§5.2 Properties of Rational Functions

Rational Function - a function of the form

$$f(x) = \frac{p(x)}{q(x)} \quad \text{where p(x) and q(x) are polynomials}$$

with $q(x) \neq 0$.

Find the Domain of a Rational Function

The domain is the set of all real numbers where the denominator $\neq 0$.

(a) The domain of $R(x) = \dfrac{2x^2 - 4}{x + 5}$ is the set of all real numbers x except -5; that is, $\{x \mid x \neq -5\}$.

(b) The domain of $R(x) = \dfrac{1}{x^2 - 4}$ is the set of all real numbers x except -2 and 2, that is, $\{x \mid x \neq -2, x \neq 2\}$.

(c) The domain of $R(x) = \dfrac{x^3}{x^2 + 1}$ is the set of all real numbers.

(d) The domain of $R(x) = \dfrac{-x^2 + 2}{3}$ is the set of all real numbers.

(e) The domain of $R(x) = \dfrac{x^2 - 1}{x - 1}$ is the set of all real numbers x except 1, that is, $\{x \mid x \neq 1\}$. ◀
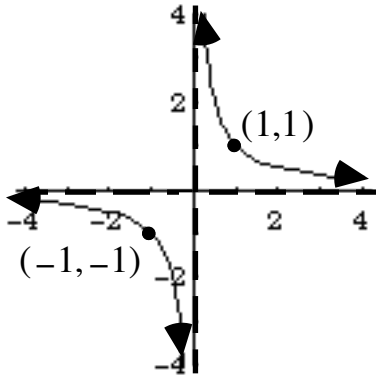
It is important to observe that the functions

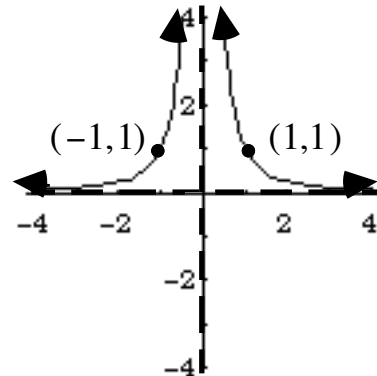$$R(x) = \frac{x^2 - 1}{x - 1} \quad \text{and} \quad f(x) = x + 1$$

are not equal, since the domain of R is $\{x \mid x \neq 1\}$ and the domain of f is the set of all real numbers.

The graphs of rational functions approach (get closer and closer to) lines called **asymptotes**.

Basic Graphs (memorize these)

$$y = \frac{1}{x}$$

$$y = \frac{1}{x^2}$$

Example 1   Use stretching/shrinking, reflecting and shifting rules to graph the following.

a.) $f(x) = \dfrac{1}{x+3} - 1$

b.) $f(x) = \dfrac{1}{(x-2)^2} + 1$

To Find the Asymptotes of a Rational Function:

(1) <u>Vertical Asymptotes</u> – When the function is in lowest terms you can find any vertical asymptotes by setting the denominator equal to 0 and solving for x to get the equation $\boxed{x = a}$.

(2) <u>Horizontal Asymptotes</u>

   Rule 1: If the numerator has lower degree than the denominator, the horizontal asymptote is $\boxed{y = 0}$.
   Rule 2: If the numerator and denominator have the <u>same degree</u> and a_n is the leading coefficient of the numerator and b_n is the leading coefficient of the denominator, the horizontal asymptote is $\boxed{y = \dfrac{a_n}{b_n}}$.

   Rule 3: If the numerator has higher degree than the denominator, there is $\boxed{\textbf{no horizontal asymptote}}$.

(3) <u>Slant Asymptotes</u> - If the numerator is of degree exactly one more than the denominator, there is an slant asymptote. To find it, <u>divide the numerator by the denominator and disregard any remainder</u>. The equation of the slant asymptote is the result of setting y = to the quotient.

Example 2 Give the equations of the vertical, horizontal and/or slant asymptotes of the rational function.

a.) $f(x) = \dfrac{3x}{(x+1)(x-2)}$

b.) $f(x) = \dfrac{(x-5)(x-2)}{x^2+9}$

c.) $f(x) = \dfrac{2x^2+3}{x-4}$

d.) $f(x) = \dfrac{2(3x-1)(x+4)}{(x+2)(5x-3)}$

Example 3 Find the x-intercepts and y-intercept of the rational function.

a.) $f(x) = \dfrac{3x}{(x+1)(x-2)}$

b.) $f(x) = \dfrac{(x-5)(x-2)}{x^2+9}$