§1.3 Complex Numbers

Complex number: a number of the form a + bi, where a and b are real numbers. a is called the <u>REAL</u> part of the complex number a + bi, bi is called the <u>IMAGINARY</u> part of the complex number a + bi.

Imaginary number: a complex number of the form a + bi, where b is nonzero.

Standard Form of a complex number:
a + bi or a + ib (Discuss $i\sqrt{5}$ & $\sqrt{5i}$)

Definition of i: $i = \sqrt{-1}$ or $i^2 = -1$

Definition of $\sqrt{-a}$ If $a > 0$, then $\sqrt{-a} = i\sqrt{a}$

Example: $\sqrt{-16}$

Simplify

example 4: a) $\sqrt{-4}$ b) $\sqrt{-8}$

OPERATIONS WITH COMPLEX NUMBERS

Addition or Subtraction of Complex Numbers:

1. Combine the real parts.
2. Combine the imaginary parts.
3. Leave the result in the form a + bi.

Note: Add (or subtract) the real numbers then add the imaginary numbers.

example 1:

a) $(3 + 5i) + (-2 + 3i)$ b) $(6 + 4i) - (3 + 6i)$

Multiplication of Complex Numbers:

1. Multiply the numbers as if they are two binomials (FOIL METHOD).
2. Substitute –1 for i^2
3. Combine the like terms and leave the result in the form a + bi.

example 2: a) $(5 + 3i)(2 + 7i)$ b) $(4 + 3i)^2$

Properties of Complex Conjugates:

If $z = a + bi$ then the conjugate $\bar{z} = a - bi$:

$$z \cdot \bar{z} = (a + bi)(a - bi) = a^2 + b^2$$

Division of Complex Numbers:

1. Write the division as a fraction.
2. Multiply the numerator and denominator by the conjugate of the denominator:

$$\frac{a+bi}{c+di} \cdot \frac{c-di}{c-di}.$$

3. Multiply and simplify in the numerator (by FOIL). Multiply and simplify in the denominator to a real number (by FOIL).
4. Write the result in the form $a + bi$.

example 3 a) $\dfrac{1+4i}{5-12i}$ b) $\dfrac{2-3i}{4-3i}$

example 5: Solve $x^2 - 4x + 8$