

§8.6 Product-to-Sum & Sum-to-Product Formulas

REMEMBER YOU KNOW ALGEBRA !

Product to Sum Identities
$\sin \alpha \cos \beta = \frac{1}{2} [\sin(\alpha + \beta) + \sin(\alpha - \beta)]$
$\cos \alpha \cos \beta = \frac{1}{2} [\cos(\alpha + \beta) + \cos(\alpha - \beta)]$
$\sin \alpha \sin \beta = \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)]$
$\cos \alpha \sin \beta = \frac{1}{2} [\sin(\alpha + \beta) - \sin(\alpha - \beta)]$

Example 1 Rewrite as a sum or difference.

a) $\sin(6\theta)\sin(4\theta)$

b) $\sin(3\theta)\cos(5\theta)$

Sum-to-Product Identities

$$\sin x + \sin y = 2 \sin\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right)$$

$$\sin x - \sin y = 2 \cos\left(\frac{x+y}{2}\right) \sin\left(\frac{x-y}{2}\right)$$

$$\cos x + \cos y = 2 \cos\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right)$$

$$\cos x - \cos y = -2 \sin\left(\frac{x+y}{2}\right) \sin\left(\frac{x-y}{2}\right)$$

Example 2 Find the exact value of $\cos 195^\circ + \cos 105^\circ$

Example 3 Express as a product: $\cos(3\theta) + \cos(2\theta)$