetric with respect to the line y = x.)





$f(x) = 3^x$



$f(x) = 10^x$

Finding the Inverse of a function: Note: the notation used is: $f^{-1}(x)$ $f(x) = y$

(1) Replace f(x) with y.
(2) Interchange the variables x and y.
(3) Solve for y and let this "new" $y = f^{-1}(x)$
(4) Verify that $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.

<u>Example</u> Find the inverse of the following functions.

a.) $f(x) = 2x - 1$

$Y = 2x - 1$

$X = 2y - 1$

$+1 \qquad +1$

$\frac{x+1}{2} = \frac{\cancel{2}y}{\cancel{2}}$

$Y = \frac{x+1}{2}$

$\boxed{f^{-1}(x) = \frac{x+1}{2}}$

b.) $f(x) = \frac{4x+6}{5}$

$Y = \frac{4x+6}{5}$

LCD's

$(5)\ \frac{X}{1} = \frac{4y+6}{5}\ (5)$

$5x = 4y + 6$

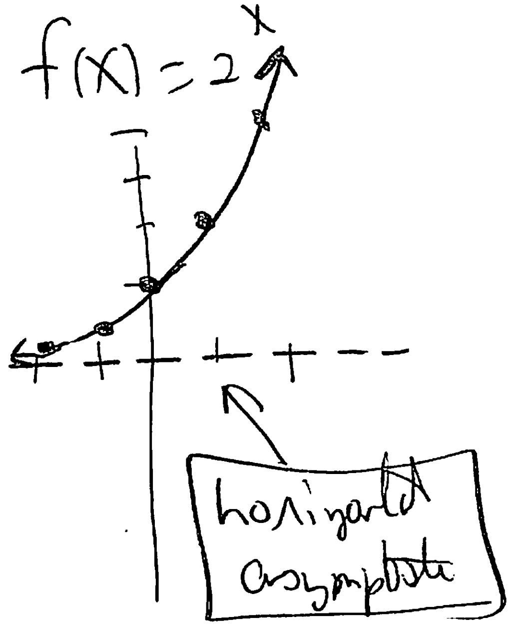
$-6 \qquad -6$

$\frac{5x-6}{4} = \frac{4y}{\cancel{4}}$

$\boxed{f^{-1}(x) = \frac{5x-6}{4}}$

$$f(x) = x^2$$

$$f(x) = 2^x$$



X	Y
0	$2^0 = 1$
1	$2^1 = 2$
2	$2^3 = 4$
3	$2^3 = 8$
4	16
5	32
-1	$\frac{2^{-1}}{1} = \frac{1}{2} = .5$
-2	$\frac{2^{-2}}{1} = \frac{1}{2^2} = \frac{1}{4} = .25$
-3	$\frac{2^{-3}}{1} = \frac{1}{2^3} = \frac{1}{8} = .1.25$

$$x^2 \cdot x^3 = x^{2+3} = x^5$$

$$\frac{x^{-n}}{1} = \frac{1}{x^{+n}}$$

$2^x = 0$?

$2^{-5} = \frac{1}{2^5} = \frac{1}{32} = .03125$

$2^{-10} = .0009$ E-4

$2^{-100} =$

§6.3 Exponential Functions

Exponential Function:

If $a > 0$, $a \neq 1$, and x is any real number, then

$\boxed{f(x) = a^x}$ defines the **exponential function** with base a.

Example 1 : Evaluate the following exponential expressions with your calculator.

a) $2^{-3.1}$ $= \boxed{.117}$

b) 2^{π} $= 8.825$

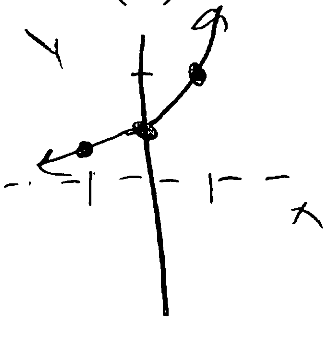
$2 \boxed{\wedge} \pi$

Graphing Exponential Functions

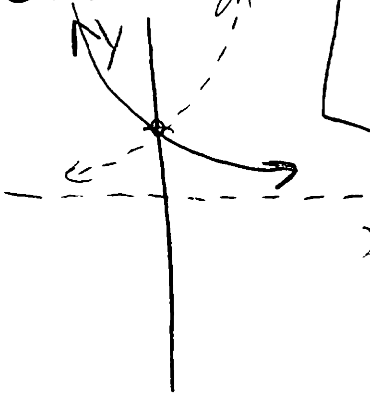
Graphs of the Form: $f(x) = a^x$

1) The point (0, 1) is on the graph. $(1, a)$, $(-1, \frac{1}{a})$
2) If $a > 1$, f is an increasing function; If $0 < a < 1$, f is an decreasing function.
3) The x-axis is a horizontal asymptote.
4) The domain is $(-\infty, \infty)$ and the range is $(0, \infty)$

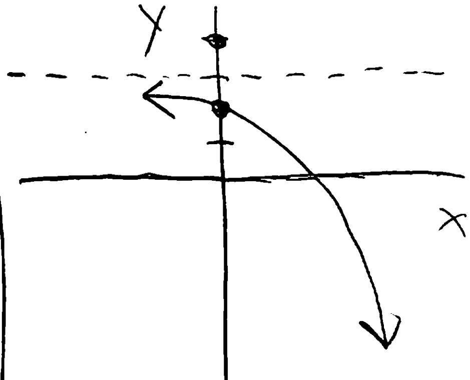
Graph: $f(x) = 2^x$

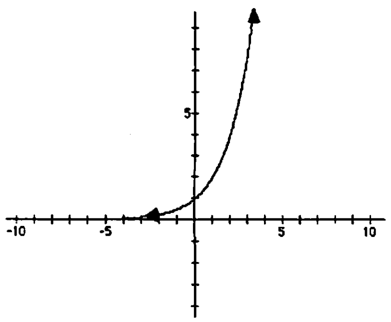


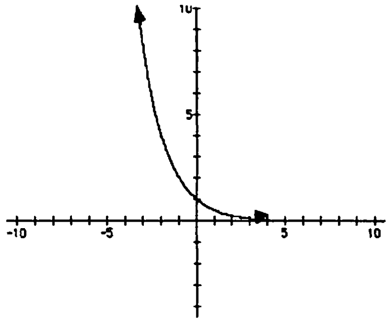
$g(x) = 2^{-x}$

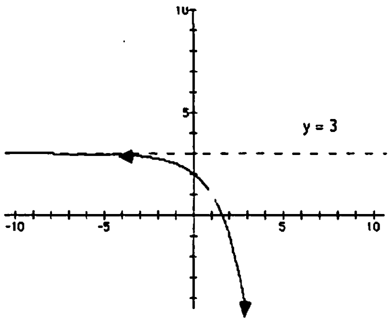


$h(x) = -2^x + 3$









Horizontal Asymptote: The line in which a graph approaches (gets closer and closer to)

Increasing Function: A function where as x-values increase so do the y-values.

Decreasing Function: A function where as x-values increase y-values decrease.

Laws of Exponents ?

$$\boxed{a^s \cdot a^t = a^{s+t}} \qquad x^2 \cdot x^3 = x^5$$

$$(a^s)^t = a^{s \cdot t} \qquad (x^2)^5 = x^{10}$$

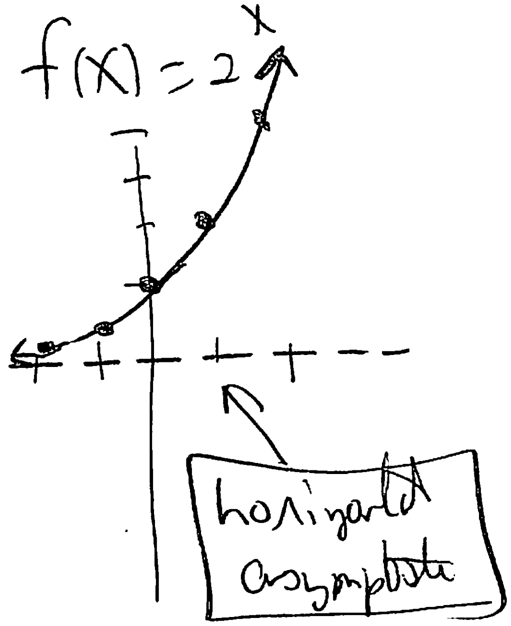
$$\boxed{a^0 = 1} \qquad (ab)^s = a^s b^s$$

$$\boxed{\frac{a^{-s}}{1} = \frac{1}{a^s}} \qquad \boxed{a^1 = a}$$

$$f(x) = x^2$$

$$f(x) = 2^x$$

X	Y
0	$2^0 = 1$
1	$2^1 = 2$
2	$2^2 = 4$
3	$2^3 = 8$
4	16
5	32
-1	$\frac{2^{-1}}{1} = \frac{1}{2} = .5$
-2	$\frac{2^{-2}}{1} = \frac{1}{2^2} = \frac{1}{4} = .25$
-3	$\frac{2^{-3}}{1} = \frac{1}{2^3} = \frac{1}{8} = .125$



$$x^2 \cdot x^3 = x^{2+3} = x^5$$

$$\frac{x^{-n}}{1} = \frac{1}{x^{+n}}$$

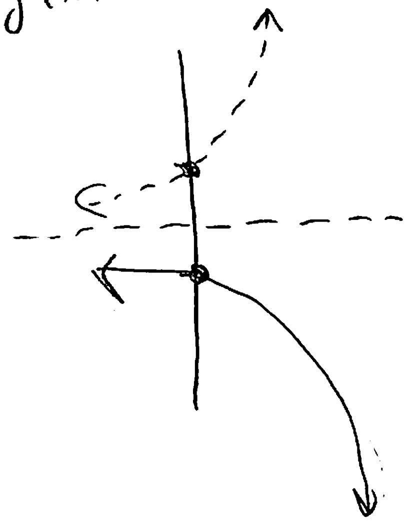
$2^x = 0$?

$2^{-5} = \frac{1}{2^5} = \frac{1}{32} = .03125$

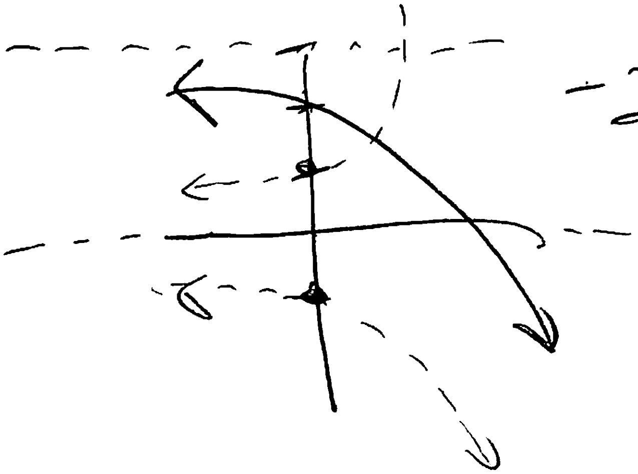
$2^{-10} = .0009$ E -4

$2^{-100} =$

$g(x) = -2^{x}$



$-2^{(x+3)}$



$-2^{x} + 3$

Exponential Equations (TYPE 1)

Example 2: Solve

a) $\left(\frac{1}{3}\right)^x = 81$

$(3^{-1})^x = 3^4$

$\cancel{3}^{-x} = \cancel{3}^{4}$

$\frac{-x}{-1} = \frac{4}{-1}$

$\boxed{x = -4}$

b) $1.5^{x+1} = \left(\frac{27}{8}\right)^x$

$\cancel{\left(\frac{3}{2}\right)}^{x+1} = \cancel{\left(\frac{3}{2}\right)}^{3x}$

$x + 1 = 3x$

$\frac{1}{2} = \frac{2x}{2}$

$\boxed{x = \frac{1}{2}}$

$\pi = 3,141529, \ldots$

The Natural Base e $e \approx 2.71828.....$

Example 3: Use a calculator to evaluate each expression.

euler

a) e^{-2} $= \boxed{.135}$

b) e^{-1} $\boxed{.368}$

c) e^{1} $= \boxed{2.718}$

d) e^{2} $= \boxed{7.389}$

$$a^{-n} = \frac{1}{a^{+n}}$$

$$5^{-1} = \frac{1}{5^{1}}$$

$3^{\square} = 3^{\square}$?

$3^{2} = 3^{3}$

exp	log
$a^5 = 4$	$\log_a(4) = 5$
$e^{-3} = b$	$\log_e(b) = -3$
$3^c = 5$	$\log_3(5) = c$
$1,2^3 = m$	$\log_{1,2}(m) = 3$
$e^b = 9$	$\log_e(9) = b$
$a^4 = 24$	$\log_a(24) = 4$

Student: Keith Barrs
Date: 11/3/09
Time: 5:28 PM

Instructor: Keith Barrs
Course: Math-1111-OL-F09
Book: Sullivan: College Algebra, 8e

Assignment: Section 6.1 Composite Functions

1. Evaluate each expression using the values in the given table.

x	-3	-2	-1	0	1	2	3
f(x)	-9	-7	-5	-3	-1	1	3
g(x)	3	2	1	0	-1	-2	-3

(a) $(f \circ g)(-2)$ (b) $(g \circ f)(0)$ (c) $(f \circ f)(3)$

(a) What is $(f \circ g)(-2)$?

$(f \circ g)(-2) =$ ☐

(b) What is $(g \circ f)(0)$?

$(g \circ f)(0) =$ ☐

(c) What is $(f \circ f)(3)$?

$(f \circ f)(3) =$ ☐

2. Evaluate each expression using the graphs of $y = f(x)$ and $y = g(x)$ shown below.

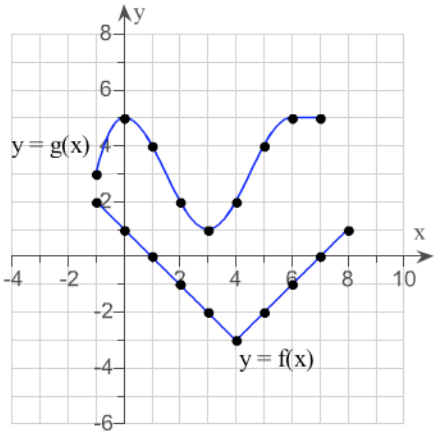
(a) $g(f(-1))$ (b) $g(f(0))$ (c) $f(g(-1))$ (d) $f(g(4))$

(a) $g(f(-1)) =$ ☐

(b) $g(f(0)) =$ ☐

(c) $f(g(-1)) =$ ☐

(d) $f(g(4)) =$ ☐



3. Given $f(x) = 9x$ and $g(x) = 3x^2 + 3$, find

(a) $(f \circ g)(4)$ (b) $(g \circ f)(2)$ (c) $(f \circ f)(1)$ (d) $(g \circ g)(0)$

(a) What is $(f \circ g)(4)$?

$(f \circ g)(4) =$ ☐

(b) What is $(g \circ f)(2)$?

$(g \circ f)(2) =$ ☐

(c) What is $(f \circ f)(1)$?

$(f \circ f)(1) =$ ☐

(d) What is $(g \circ g)(0)$?

$(g \circ g)(0) =$ ☐

4. Given $f(x) = 3\sqrt{x}$ and $g(x) = 5x$, find the following expressions.

(a) $(f \circ g)(4)$ (b) $(g \circ f)(2)$ (c) $(f \circ f)(1)$ (d) $(g \circ g)(0)$

(a) $(f \circ g)(4) =$ ☐
(Type an exact answer, using radicals as needed. Simplify your answer.)

(b) $(g \circ f)(2) =$ ☐
(Type an exact answer, using radicals as needed. Simplify your answer.)

(c) $(f \circ f)(1) =$ ☐
(Type an exact answer, using radicals as needed. Simplify your answer.)

(d) $(g \circ g)(0) =$ ☐
(Type an exact answer, using radicals as needed. Simplify your answer.)

5. For $f(x) = 9x$ and $g(x) = \frac{1}{9}x$, find $(f \circ g)(x)$ and $(g \circ f)(x)$. Then determine whether $(f \circ g)(x) = (g \circ f)(x)$.

What is $(f \circ g)(x)$?

$(f \circ g)(x) =$ ☐

What is $(g \circ f)(x)$?

$(g \circ f)(x) =$ ☐

Does $(f \circ g)(x) = (g \circ f)(x)$?

○ Yes
○ No

---

6. For $f(x) = x^{19}$ and $g(x) = \sqrt[19]{x}$, find $(f \circ g)(x)$ and $(g \circ f)(x)$. Then determine whether $(f \circ g)(x) = (g \circ f)(x)$.

What is $(f \circ g)(x)$?

$(f \circ g)(x) =$ ☐

What is $(g \circ f)(x)$?

$(g \circ f)(x) =$ ☐

Does $(f \circ g)(x) = (g \circ f)(x)$?

○ Yes
○ No

Student: Keith Barrs
Date: 11/3/09
Time: 5:28 PM

Instructor: Keith Barrs
Course: Math-1111-OL-F09
Book: Sullivan: College Algebra, 8e

Assignment: Section 6.1 Composite Functions

7. For $f(x) = 4x - 7$ and $g(x) = \frac{1}{4}(x + 7)$, find $(f \circ g)(x)$ and $(g \circ f)(x)$. Then determine whether $(f \circ g)(x) = (g \circ f)(x)$.

What is $(f \circ g)(x)$?

$(f \circ g)(x) =$ ☐

What is $(g \circ f)(x)$?

$(g \circ f)(x) =$ ☐

Does $(f \circ g)(x) = (g \circ f)(x)$?

○ Yes
○ No

§6.4 Logarithmic Functions

Logarithm :

For all real numbers y, and all positive numbers a and x, where $a \neq 1$:

exp

$y = \log_a(x)$ → if and only if $\boxed{x = a^y}$.

Examples textbook.

Note that your calculator has the ability to evaluate two types of logs.

Common Logs	**log (base 10)**	**-> log x** $= \log_{10} x$
Natural Logs	**log (base e)**	**-> ln x**

Example 1: Evaluating Logarithms on a Calculator

a) $\log_{10} 2.5$ $= \boxed{.398}$

b) $\log_{10}\left(\frac{1}{3}\right)$ $= \boxed{-.477}$

c) $\ln_e 0.3$ $\boxed{-1.204}$

$\log .3 = \boxed{-.523}$

d) $\ln_e(-1) = x$

$e^x = -1$

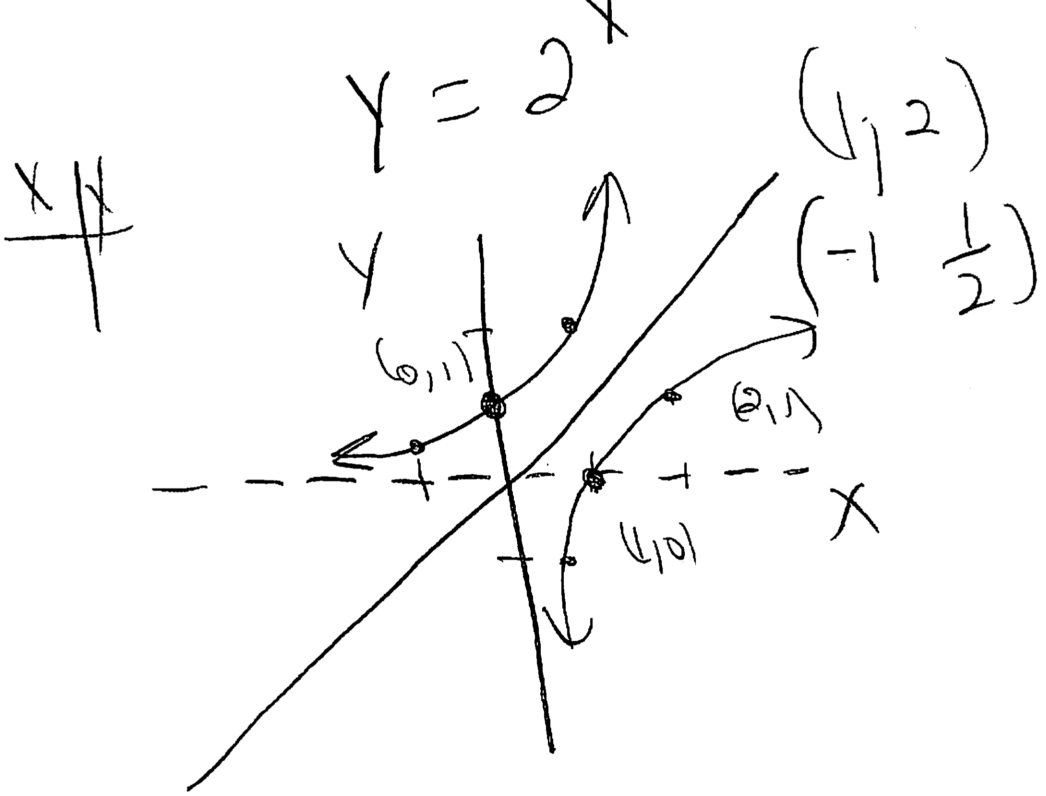
Properties of Logarithms: (also true for natural logarithms)

1) $\log_a 1 = 0$ because $a^0 = 1$

2) $\log_a(a^1) = 1$ because $a^1 = a$

3) $\log_a(a^x) = x$ because $a^x = a^x$

4) $\log_a(x) = \log_a(y)$, then $x = y$

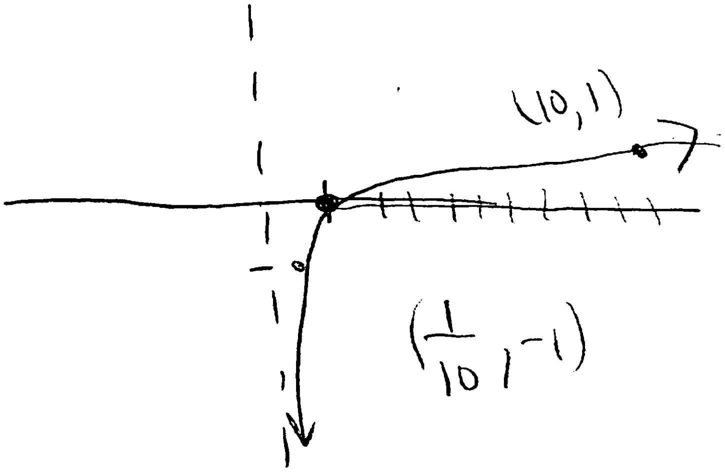

Y = 2^X
X | Y
Y
X
(1, 2)
(-1 1/2)
(0,1)
(2,1)
(1,0)

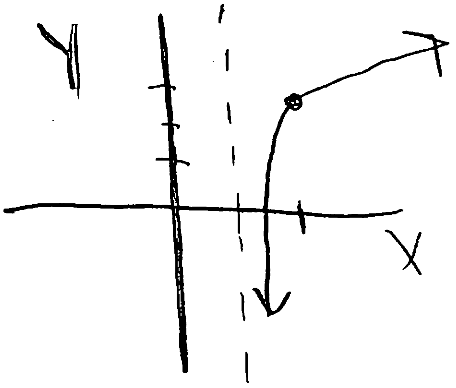
Graphs of the Form: $f(x) = \log_a x$

1) The points (1, 0), (a, 1), $\left(\frac{1}{a}, -1\right)$ is on the graph.
2) If a > 1, f is an increasing function; If 0 < a < 1, f is a decreasing function.
3) The y-axis is a vertical asymptote.
4) The domain is $(0, \infty)$ and the range is $(-\infty, \infty)$.

Examples: a) $y = \log_{10} x$ b) $y = \log_2(x-1) + 3$

(Graph)





Solving Logarithmic Equations

Solve: a) $\log_3(4x - 7) = 2$

$$3^2 = 4x - 7$$
$$9 = 4x - 7$$
$$+7 \quad\quad +7$$

$$\frac{16}{4} = \frac{4x}{4}$$

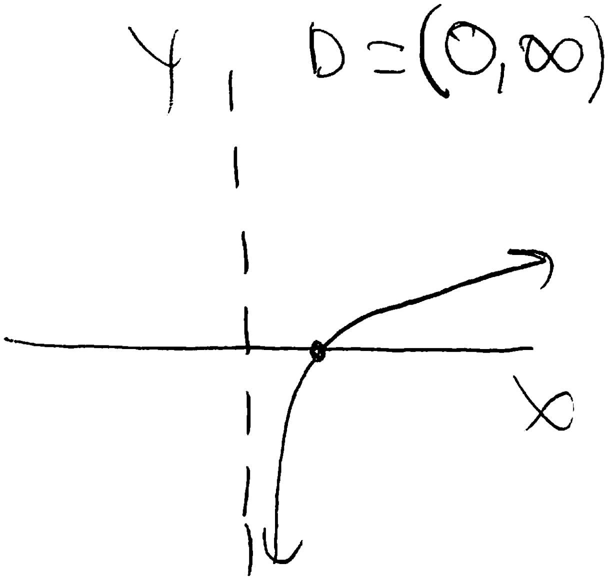
$$\boxed{x = 4}$$

Solve: b) $\log_x(64) = 2$

$\sqrt{x^2} = \sqrt{64}$

$x = \overset{+}{\cancel{-}}8$

$x = +8$



$D = (0, \infty)$

$\log_{\text{positive}}(\text{positive}) = \text{neg or pos}$

Solve: c) $e^{2x} = 5$

$\frac{\ln_e(5)}{2} = \frac{2x}{2}$

$x = \frac{\ln_e(5)}{2}$

$x = .805$

$2^x = 16$

$\cancel{2}^x = \cancel{2}^4$

$x = 4$

§6.5 Properties of Logarithms

Properties of Logarithms:

(also true for natural logarithms)

1) $\log_a 1 = 0$ because $a^0 = 1$

2) $\log_a a = 1$ because $a^1 = a$

3) $\log_a(a^x) = x$ because $a^x = a^x$

4) $\log_a(x) = \log_a(y)$, then $x = y$

$$\frac{a^{-n}}{1} = \frac{1}{a^{+n}}$$

$$\frac{1}{8} = \frac{1}{2^3} = 2^{-3}$$

Example : Solve for x.

a) $\cancel{\log}_2(x) = \cancel{\log}_2(3)$

$\boxed{x = 3}$

b) $\log_4(4^{\textcircled{1}}) = x$

$\boxed{x = 1}$

c) $\log_2\left(\frac{1}{8}\right) = x$

$\log_2 2^{-3} = x$

$\boxed{x = -3}$

Example : Rewrite using Properties of Natural Logarithms

a) $\ln\frac{1}{e}$

$\ln_e e^{-1} = x$

$\boxed{x = -1}$

b) $\ln_e(e^3) = x$

$\boxed{x = 3}$

c) $\ln_e(e^0) = x$

$\boxed{x = 0}$

Properties of Logarithms:

For any positive real numbers x and y, real number r, and any positive real number a,($a \neq 1$):

Product Rule a) $\boxed{\log_a(xy) = \log_a x + \log_a y}$

Quotient Rule b) $\boxed{\log_a \frac{x}{y} = \log_a x - \log_a y}$

Power Rule c) $\boxed{\log_a x^r = r \log_a x}$

Example : Rewrite the logarithm in terms of ln 2 and ln 3.

a) $\ln 6 = \ln(2 \cdot 3)$

$= \boxed{\ln 2 + \ln 3}$

$1.791 = (0.693) + (1.098)$

b) $\ln \frac{2}{27} =$

$= \ln 2 - \ln 27$

$= \ln 2 - \ln 3^3$

$= \boxed{\ln 2 - 3 \ln 3}$

c) $\ln \frac{9}{4} =$

$\ln 9 - \ln 4$

$\ln 3^2 - \ln 2^2$

$\boxed{2 \ln 3 - 2 \ln 2}$

Example : Rewrite using the properties of logarithms.

a) $\log_{10}(5x^3y)$

$= \log_{10} 5x^3 + \log_{10} y$

$= \log_{10} 5 + \log_{10} x^3 + \log_{10} y$

$= \boxed{\log_{10} 5 + 3\log_{10} x + \log_{10} y}$

b) $\ln \frac{\sqrt{(3x-5)}}{7}$

$= \ln \sqrt{(3x-5)} - \ln 7$

$= \ln (3x-5)^{\frac{1}{2}} - \ln 7$

$= \boxed{\frac{1}{2} \ln (3x-5) - \ln 7}$

Example : Rewrite in condensed form.

a) $\log_a 7 + 4\log_a 3$

$= \log_a 7 + \log_a 3^4$

$= \log_a 7 + \log_a 81$

$= \log_a 7 \cdot 81 = \boxed{\log_a 567}$

b) $\frac{2}{3}\ln 8 - \ln(3^4 - 8)$

$= \ln 8^{\frac{2}{3}} - \ln(81-8)$

$= \ln (\sqrt[3]{8})^2 - \ln(73)$

$= \ln 2^2 - \ln 73$

$= \ln 4 - \ln 73$

$= \boxed{\ln \frac{4}{73}}$

$\sqrt{x} = x^{\frac{1}{2}}$

$\sqrt[3]{x} = x^{\frac{1}{3}}$

$\sqrt[4]{x} = x^{\frac{1}{4}}$

$\sqrt{x^3} = x^{\frac{3}{2}}$

Change of Base Formula :

Let a, b and x be positive real numbers such that $a \neq 1$ and $b \neq 1$. Then

$$\log_a(x) = \frac{\log_b(x)}{\log_b(a)} \qquad \left(\log_a x = \frac{\log_{10} x}{\log_{10} a} \quad \text{or} \quad \log_a x = \frac{\ln x}{\ln a}\right)$$

Example : Changing Bases Using Common Logarithms & Natural Logarithm

a) $\log_4(30)$

$= \dfrac{\log_{10}(30)}{\log_{10}(4)}$

$= \log(30)/\log(4)$

$= \boxed{2,453}$

b) $\log_2 14$

$= \dfrac{\log_{10}(14)}{\log_{10}(2)}$

$= \boxed{3.807}$

c) $\log_4 30$

$= \dfrac{\ln 30}{\ln 4}$

$= \boxed{2.453}$

d) $\log_2 14$

$= \dfrac{\ln 14}{\ln 2}$

$= \boxed{3,807}$

$l\infty(\mathrm{pos})(\mathrm{pos})$

§6.6 Logarithmic and Exponential Equations

Properties of Exponential and Logarithmic Functions:

For $a > 0$ and $a \neq 1$:

1) $a^x = a^y$ if and only if $x = y$.
2) If $x > 0$ and $y > 0$, $\log_a x = \log_a y$ if and only if $x = y$.

Example: Solve using the properties above.

a) $2^x = 16$

$\cancel{2}^x = \cancel{2}^4$

$\boxed{x = 4}$

$4^x = 72$

b) $\cancel{\log_5}(x+1) = \cancel{\log_5}(10)$

$x+1 = 10$

$\boxed{x = 9}$

(**Note**: remember that the domain of $y = \log_b x$ is $(0, \infty)$. For this reason it is always necessary to check that the solution of a logarithmic equation results in logarithms of positive numbers in the original equation.)

Solving Exponential and Logarithmic Equations (TYPE 2)

An exponential or logarithmic equation may be solved by changing the equation into one of the following **FORMS**, where a and b are real numbers, $a > 0$, and $a \neq 1$.

Exp

1) $\boxed{a^{f(x)} = b}$ Solve by taking the logarithms of each side. (Natural logarithms are often a good choice.)

Log

2) $\log_a f(x) = \log_a g(x)$ From the given equation, $f(x) = g(x)$, which is solved algebraically.

log

3) $\log_a f(x) = b$ Solve by using the definition of logarithm to write the expression in exponential form as $f(x) = a^b$.

Examples: Solve the following equations for x.

a) $4^x = 72$

$$\ln 4^x = \ln 72$$

$$\frac{x \ln 4}{\ln 4} = \frac{\ln 72}{\ln 4}$$

$$\boxed{x = 3.085}$$

d) $\log_4(x) = 3$

$$4^3 = x$$

$$\boxed{x = 64}$$

$a^{f(x)} = b$

b) $4e^{2x} = 5$

$$\frac{4e^{2x}}{4} = \frac{5}{4}$$

$$\log_a a^x = \boxed{x}$$

$$e^{2x} = \frac{5}{4}$$

$$\ln_e e^{\boxed{2x}} = \ln_e \frac{5}{4}$$

$$\frac{2x}{2} = \frac{\ln \frac{5}{4}}{2}$$

$$x = \frac{\ln \frac{5}{4}}{2} = \boxed{.112}$$

c) $e^{2x} - 3e^{x} + 2 = 0$

$$(e^x - 2)(e^x - 1) = 0$$

$e^x - 2 = 0$	$e^x - 1 = 0$
$\boxed{e^x = 2}$	$e^x = 1$
$\ln_e e^x = \ln 2$	$\ln_e e^x = \ln 1$
$x = \ln 2$ $\boxed{x = .693}$	$\boxed{x = 0}$

e) $5 + 2\ln x = 4$

$$-5 \qquad -5$$

$$\frac{2\ln x}{2} = \frac{-1}{2}$$

$$\log_a f(x) = b$$

$$\ln_e x = -\frac{1}{2}$$

$$e^{-\frac{1}{2}} = x$$

$$x = e^{-\frac{1}{2}} = \boxed{.607}$$

$$\log_a(x \cdot y) = \log_a x + \log_a y$$

f) $\log(5x) + \log(x - 1) = 2$

$$\log_{10}(5x)(x-1) = 2$$

$$\log_{10}(5x^2 - 5x) = 2$$

$$10^2 = 5x^2 - 5x$$

$$\frac{0}{5} = \frac{5x^2}{5} - \frac{5x}{5} - \frac{100}{5}$$

$$0 = x^2 - x - 20$$

$$0 = (x + 4)(x - 5)$$

$x + 4 = 0$	$x - 5 = 0$
$\cancel{x = -4}$	$\boxed{x = 5}$ ✓

17. $$\begin{aligned}\log_2 6 \cdot \log_6 4 &= \log_6 4^{\log_2 6}\\ &= \log_6 \left(2^2\right)^{\log_2 6}\\ &= \log_6 2^{2\log_2 6}\\ &= \log_6 2^{\log_2 6^2}\\ &= \log_6 6^2\\ &= 2\end{aligned}$$

$$x^2 - 3x + 2 = 0$$

$$(x-2)(x-1) = 0$$

$$x-2=0 \quad \Big| \quad x-1=0$$

$$\boxed{x=2} \qquad \boxed{x=1}$$

$$x^2 \cdot x^5 = x^7$$

$$e^x \cdot e^x = e^{2x}$$

§6.7 Compound Interest

Simple Interest Formula

If a principle of P dollars is borrowed for a period of t years at a per annum interest rate r, expressed as a decimal, the interest I charged is $\boxed{I = Prt}$

$P = 1000 \quad r = .08 \quad t = 5 \quad I =$

Formulas for Compound Interest:

After **t** years, the balance **A** in an account with principal **P** and annual interest rate **r** (in decimal form) is given by the following formulas:

1. For **n** compoundings per year: $\boxed{A = P\left(1+\frac{r}{n}\right)^{(n\cdot t)}}$
2. For continuous compounding: $\boxed{A = Pe^{(r\cdot t)}}$

Example (future value): A total of \$12,000 is invested at an annual interest rate of 9%. Find the balance after 5 years if it is compounded:

a) quarterly. $n = 4$

$A =$
$P =$
$n =$
$t =$
$r =$

$$A = P\left(1+\frac{r}{n}\right)^{(n\cdot t)}$$

$$= 12000\left(1+\frac{.09}{4}\right)^{(4\cdot 5)}$$

$$= \boxed{18726.11}$$

b) continuously.

$$A = Pe^{(r\cdot t)}$$

$$= 12000e^{\wedge}(.09\cdot 5)$$

$$= \boxed{18819.75}$$

Compound Interest (rate of interest):

Example : What annual rate of interest compounded annually should you seek if you want to double your investment in 5 years?

$$A = P\left(1+\frac{r}{n}\right)^{(n \cdot t)}$$

$$\frac{2\cancel{P}}{\cancel{P}} = \frac{\cancel{P}\left(1+\frac{r}{1}\right)^{(1 \cdot 5)}}{\cancel{P}}$$

$$2 = (1+r)^5$$

$$\sqrt[5]{2} = \sqrt[\cancel{5}]{(1+r)^{\cancel{5}}}$$

$$\sqrt[5]{2} = 1+r$$

$$r = \sqrt[5]{2} - 1$$

Continuous Compounding:

Example : How long will it take for the money in an account that is compounded continuously at 5% to double ? Triple?

$$A = Pe^{(r \cdot t)}$$

$$\frac{2\cancel{P}}{\cancel{P}} = \frac{\cancel{P}e^{(r \cdot t)}}{\cancel{P}}$$

$$2 = e^{(.05t)}$$

$$\ln_e 2 = \ln_e e^{.05t}$$

$$\frac{.05t}{.05} = \frac{\ln 2}{.05}$$

$$\boxed{t = 13.86}$$