§6.2 One-to-One Functions; Inverse Functions

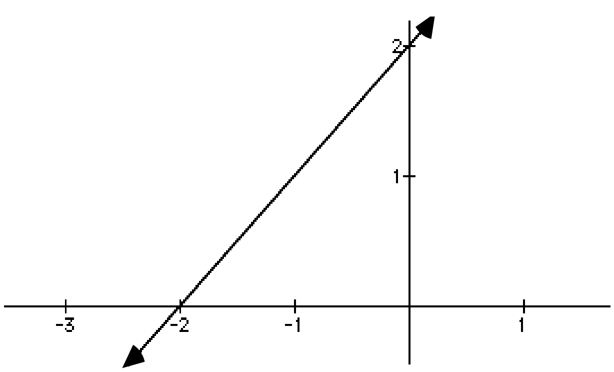
Example: Let $f(x) = 8x$ and $g(x) = \frac{1}{8}x$

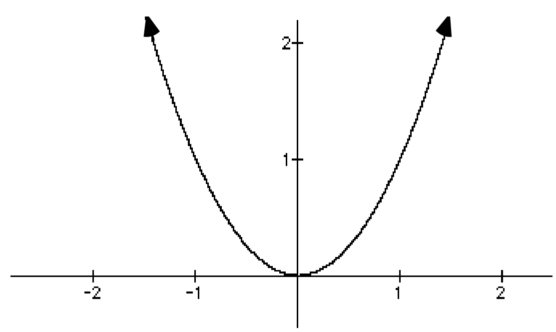
Find f(12) and g(96) ? What do you notice about these results?

Horizontal Line Test:

A function f has a inverse function if and only if no horizontal line intersects the graph of f at more than one point.

Example: Do the following graphs of functions have inverses ?





Inverse Function (Verifying)

Let f and g be two functions such that:

$(f \circ g)(x) = x$ for every x in the domain of g,

<u>and</u> $(g \circ f)(x) = x$ for every x in the domain of f.

The function g is the **inverse** of the function f and is denoted by $f^{-1}(x)$ where

$$f(f^{-1}(x)) = x \text{ and } f^{-1}(f(x)) = x.$$

Example: Let $f(x) = x^3 - 1$, and let $g(x) = \sqrt[3]{x+1}$.

Is g the inverse of f ?

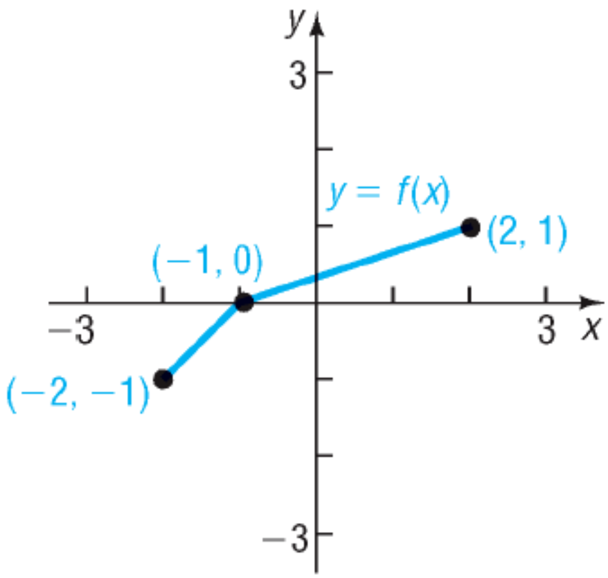
What's the inverse of a function defined by a set of ordered pairs?

Find the inverse of:

$\{(-3, -27), (-2, -8), (-1, -1), (2, 8), (3, 27)\}$

Graphs of Inverses:

(A graph and it's inverse are symmetric with respect to the line y = x.)



Finding the Inverse of a function: Note: the notation used is: $f^{-1}(x)$

(1) Replace f(x) with y.
(2) Interchange the variables x and y.
(3) Solve for y and let this "new" $y = f^{-1}(x)$
(4) Verify that $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.

<u>Example</u> Find the inverse of the following functions.

a.) $f(x) = 2x - 1$

b.) $f(x) = \dfrac{4x+6}{5}$