§6.6 Logarithmic and Exponential Equations

Properties of Exponential and Logarithmic Functions:

For $a > 0$ and $a \neq 1$:

1) $a^x = a^y$ if and only if $x = y$.
2) If $x > 0$ and $y > 0$, $\log_a x = \log_a y$ if and only if $x = y$.

Example: Solve using the properties above.

a) $2^x = 16$

b) $\log_5(x+1) = \log_5(10)$

(**Note**: remember that the domain of $y = \log_b x$ is $(0, \infty)$. For this reason it is always necessary to check that the solution of a logarithmic equation results in logarithms of positive numbers in the original equation.)

Solving Exponential and Logarithmic Equations (TYPE 2)

An exponential or logarithmic equation may be solved by changing the equation into one of the following **FORMS**, where a and b are real numbers, $a > 0$, and $a \neq 1$.

1) $a^{f(x)} = b$ Solve by taking the logarithms of each side. (Natural logarithms are often a good choice.)

2) $\log_a f(x) = \log_a g(x)$ From the given equation, $f(x) = g(x)$, which is solved algebraically.

3) $\log_a f(x) = b$ Solve by using the definition of logarithm to write the expression in exponential form as $f(x) = a^b$.

Examples: Solve the following equations for x.

a) $4^x = 72$

d) $\log_4 x = 3$

b) $4e^{2x} = 5$

e) $5 + 2\ln x = 4$

c) $e^{2x} - 3e^{x} + 2 = 0$

f) $\log 5x + \log(x - 1) = 2$