§6.5 Properties of Logarithms

Properties of Logarithms:
(also true for natural logarithms)

1) $\log_a 1 = 0$ because $a^0 = 1$

2) $\log_a a = 1$ because $a^1 = a$

3) $\log_a a^x = x$ because $a^x = a^x$

4) $\log_a x = \log_a y$, then $x = y$

Example : Solve for x.

a) $\log_2 x = \log_2 3$ b) $\log_4 4 = x$ c) $\log_2 \frac{1}{8} = x$

Example : Rewrite using Properties of Natural Logarithms

a) $\ln \frac{1}{e}$ b) $\ln e^3$ c) $\ln e^0$

Properties of Logarithms:

For any positive real numbers x and y, real number r, and any positive real number a, $(a \neq 1)$:

Product Rule a) $\log_a xy = \log_a x + \log_a y$

Quotient Rule b) $\log_a \frac{x}{y} = \log_a x - \log_a y$

Power Rule c) $\log_a x^r = r \log_a x$

Example : Rewrite the logarithm in terms of ln 2 and ln 3.

a) $\ln 6$

b) $\ln \frac{2}{27}$

Example : Rewrite using the properties of logarithms.

a) $\log_{10} 5x^3 y$

b) $\ln \frac{\sqrt{3x-5}}{7}$

Example : Rewrite in condensed form.

a) $\log_a 7 + 4\log_a 3$

b) $\frac{2}{3}\ln 8 - \ln\left(3^4 - 8\right)$

Change of Base Formula :

Let a, b and x be positive real numbers such that $a \neq 1$ and $b \neq 1$. Then

$$\log_a x = \frac{\log_b x}{\log_b a} \qquad \left(\log_a x = \frac{\log_{10} x}{\log_{10} a} \quad \text{or} \quad \log_a x = \frac{\ln x}{\ln a} \right)$$

Example : Changing Bases Using Common Logarithms & Natural Logarithm

a) $\log_4 30$ b) $\log_2 14$ c) $\log_4 30$ d) $\log_2 14$