

§1.5 Solving Inequalities

Interval Notation

Interval	Inequality	Graph
The open interval (a, b)	$a < x < b$	
The closed interval $[a, b]$	$a \leq x \leq b$	
The half-open interval $[a, b)$	$a \leq x < b$	
The half-open interval $(a, b]$	$a < x \leq b$	
The interval $[a, \infty)$	$x \geq a$	
The interval (a, ∞)	$x > a$	
The interval $(-\infty, a]$	$x \leq a$	
The interval $(-\infty, a)$	$x < a$	
The interval $(-\infty, \infty)$	All real numbers	

Examples 1 Write in interval notation and graph.

a.) $1 \leq x \leq 3$

b.) $-4 < x < 0$

c.) $x > 5$

d.) $x \leq 1$

Properties of Inequalities - Let a, b and c be real numbers.

- 1.) $a < b$ and $a + c < b + c$ are equivalent.
(addition property)
- 2.) If $c > 0$, then $a < b$ and $ac < bc$ are equivalent.
(multiplication property)
- 3.) If $c < 0$, then $a < b$ and $ac > bc$ are equivalent.
(multiplication property) *

Note: Replacing $<$ with $>$, \leq or \geq results in equivalent properties.

* **Note:** When multiplying or dividing both sides of the inequality by a negative number, we must reverse the direction of the inequality symbol.

Linear Inequalities: an inequality that can be written in the form $ax + b > 0$ where $a \neq 0$. (Note: Any inequality symbol may be used $<$, $>$, \leq , \geq .)

Use the properties of inequalities to solve linear inequalities by isolating the variable.

Example 2 Solve.

a.) $3 - 2x < 5$

b.) $4x + 7 \geq 2x - 3$

Double Inequalities: Isolate the variable in the middle. Perform operations on each part of the inequality.

c.) $-5 < 3x - 2 < 1$

d.) $-1 \leq \frac{3 - 5x}{2} \leq 9$