§6.7 Compound Interest

Simple Interest Formula

If a principle of P dollars is borrowed for a period of t years at a per annum interest rate r, expressed as a decimal, the interest I charged is **$I = Prt$**

Formulas for Compound Interest:

After **t** years, the balance **A** in an account with principal **P** and annual interest rate **r** (in decimal form) is given by the following formulas:

1. For **n** compoundings per year: $$A = P\left(1 + \frac{r}{n}\right)^{(n \cdot t)}$$

2. For continuous compounding: $$A = Pe^{(r \cdot t)}$$

Example (future value): A total of $12,000 is invested at an annual interest rate of 9%. Find the balance after 5 years if it is compounded:

a) quarterly.

b) continuously.

Compound Interest (rate of interest):

Example : What annual rate of interest compounded annually should you seek if you want to double your investment in 5 years?

Continuous Compounding:

Example : How long will it take for the money in an account that is compounded continuously at 5% to double ? Triple?

§6.7 Compound Interest

Simple Interest Formula

If a principle of P dollars is borrowed for a period of t years at a per annum interest rate r, expressed as a decimal, the interest I charged is **$I = Prt$**

$P = 1000 \quad r = .08 \quad t = 5 \quad I =$

Formulas for Compound Interest:

After **t** years, the balance **A** in an account with principal **P** and annual interest rate **r** (in decimal form) is given by the following formulas:

1. For **n** compoundings per year: $$A = P\left(1 + \frac{r}{n}\right)^{(n \cdot t)}$$

2. For continuous compounding: $$A = Pe^{(r \cdot t)}$$

Example (future value): A total of $12,000 is invested at an annual interest rate of 9%. Find the balance after 5 years if it is compounded:

a) quarterly. $n = 4$

$A =$
$P =$
$n =$
$t =$
$r =$

$A = P\left(1 + \frac{r}{n}\right)^{(n \cdot t)}$

$= 12000\left(1 + \frac{.09}{4}\right)^{(4 \cdot 5)}$

$= 18726.11$

b) continuously.

$A = Pe^{(r \cdot t)}$

$= 12000e^{\wedge(.09 \cdot 5)}$

$= 18819.75$

Compound Interest (rate of interest):

Example : What annual rate of interest compounded annually should you seek if you want to double your investment in 5 years?

$A = P\left(1+\frac{r}{n}\right)^{(n\cdot t)}$

$\frac{2P}{P} = \frac{P\left(1+\frac{r}{1}\right)^{(1\cdot 5)}}{P}$

$2 = (1+r)^5$

$\sqrt[5]{2} = \sqrt[5]{(1+r)^5}$

$\sqrt[5]{2} = 1+r$

$r = \sqrt[5]{2} - 1$

Continuous Compounding:

Example : How long will it take for the money in an account that is compounded continuously at 5% to double ? Triple?

$A = Pe^{(r\cdot t)}$

$\frac{2P}{P} = \frac{Pe^{(r\cdot t)}}{P}$

$2 = e^{(.05t)}$

$\ln_e 2 = \ln_e e^{.05t}$

$\frac{.05t}{.05} = \frac{\ln 2}{.05}$

$t = 13.86$