§8.3 Trigonometric Identities

Fundamental Trigonometric Identities

Reciprocal Identities

$$\sin\theta = \frac{1}{\csc\theta} \qquad \cos\theta = \frac{1}{\sec\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Quotient or Ratio Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad\qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1 \qquad \tan^2\theta + 1 = \sec^2\theta \qquad 1 + \cot^2\theta = \csc^2\theta$$

Cofunction Identities

$$\sin(90° - \theta) = \cos\theta \qquad \cos(90° - \theta) = \sin\theta$$
$$\tan(90° - \theta) = \cot\theta$$

$$\csc(90° - \theta) = \sec\theta \qquad \sec(90° - \theta) = \csc\theta \qquad \cot(90° - \theta) = \tan\theta$$

Even and Odd Trigonometric Functions

The cosine and secant functions are **even**.

$$\cos(-t) = \cos t \qquad \sec(-t) = \sec t$$

The sine, cosecant, tangent, and cotangent functions are **odd**.

$$\sin(-t) = -\sin(t) \qquad \csc(-t) = -\csc(t)$$

$$\tan(-t) = -\tan(t) \qquad \cot(-t) = -\cot(t)$$

Example 1   Simplify : $\dfrac{\cot\theta}{\csc\theta}$

Example 2   Simplify: $\dfrac{\cos\theta}{1+\sin\theta}$

Example 3   Simplify: $\dfrac{1+\sin u}{\sin u}+\dfrac{\cot u-\cos u}{\cos u}$

Example 4   Simplify: $\dfrac{\sin^2\theta - 1}{\tan\theta\sin\theta - \tan\theta}$

Establish (prove)Trigonometric Identities

- an <u>identity</u> is an equation which is true for all values for which the equation is defined
- to verify an identity, generally work with one side of the equation and show that it equals the other side
- some suggestions to consider when verifying identities:

1.) simplify the more complex side

2.) perform algebraic operations including squaring, factoring, adding or subtracting fractions, multiplying the numerator and denominator by a nonzero factor

3.) rewrite in terms of sine and cosine

4.) rewrite in terms of a single trigonometric function

5.) use other identities (reciprocal identities, ratio identities, Pythagorean identities)

Examples Verify each identity.

a.) $\csc\theta \cdot \tan\theta = \sec\theta$

b.) $\sin^2(-\theta) + \cos^2(-\theta) = 1$

c.) $\dfrac{\sin^2(-\theta) - \cos^2(-\theta)}{\sin(-\theta) - \cos(-\theta)} = \cos\theta - \sin\theta$

d.) $\dfrac{1 + \tan u}{1 + \cot u} = \tan u$

e.) $\dfrac{\sin \theta}{1 + \cos \theta} + \dfrac{1 + \cos \theta}{\sin \theta} = 2 \csc \theta$

f.) $\dfrac{\tan v + \cot v}{\sec v \csc v} = 1$