§5.3 Graphs of Rational Functions

Guidelines for Graphing Rational Functions

1. Find and plot the x-intercepts.
   (Set numerator = 0 and solve for x)
2. Find and plot the y-intercepts.
   (Let x = 0 and solve for y)
3. Find and plot the Vertical Asymptotes.
   (Set denominator = 0 and solve for x)
4. Find and plot the Horizontal Asymptotes.
   (Top heavy, Bottom heavy or Same)
5. Find and plot the Slant Asymptotes.
   (Divide numerator by denominator.)
6. Find where the graph will intersect its nonvertical asymptote by solving f(x) = k, where k is the y-value of the horizontal asymptote, or f(x) = mx + b, where y = mx + b is the equation of the oblique asymptote.
7. Plot at least one point between and beyond each x-intercept and vertical asymptotes.

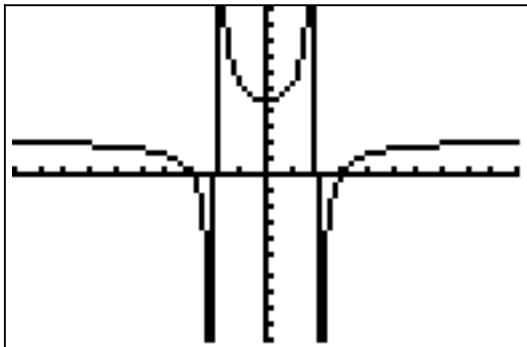**Use smooth curves to complete the graph between and beyond the vertical asymptotes.**

Examples Sketch the graph and provide information about intercepts and asymptotes.
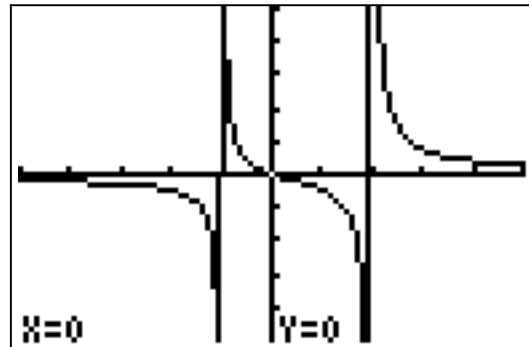
a.) $f(x) = \dfrac{2(x^2 - 9)}{x^2 - 4}$

b.) $f(x) = \dfrac{x}{x^2 - x - 2}$
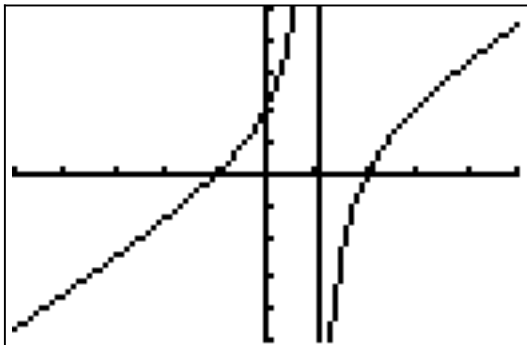
c.) $f(x) = \dfrac{x^2 - x - 2}{x - 1}$

a)

b)

c)

Guidelines for Graphing Rational Functions

example   a.) $f(x) = \dfrac{2(x^2 - 9)}{x^2 - 4}$

1.   Find and plot the x-intercepts. (Set numerator = 0 and solve for x).

$2(x^2 - 9) = 0$

$x^2 - 9 = 0$

$x^2 = 9$

$x = \pm 3$

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2.   Find and plot the y-intercepts. (Let x = 0 and solve for y)

$f(0) = \dfrac{2(0^2 - 9)}{0^2 - 4} \quad = \quad \dfrac{9}{2}$

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3.   Find and plot the Vertical Asymptotes. (Set denominator = 0 and solve for x)

$x^2 - 4$

$x^2 = 4$

$x = \pm 2$

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4.   Find and plot the Horizontal Asymptotes. (Top heavy, Bottom heavy or Same)

$f(x) = \dfrac{2(x^2 - 9)}{x^2 - 4}$ Rule 2   Numerator and denominator have the <u>same degree.</u>

$y = 2$ H.A.

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5.   Find and plot the Slant Asymptotes. (Divide numerator by denominator.)

None !   Only have these if Numerator is exactly 1 degree higher than denominator!

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6.   Find where the graph will intersect its nonvertical asymptote by solving f(x) = k,   where k is the y-value of the horizontal asymptote, or f(x) = mx + b, where y = mx + b is the equation of the oblique asymptote.

Solve $2 = \dfrac{2(x^2 - 9)}{x^2 - 4}$ (No solution!)   No oblique asymptotes.

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7.   Plot at least one point between and beyond each x-intercept and vertical asymptotes.

Remember Test Points ?

Choose test points carefully!

x = -4	x = -2.5	x = 0	x = 2.5	x = 4			x = -1	x = 1	
y = 1.16	y = -2.4	y = 4.5	y = -2.4	y = 1.16			y = 5.3	y = 5.3	

Note: YOU STILL MAY HAVE TO PLOT ADDITIONAL POINTS !

Use smooth curves to complete the graph between and beyond the vertical asymptotes.

Example   Sketch the graph and provide information about intercepts and asymptotes.

$$f(x) = \frac{x}{x^2 - x - 2}$$

1.   Find and plot the x-intercepts. (Set numerator = 0 and solve for x)

$x = 0$

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2.   Find and plot the y-intercepts. (Let x = 0 and solve for y)

$$f(0) = \frac{0}{0^2 - 0 - 2} = 0$$

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3.   Find and plot the Vertical Asymptotes. (Set denominator = 0 and solve for x)

$x^2 - x - 2 = 0$
$(x + 1)(x - 2) = 0$
$x = -1$ and $x = 2$

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4.   Find and plot the Horizontal Asymptotes. (Top heavy, Bottom heavy or Same)

(Rule 1) $y = 0$

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5.   Find and plot the Slant Asymptotes. (Divide numerator by denominator.)
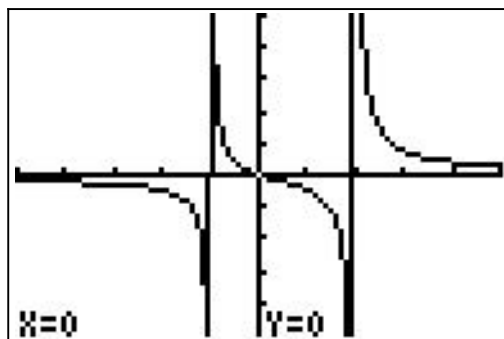None

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6.   Plot at least one point between and beyond each x-intercept and vertical asymptotes.

choose:

x = -2	x = -.5	x = 1	x = 3
y = -.5	y = .4	y = -.5	y = .75

Note: YOU MAY WANT TO PICK MORE POINTS TO GET A BETTER GRAPH !

ANSWER:

Example   Sketch the graph and provide information about intercepts and asymptotes.

$$f(x) = \frac{x^2 - x - 2}{x - 1}$$

1.   Find and plot the x-intercepts. (Set numerator = 0 and solve for x)

$x^2 - x - 2 = 0$
$(x + 1)(x - 2) = 0$
$x = -1$ and $x = 2$

2.   Find and plot the y-intercepts. (Let x = 0 and solve for y)

$$f(0) = \frac{0^2 - 0 - 2}{0 - 1} = \frac{-2}{-1} = 2$$

3.   Find and plot the Vertical Asymptotes. (Set denominator = 0 and solve for x)

$(x - 1) = 0$ $x = 1$

4.   Find and plot the Horizontal Asymptotes. (Top heavy, Bottom heavy or Same)

(Rule 3) Top Heavy   none !

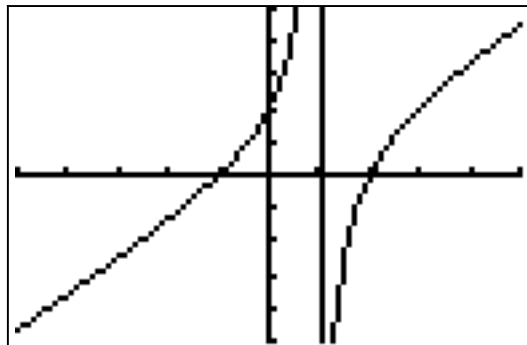5.   Find and plot the Slant Asymptotes. (Divide numerator by denominator.)

$x - 1 \overline{)x^2 - x - 2}$ $y = x$
   $\underline{-x^2 + x}$
        0

6.   Plot at least one point between and beyond each x-intercept and vertical asymptotes.

choose:

x = -2	x = 0	x = 1.5	x = 3
y = -1.3	y = 2	y = -2.5	y = 2

Note: YOU MAY WANT TO PICK MORE POINTS TO GET A BETTER GRAPH !

ANSWER: