

§7.3 Properties of the Trigonometric Functions

Domain(Range) and Period of Sine and Cosine

Domain of the Sine and Cosine is all real numbers:

$$-\infty < t < \infty$$

Range of Sine	$-1 \leq y \leq 1$
	$-1 \leq \sin t \leq 1$

Range of Cosine	$-1 \leq x \leq 1$
	$-1 \leq \cos t \leq 1$

Definition of a periodic function

A function f is **periodic** if there exists a positive real number c such that

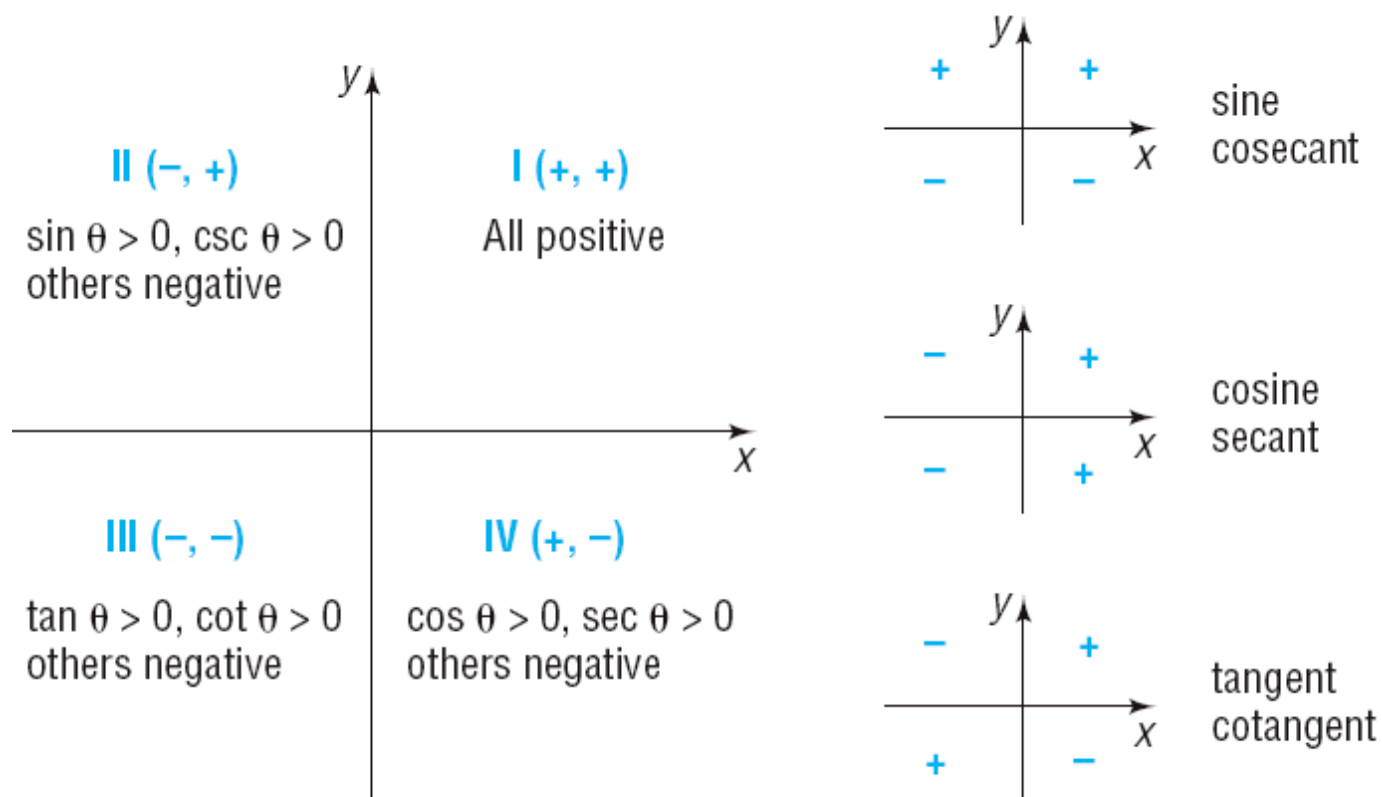
$$f(t + c) = f(t)$$

for all t in the domain of f . The smallest number c for which f is periodic is called the **period** of f .

Examples: Find

a) $\sin \frac{17\pi}{4}$ b) $\cos 5\pi$

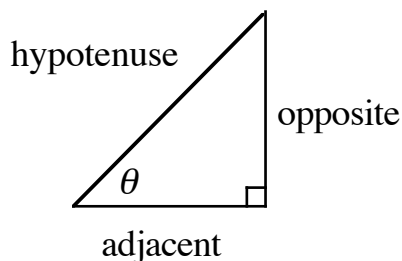
Determine the signs of the Trig Functions in a Given Quadrant



If $\sin \theta < 0$ and $\cos \theta < 0$, name the quadrant in which the angle lies.

Find the Values of the Trig Functions Using Fundamental Identities

Right Triangle Trigonometry



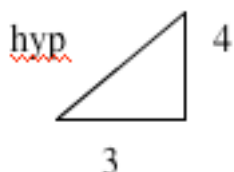
Right Triangle Definitions of Trigonometric Functions

Let θ be an acute angle of a right triangle. Then:

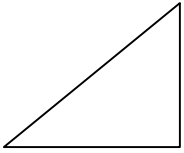
$$\sin \theta = \frac{\text{opp}}{\text{hyp}} \quad \cos \theta = \frac{\text{adj}}{\text{hyp}} \quad \tan \theta = \frac{\text{opp}}{\text{adj}}$$

$$\csc \theta = \frac{\text{hyp}}{\text{opp}} \quad \sec \theta = \frac{\text{hyp}}{\text{adj}} \quad \cot \theta = \frac{\text{adj}}{\text{opp}}$$

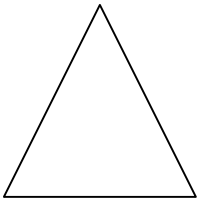
Example Evaluate the six trig functions for this triangle:



Example : Find the values of $\sin 45^\circ$, $\cos 45^\circ$, and $\tan 45^\circ$ using right triangle trigonometry.



Example : Find the values of $\sin 30^\circ$, $\cos 30^\circ$, $\sin 60^\circ$, and $\cos 60^\circ$ using right triangle trigonometry.



Trigonometric Identities

Reciprocal Identities

$$\sin \theta = \frac{1}{\csc \theta} \quad \cos \theta = \frac{1}{\sec \theta} \quad \tan \theta = \frac{1}{\cot \theta}$$

$$\csc \theta = \frac{1}{\sin \theta} \quad \sec \theta = \frac{1}{\cos \theta} \quad \cot \theta = \frac{1}{\tan \theta}$$

Quotient or Ratio Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1 \quad \tan^2 \theta + 1 = \sec^2 \theta$$
$$1 + \cot^2 \theta = \csc^2 \theta$$

Example: Find the exact values of the remaining four trig functions of θ using identities.

$$\sin \theta = \frac{\sqrt{5}}{5} \quad \text{and} \quad \cos \theta = \frac{2\sqrt{5}}{5}$$

Example : Given that $\sin \theta = \frac{1}{3}$ and $\cos \theta < 0$, find the exact value of each of the remaining five trig functions of θ .

Even and Odd Trigonometric Functions

The cosine and secant functions are **even**.

$$\cos(-t) = \cos t \qquad \sec(-t) = \sec t$$

The sine, cosecant, tangent, and cotangent functions are **odd**.

$$\sin(-t) = -\sin(t) \qquad \csc(-t) = -\csc(t)$$

$$\tan(-t) = -\tan(t) \qquad \cot(-t) = -\cot(t)$$

Find the exact value of:

a) $\sin(-45^\circ)$ b) $\cos(-\pi)$