

Math 1113 Sample Test 1 Solutions

1) (C) polynomial degree 6

2) not a polynomial (B)
negative power of x

3) shift right 6 and up 9
(A)

4) $x = -2$ $x = 2$ $x = 8$

$$(x+2)(x-2)(x-8)$$

5) $x = -4$

$x = 1$

$x = 1$

$$(x+4)(x-1)^2$$

6) (C) ok

(e) ok

(f) ok

7) $(x-1)(x+6) = 0$

$$x \neq 1 \quad x \neq -6$$

8) a) Domain $x \neq -2$	Range $y \neq -4$	9) a) Domain $x \neq 2$ $x \neq -2$	Range $y < -2$ $y \geq -1$
b) $x_{int} = 0$ $y_{int} = 0$		b) $x_{int} = 1$ $x = -1$	e) none
c) $y = -4$ H.A.		$y_{int} = -1$	
d) $x = -2$ V.A.		c) H.A. $y = -2$	
e) none		d) $x = -2$ $x = 2$ V.A.	

10) shift right 2 and up 2 (D)

11) V.A. $x + 6 = 0$ $x = -6$	H.A. $y = \frac{2}{1} = 2$	O.A. none
$R(x) = \frac{2x}{x+6}$		

12) V.A. $x^2 - 8x + 15 = 0$ $(x-3)(x-5) = 0$ $x = 3$ $x = 5$	H.A. top heavy none S.A. $y = x + 8$	Solution to S.A. on the last page of this document
$R(x) = \frac{x^3 - 27}{x^2 - 8x + 15}$		

13) $T(x) = \frac{x^3}{x^4 - 81}$ S.A. None	V.A. $x^4 - 81 = 0$ $(x^2 - 9)(x^2 + 9) = 0$ $(x-3)(x+3)(x^2 + 9) = 0$ $x = 3$ $x = -3$	H.A. bottom heavy $y = 0$
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$$14) R(x) = \frac{x+11}{x(x+16)}$$

a) $x \neq 0$
 $x \neq -16$ (C)

c) H.A. $y=0$
 S.A. none

b) V.A.
 $x=0$ $x=-16$

d) (C)

$$15) R(x) = \frac{10x+10}{3x+9}$$

a) Domain
 $x \neq -3$

c) H.A. $y = \frac{10}{3}$
 S.A. none

b) V.A. $x = -3$

d) (A)

$$16) R(x) = \frac{x^4 + x^2 + 10}{x^2 - 64}$$

a) Domain $x \neq 8$
 $x \neq -8$

c) H.A. top heavy
 none

S.A. none

b) V.A.

$x=8$ $x=-8$

d) (A)

$$17) R(x) = \frac{8}{(x-5)(x^2-64)}$$

a) Domain

$$x \neq 5, x \neq 8, x \neq -8$$

c) H.o.b. bottom.

heavy

$$y=0$$

S.A, none

b) V.A. $x=5$

$$x=8$$

$$x=-8$$

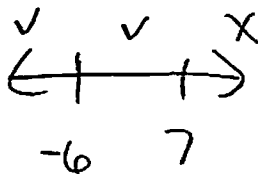
d) (C)

18) Solve

$$(x+6)^2(x-7) < 0$$

$$x+6=0 \quad x-7=0$$

$$x=-6 \quad x=7$$



test pts

$$x=-5$$

$$x=0$$

$$x=8$$

$$y=-12$$

$$y=-252$$

$$y=196$$

(yes)

(yes)

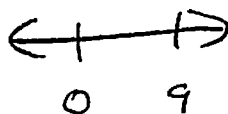
(no)

$$(-\infty, -6) \cup (-6, 7)$$

$$19) x^3 - 9x^2 < 0$$

$$x^2(x-9) < 0$$

$$x=0 \quad x=9$$



$$(-\infty, 0) \cup (0, 9)$$

(D)

test pts

$$x=-1$$

$$x=1$$

$$x=10$$

$$y=-10$$

$$y=-8$$

$$y=100$$

(yes)

(yes)

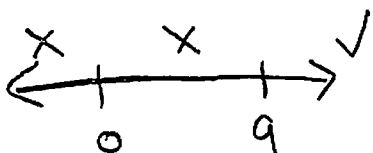
(no)

$$20) 4x^3 > 36x^2$$

$$4x^3 - 36x^2 > 0$$

$$4x^2(x-9) > 0$$

$$x=0 \quad x=9$$

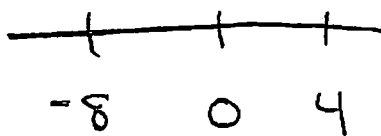


test pts

$x = -1$	$x = 1$	$x = 10$
$y = -40$	$y = -32$	$y = 400$
(no)	(no)	(yes)

$$(9, \infty)$$

$$21) \frac{(x-4)(x+8)}{x} \leq 0$$



$$x-4=0 \quad x+8=0$$

$$\underline{x=4} \quad \underline{x=-8} \quad \underline{x=0}$$

test pts

$x = -9$	$x = -7$	$x = 1$	$x = 5$
$y = -1.4$	$y = 1.5$	$y = -27$	$y = 26$
(yes)	(no)	(yes)	(no)

$$(-\infty, -8] \cup (0, 4] \quad \text{D}$$

$$22) \frac{x+17}{x-4} \geq 1$$

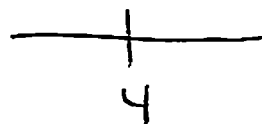
$$\frac{x+17}{x-4} - 1 \geq 0$$

$$\frac{x+17}{x-4} - \frac{x-4}{x-4} \geq 0$$

$$\frac{x+17-x+4}{x-4} \geq 0$$

$$\frac{21}{x-4} \geq 0$$

$x=4$ critical #



test pts

$x = 3$	$x = 5$
$y = -21$	$y = 21$
(no)	(yes)

$$(4, \infty) \quad \text{D}$$

$$23) \quad \frac{6}{x-8} < \frac{7}{6x-7}$$

$$\frac{6}{x-8} - \frac{7}{6x-7} < 0$$

$$\text{LCD } (x-8)(6x-7)$$

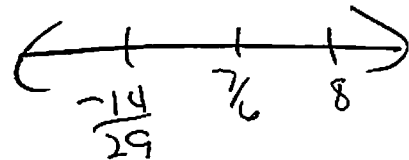
$$\frac{6(6x-7)}{(x-8)(6x-7)} - \frac{7(x-8)}{(6x-7)(x-8)}$$

$$\frac{36x-42-7x+56}{(x-8)(6x-7)} < 0$$

$$\frac{29x+14}{(x-8)(6x-7)} < 0$$

Critical Numbers

$$\begin{array}{l|l|l} 29x+14 \leq 0 & x-8 \geq 0 & 6x-7 \geq 0 \\ x = -\frac{14}{29} & x = 8 & x = \frac{7}{6} \end{array}$$



test pts

$x = -1$	$x = 0$	$x = 2$	$x = 9$
$y = -1.2$	$y = .25$	$y = -2.4$	$y = 5.8$
(yes)	(no)	(yes)	(no)

$$\left(-\infty, -\frac{14}{29}\right) \cup \left(\frac{7}{6}, 8\right) \quad \text{C}$$

$$24) \quad f(x) = 3x^3 - 2x^2 - 4x - 6$$

$$x-2 \quad \underline{k=2}$$

$$\begin{aligned} f(2) &= 3(2)^3 - 2(2)^2 - 4(2) - 6 \\ &= 24 - 8 - 8 - 6 \\ &= 2 \end{aligned}$$

$x-2$ a factor?

(NO) $\neq 0$

25) (7) max zeros

$$f(x) = -3x^7 + x^3 - x^2 + 3$$

(3 or 1) + real zeros

$$f(-x) = 3x^7 - x^3 - x^2 + 3$$

(2 or 0) \rightarrow real zeros

$$26) f(x) = 6x^4 + 8x^2 - 2$$

max zeros (4)

+ real zeros (1) sign change

- real zeros

$$f(-x) = 6x^4 + 8x^2 - 2$$

(1) sign change

$$27) f(x) = 2x^4 - 3x^3 + x^2 - x + 1$$

$$p = 1 \rightarrow \pm 1$$

$$q = 2 \rightarrow \pm 1, \pm 2$$

$$\frac{p}{q} = \pm 1, \pm \frac{1}{2} \quad (C)$$

$$28) f(x) = 33x^4 - x^2 + 121 \quad (C)$$

$$p = 121 \rightarrow \pm 1, \pm 11, \pm 121$$

$$q = 33 \rightarrow \pm 1, \pm 3, \pm 11, \pm 33$$

$$\frac{p}{q} = \pm 1, \pm \frac{1}{3}, \pm \frac{1}{11}, \pm 11, \pm \frac{11}{3}, \pm 121, \pm \frac{121}{3}, \pm \frac{1}{33}$$

$$29) f(x) = x^3 + 5x^2 - 17x - 21$$

$$P = 21 \Rightarrow \pm 1, \pm 3, \pm 7, \pm 21$$

$$Q = 1 \Rightarrow \pm 1$$

$$\begin{array}{r|rrrr} -1 & 1 & 5 & -17 & -21 \\ & & -1 & -4 & 21 \\ \hline & 1 & 4 & -21 & 0 \end{array}$$

$$x^2 + 4x - 21$$

$$(x + 7)(x - 3)$$

+ real o's ①

- real o's

$f(-x) = -, +, --$ ② or ③

$$\frac{P}{Q} = \pm 1, \pm 3, \pm 7, \pm 21$$

Zeros

$$x = -1$$

$$x = -7$$

$$x = 3$$

Factors

$$(x + 1)(x + 7)(x - 3)$$

$$30) f(x) = 3x^4 + 2x^3 - 7x^2 - 4x + 2$$

$$P = 2 \Rightarrow \pm 1, \pm 2$$

$$Q = 3 \Rightarrow \pm 1, \pm 3$$

+ real o's ② or ③

- real o's $f(-x) = +, -, -, +, +$

② or ③

$$\frac{P}{Q} = \pm 1, \pm \frac{1}{3}, \pm 2, \pm \frac{2}{3}$$

$$\begin{array}{r|rrrrr} -1 & 3 & 2 & -7 & -4 & 2 \\ & & -3 & 1 & 6 & -2 \\ \hline & 3 & -1 & -6 & 2 & 0 \end{array}$$

$$\begin{array}{r|rrrrr} \frac{1}{3} & 3 & -1 & -6 & 2 & 0 \\ & & 1 & 0 & -2 & 0 \\ \hline & 3 & 0 & -6 & 0 & 0 \end{array}$$

$$3x^2 - 6 = 0$$

$$3x^2 = 6$$

$$x^2 = 2$$

$$x = \pm\sqrt{2}$$

Zeros

$$x = -1$$

$$x = \frac{1}{3}$$

$$x = \sqrt{2}$$

$$x = -\sqrt{2}$$

Factors

$$(x + 1)\left(x - \frac{1}{3}\right)(x - \sqrt{2})(x + \sqrt{2})$$

$$31) f(x) = 8x^4 - 3x^2 + 4x - 1 \quad [0, 3]$$

$$\left. \begin{aligned} f(0) &= 8(0)^4 - 3(0)^2 + 4(0) - 1 = -1 \text{ negative} \\ f(3) &= 8(3)^4 - 3(3)^2 + 4(3) - 1 = 632 \text{ positive} \end{aligned} \right\} \begin{array}{l} \text{Sign} \\ \text{change!} \\ \text{so } \boxed{\text{yes}}! \end{array}$$

32) degree 3

$$x = 7 \quad x = -9 - i \quad \text{or} \quad \boxed{x = -9 + i} \text{ also zero}$$

33) degree 6

$$x = 3 \quad x = -9 \quad x = -1 + i \quad x = -2 - i$$

$$\text{or} \quad \boxed{x = -1 - i} \quad \boxed{x = -2 + i}$$

$$34) f(x) = x^3 - 7x^2 + 25x - 175$$

$$\boxed{x = -5i} \quad \text{or} \quad \boxed{x = +5i} \text{ other complex zero.}$$

$$\text{then } (x+5i)(x-5i) = x^2 + 25 \text{ is a factor}$$

$$\begin{array}{r} x^2 + 0x + 25 \overline{) x^3 - 7x^2 + 25x - 175} \\ \underline{\ominus x^3 \oplus 0x^2 \oplus 25x} \\ -7x^2 - x - 175 \\ \underline{\oplus 7x^2 \oplus 0x \oplus 175} \\ 0 \end{array}$$

$$x - 7 = 0$$

$$\boxed{x = 7}$$

$$\boxed{x = +5i}$$

+ real o's
③ or ①

- real o's
f(-x) =

- - - -
none

$$35) h(x) = 3x^4 + 10x^3 + 4x^2 + 40x - 32$$

$$\boxed{x = -2i} \quad \text{or} \quad \boxed{x = +2i} \quad \text{also zero!}$$

$$(x+2i)(x-2i) = x^2 + 4 \quad \text{is a factor}$$

$$\begin{array}{r}
 \overline{) 3x^4 + 10x^3 + 4x^2 + 40x - 32} \\
 \underline{\ominus 3x^4 + 10x^3 + 12x^2} \\
 10x^3 - 8x^2 + 40x \\
 \underline{\ominus 10x^3 + 10x^2 + 40x} \\
 -8x^2 + 0x - 32 \\
 \underline{\oplus 8x^2 + 0x + 32} \\
 \oplus 0
 \end{array}$$

+ real o's ①

- real o's

$$f(-x) = +, -, +, -, -$$

③ or ①

$$3x^2 + 10x - 8 = 0$$

$$(3x-2)(x+4) = 0$$

$$3x-2=0 \quad | \quad x+4=0$$

$$\boxed{x = \frac{2}{3}} \quad | \quad \boxed{x = -4}$$

$$\boxed{x = +2i}$$

$$\boxed{x = -2i}$$

$$36) h(x) = x^4 - 12x^3 + 40x^2 + 16x - 240$$

degree 4

$$\boxed{x = 4-2i} \quad \text{so} \quad \boxed{x = 4+2i}$$

~~then~~ x

$$\begin{array}{r}
 4-2i \overline{) 1 \quad -12 \quad 40 \quad 16 \quad -240} \\
 \underline{4-2i \quad -36+8i \quad 32+24i \quad 240} \\
 1 \quad -8-2i \quad 4+8i \quad 48+24i \quad \text{⊙}
 \end{array}$$

$$\begin{array}{r}
 4+2i \overline{) 1 \quad -8-2i \quad 4+8i \quad 48+24i} \\
 \underline{4+2i \quad -16-8i \quad -48-24i} \\
 1 \quad -4 \quad -12 \quad 0
 \end{array}$$

$$x^2 - 4x - 12 = 0$$

$$(x-6)(x+2) = 0$$

$$x-6=0 \quad | \quad x+2=0$$

$$\boxed{x = 6} \quad | \quad \boxed{x = -2}$$

+ real o's

③ or ①

- real o's

$$f(-x) = +, +, +, -$$

①

$$37) f(x) = x^4 + 12x^3 - 9x^2 + 48x - 52$$

$$p = 52 \Rightarrow \pm 1, \pm 2, \pm 4, \pm 13, \pm 26, \pm 52$$

$$q = 1 \Rightarrow \pm 1$$

$$\frac{p}{q} = \pm 1, \pm 2, \pm 4, \pm 13, \pm 26, \pm 52$$

Descartes Rule

+ need 0 or (3 sign changes)
 (3) or (1) ✓

- need 0 or

$f(-x) = +, -, -, -, -$

(1) ✓

$$\begin{array}{r|rrrrr} \textcircled{1} & 1 & 12 & -9 & 48 & -52 \\ & & 1 & 13 & 4 & 52 \\ \hline & 1 & 13 & 4 & 52 & \textcircled{0} \end{array}$$

$$\begin{array}{r|rrrr} \textcircled{-13} & 1 & 13 & 4 & 52 \\ & & -13 & 0 & -52 \\ \hline & 1 & 0 & 4 & \textcircled{0} \end{array}$$

$$x^2 + 4 = 0$$

$$x^2 = -4$$

$$x = \pm 2i$$

zeros

$$x = 1 \quad x = -13 \quad x = 2i \quad x = -2i$$

factors

$$(x-1)(x+13)(x-2i)(x+2i)$$

12)

$$X^2 - 8x + 15 \quad \begin{array}{r} X + 8 \\ \hline X^3 + 0x^2 + 0x - 27 \end{array}$$

$$\ominus X^3 \oplus 8x^2 \ominus 15x$$

$$y = x + 8$$

$$8x^2 - 15x - 27$$

$$\ominus 8x^2 \oplus 64x \ominus 120$$

$$+ 49x - 147$$

$$\frac{X^3 - 27}{X^2 - 8x + 15} = \frac{X^3 - 3^3}{(X-3)(X-5)} = \frac{\cancel{(X-3)}(X^2 + 3x + 9)}{\cancel{(X-3)}(X-5)}$$

∴ 3 is NOT a Vertical Asymptote!

ONLY 5 is a V.A.