# §5.1 Polynomial Functions and Models

A polynomial function is a function of the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0$$

where $a_n, a_{n-1}, \ldots, a_1, a_0$ are real numbers and n is a nonnegative integer.

Determine which of the following are polynomial functions. For those that are, state the degree; for those that are not, tell why not.

(a) $f(x) = 2 - 3x^4$

(b) $g(x) = \sqrt{x}$

(c) $h(x) = \dfrac{x^2 - 2}{x^3 - 1}$

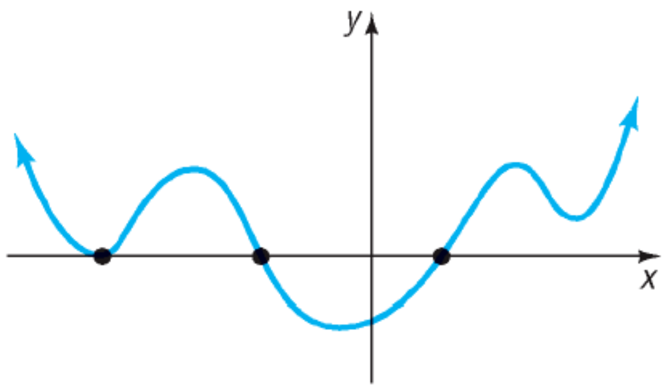
(d) $F(x) = 0$

(e) $G(x) = 8$

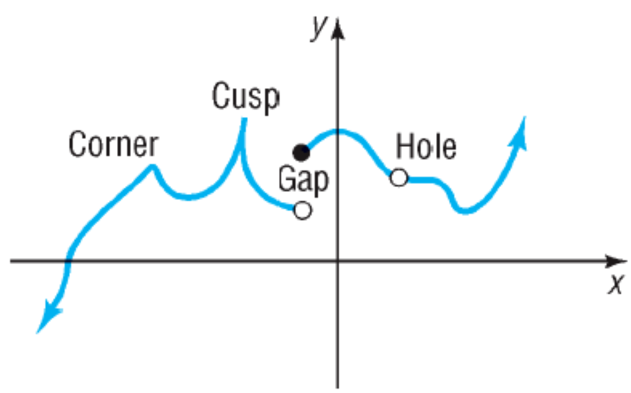
(f) $H(x) = -2x^3(x - 1)^2$

# Summary of the Properties of the Graphs of Polynomial Functions

| Degree | Form | Name | Graph |
|---|---|---|---|
| No degree | $f(x) = 0$ | Zero function | The $x$-axis |
| 0 | $f(x) = a_0, \quad a_0 \neq 0$ | Constant function | Horizontal line with $y$-intercept $a_0$ |
| 1 | $f(x) = a_1x + a_0, \quad a_1 \neq 0$ | Linear function | Nonvertical, nonhorizontal line with slope $a_1$ and $y$-intercept $a_0$ |
| 2 | $f(x) = a_2x^2 + a_1x + a_0, \quad a_2 \neq 0$ | Quadratic function | Parabola: Graph opens up if $a_2 > 0$; graph opens down if $a_2 < 0$ |



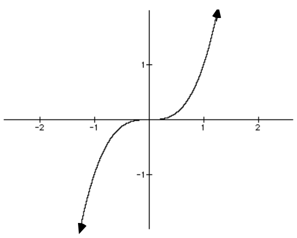
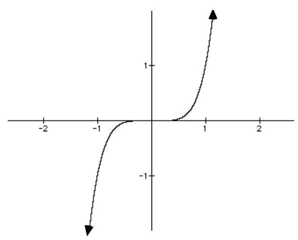
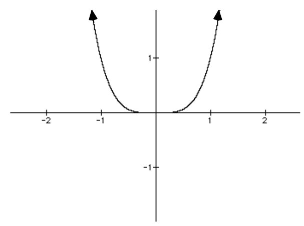
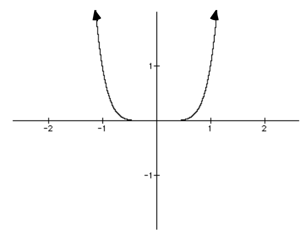
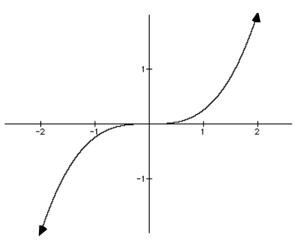
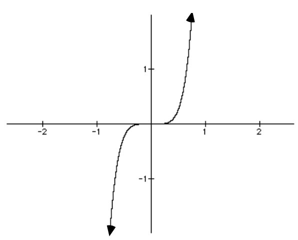
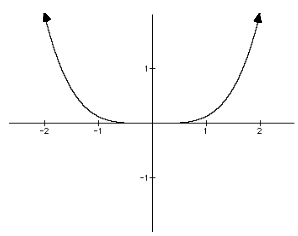
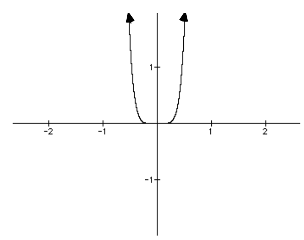

**(a)** Graph of a polynomial function: smooth, continuous




**(b)** Cannot be the graph of a polynomial function

Polynomials are **<u>continuous</u>** (no breaks in the graph) and **<u>smooth</u>** (no sharp angles, only rounded curves)

## Graphing Functions of the Form: $P(x) = ax^n$

| $P(x) = x^3$ | $P(x) = x^5$ | $P(x) = x^4$ | $P(x) = x^6$ |
|---|---|---|---|
|  |  |  |  |
|  |  |  |  |
| $P(x) = \frac{1}{3}x^3$ | $P(x) = 8x^5$ | $P(x) = \frac{1}{8}x^4$ | $P(x) = 9x^6$ |

<u>Note</u>: The graph of $y = x^n$ is similar to the graph of

$$\begin{cases} y = x^2 \text{ if n is even} \\ y = x^3 \text{ if n is odd} \end{cases}$$, except that the greater n is, the flatter

the graph is on [-1, 1] and the steeper it is on

$(-\infty,-1)\cup(1,\infty)$.

## Examining Vertical and Horizontal Translations (Shifts):

Example 1: Graph

a.) $y = -(x + 2)^4 + 6$

b.) $y = -3 - (x - 1)^3$

## Finding a polynomial from its Zeros:

Example Find a polynomial of degree 3 whose zeros are -4, -2, and 3.

## Identifying Zeros and Their Multiplicities

For the polynomial, list all zeros and their multiplicities.

$$f(x) = -2(x-2)(x+1)^3(x-3)^4$$

| **If $r$ Is a Zero of Even Multiplicity** | |
|---|---|
| Sign of $f(x)$ does not change from one side of $r$ to the other side of $r$. | Graph **touches** $x$-axis at $r$. |

| **If $r$ Is a Zero of Odd Multiplicity** | |
|---|---|
| Sign of $f(x)$ changes from one side of $r$ to the other side of $r$. | Graph **crosses** $x$-axis at $r$. |

## Theorem

**Turning Points**

If $f$ is a polynomial function of degree $n$, then $f$ has at most $n - 1$ turning points.

If the graph of a polynomial function $f$ has $n - 1$ turning points, the degree of $f$ is at least $n$.

# **Example** Graphing a Polynomial using x-intercepts

For the polynomial: $f(x) = x^2(x - 2)$

(a) Find the $x$- and $y$-intercepts of the graph of $f$.

(b) Use the $x$-intercepts to find the intervals on which the graph of $f$ is above the $x$-axis and the intervals on which the graph of $f$ is below the $x$-axis.

(c) Locate other points on the graph and connect all the points plotted with a smooth, continuous curve.

| | | | |
|---|---|---|---|
| **Interval** | $(-\infty, 0)$ | $(0, 2)$ | $(2, \infty)$ |
| **Number Chosen** | $-1$ | $1$ | $3$ |
| **Value of $f$** | $f(-1) = -3$ | $f(1) = -1$ | $f(3) = 9$ |
| **Location of Graph** | Below $x$-axis | Below $x$-axis | Above $x$-axis |
| **Point on Graph** | $(-1, -3)$ | $(1, -1)$ | $(3, 9)$ |