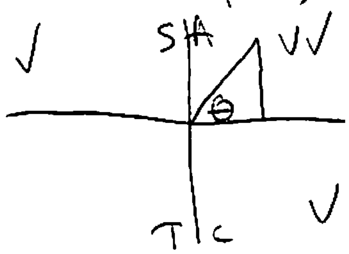
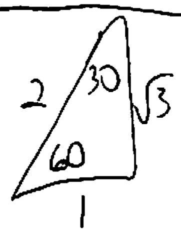
Math 1113 Sample Test 3 Solutions

1) Find exact value in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \boxed{60^\circ \text{ or } \frac{\pi}{3}}$$

$\sin\theta = \frac{\sqrt{3}}{2}$?



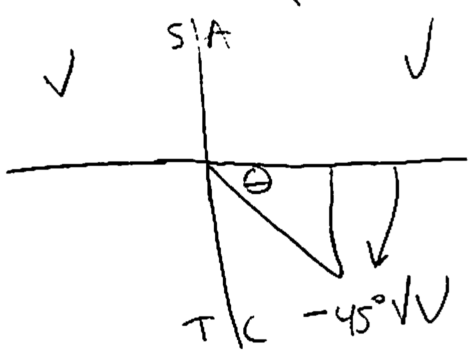


2) Use calculator find to 2 decimal places

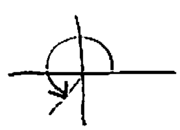
$$\cos^{-1}\left(\frac{2}{3}\right) = .84 \text{ radians}$$

3) Find exact value.

$$\sin^{-1}\left(\sin\frac{5\pi}{4}\right) = \sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \boxed{-45^\circ \text{ or } -\frac{\pi}{4}}$$

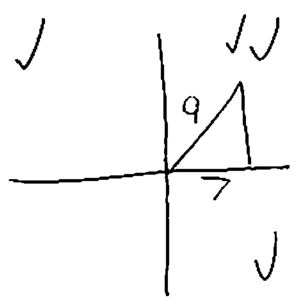


$$\sin\left(\frac{5\pi}{4}\right) = \sin(225^\circ)$$
$$= -\frac{\sqrt{2}}{2}$$



4) Find the exact value.

$$\cos\left(\cos^{-1}\left(\frac{7}{9}\right)\right) = \frac{7}{9}$$



5) Find the exact value.

$$\csc\left[\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)\right]$$

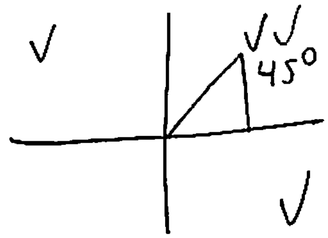
$$\csc(45^\circ) = \frac{1}{\sin 45^\circ}$$

$$= \frac{1}{\frac{\sqrt{2}}{2}} = \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{2}$$

$$= \boxed{\sqrt{2}}$$

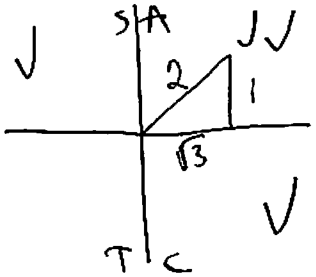
$$\sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = 45^\circ$$

$$\sin\theta = \frac{\sqrt{2}}{2}$$



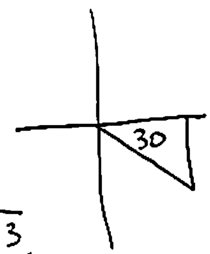
6) Find the exact value.

$$\cos^{-1}\left(\cos\frac{11\pi}{6}\right) = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$



$$\boxed{30^\circ \text{ or } \frac{\pi}{6}}$$

$$\cos\left(\frac{11\pi}{6}\right)$$



$$\cos(330^\circ) = {}^{+}\frac{\sqrt{3}}{2}$$

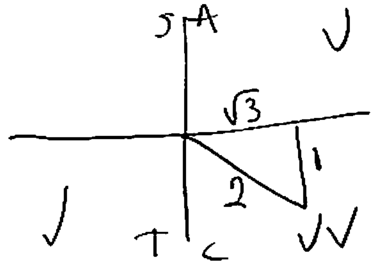
7) $$\tan\left[\sin^{-1}\left(-\frac{1}{2}\right)\right]$$

$$\tan\left(-\frac{\pi}{6}\right) = \frac{-1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \boxed{-\frac{\sqrt{3}}{3}}$$

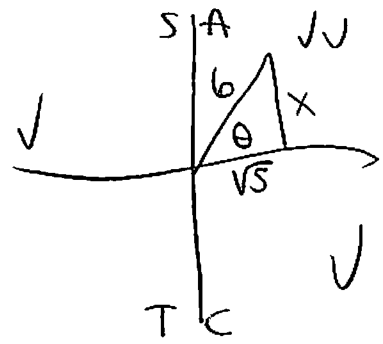
$$\sin^{-1}\left(\frac{-1}{2}\right) =$$

$$\sin\theta = \frac{-1}{2}$$



8) $\cot\left[\cos^{-1}\left(\frac{\sqrt{5}}{6}\right)\right]$

$\cos^{-1}\left(\frac{\sqrt{5}}{6}\right)$



$$6^2 = x^2 + (\sqrt{5})^2$$
$$36 = x^2 + 5$$
$$x^2 = 31$$
$$x = \sqrt{31}$$

$$\cot = \frac{\text{adj}}{\text{opp}}$$
$$= \frac{\sqrt{5}}{\sqrt{31}}$$
$$= \frac{\sqrt{5}}{\sqrt{31}} \cdot \frac{\sqrt{31}}{\sqrt{31}}$$
$$= +\frac{\sqrt{155}}{31}$$

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9) $$\frac{\sin\theta}{1-\cos\theta} \cdot \frac{(1+\cos\theta)}{(1+\cos\theta)} = \frac{\sin\theta(1+\cos\theta)}{1-\cos\theta+\cos\theta-\cos^2\theta}$$

$$= \frac{\sin\theta(1+\cos\theta)}{1-\cos^2\theta} = \frac{\cancel{\sin\theta}(1+\cos\theta)}{\cancel{\sin^2\theta}} = \boxed{\frac{1+\cos\theta}{\sin\theta}}$$

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10) $$\frac{(\sin\theta+\cos\theta)(\sin\theta+\cos\theta)-1}{\sin\theta\cos\theta}$$

$$= \frac{\overbrace{\sin^2\theta+\sin\theta\cos\theta+\sin\theta\cos\theta+\cos^2\theta}^{1}-1}{\sin\theta\cos\theta}$$

$$= \frac{\overset{0}{1-1}+2\cancel{\sin\theta}\cancel{\cos\theta}}{\cancel{\sin\theta}\cancel{\cos\theta}} = 2$$

11) $(\tan\theta + \cot\theta)\sin\theta = \sec(\theta)$ (prove)

$$= \left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)\sin\theta$$

LCD = $\sin\theta\cos\theta$

$$= \left(\frac{\sin\theta}{\cos\theta}\cdot\frac{\sin\theta}{\sin\theta} + \frac{\cos\theta}{\sin\theta}\cdot\frac{\cos\theta}{\cos\theta}\right)\sin\theta$$

$$= \left(\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}\right)\sin\theta$$

$$= \left(\frac{1}{\cancel{\sin\theta}\cos\theta}\right)\frac{\cancel{\sin\theta}}{1} = \frac{1}{\cos\theta} = \sec\theta!$$ $\boxed{\text{Choice C}}$

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12) $(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

prove

FOIL

$$\sec^2\theta - \cancel{\sec\theta\tan\theta} + \cancel{\sec\theta\tan\theta} - \tan^2\theta$$

$$\sec^2\theta - \tan^2\theta = 1$$

$$1 = 1$$

Choice D

from Identity

$\tan^2\theta + 1 = \sec^2\theta$

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13) $\frac{2\csc\theta}{\sec\theta} + \frac{3\cos\theta}{\sin\theta} = 5\cot\theta$ Prove

Choice C

$$= \frac{\frac{2}{\sin\theta}}{\frac{1}{\cos\theta}} + \frac{3\cos\theta}{\sin\theta}$$

$$= \frac{2\cos\theta}{\sin\theta} + \frac{3\cos\theta}{\sin\theta} = 2\cot\theta + 3\cot\theta = 5\cot\theta$$

14) $\dfrac{\csc\theta - \sin\theta}{\csc\theta + \sin\theta} = \dfrac{\cos^2\theta}{1+\sin^2\theta}$ choice D

LCD = $\sin\theta$

$$= \frac{\left(\frac{1}{\sin\theta} - \frac{\sin\theta}{1}\right)\sin\theta}{\left(\frac{1}{\sin\theta} + \frac{\sin\theta}{1}\right)\sin\theta} = \frac{1-\sin^2\theta}{1+\sin^2\theta} = \frac{\cos^2\theta}{1+\sin^2\theta}$$

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use sum or difference formulas.

15) $\sin 75^\circ$

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\alpha = 30^\circ$

$\beta = 45^\circ$

$= \sin(30^\circ + 45^\circ)$

$= \sin(30^\circ)\cos(45^\circ) + \cos 30^\circ \sin 45^\circ$

$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \frac{\sqrt{2}+\sqrt{6}}{4}$ or $\frac{1}{4}(\sqrt{6}+\sqrt{2})$

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16) Find exact value

$\sin 40^\circ \cos 5^\circ + \cos 40^\circ \sin 5^\circ = \sin(40^\circ + 5^\circ) = \sin 45^\circ = \frac{\sqrt{2}}{2}$

$\sin\alpha\cos\beta + \cos\alpha\sin\beta = \sin(\alpha+\beta)$

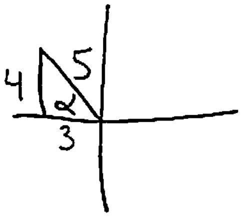
$\alpha = 40^\circ \quad \beta = 5^\circ$

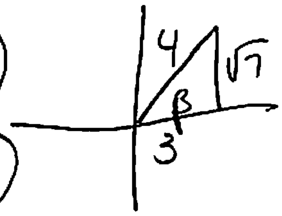
17) Find the exact value given:

$\tan\alpha = \frac{-4}{3}$ $\quad \frac{\pi}{2} < \alpha < \pi$ $\qquad \cos\beta = \frac{3}{4}$ $\quad 0 < \beta < \frac{\pi}{2}$

$\sin\alpha = +\frac{4}{5}$ $\qquad \sin\beta = +\frac{\sqrt{7}}{4}$

$\cos\alpha = -\frac{3}{5}$ $\qquad \tan\beta = +\frac{\sqrt{7}}{3}$





$4^2 = 3^2 + x^2$

$x^2 = 7$

$x = \sqrt{7}$

a) $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$= \left(\frac{4}{5}\right)\left(\frac{3}{4}\right) + \left(\frac{-3}{5}\right)\left(\frac{\sqrt{7}}{4}\right)$

$= \frac{12}{20} - \frac{3\sqrt{7}}{20} = \boxed{\frac{12-3\sqrt{7}}{20}}$

b) $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$= \left(\frac{-3}{5}\right)\left(\frac{3}{4}\right) - \left(\frac{4}{5}\right)\left(\frac{\sqrt{7}}{4}\right)$

$= \frac{-9}{20} - \frac{4\sqrt{7}}{20} = \boxed{\frac{-9-4\sqrt{7}}{20}}$

c) $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$= \left(\frac{4}{5}\right)\left(\frac{3}{4}\right) - \left(\frac{-3}{5}\right)\left(\frac{\sqrt{7}}{4}\right)$

$= \frac{12}{20} + \frac{3\sqrt{7}}{20} = \boxed{\frac{12+3\sqrt{7}}{20}}$

d) $\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta} = \frac{\frac{-4}{3} - \frac{\sqrt{7}}{3}}{1+\left(\frac{-4}{3}\right)\left(\frac{\sqrt{7}}{3}\right)}$

$= \frac{\frac{-4-\sqrt{7}}{3}}{\frac{9}{9} - \frac{4\sqrt{7}}{9}} = \frac{-4-\sqrt{7}}{\cancel{3}} \cdot \frac{\overset{3}{\cancel{9}}}{9-4\sqrt{7}}$

$= \boxed{\frac{-12-3\sqrt{7}}{9-4\sqrt{7}}} = \boxed{\frac{-(12+3\sqrt{7})}{(9-4\sqrt{7})}}$

18) prove

$$\cos\left(\frac{3\pi}{2}+\theta\right)=\sin\theta$$

$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$

$$\cos\left(\frac{3\pi}{2}+\theta\right)=\cos\frac{3\pi}{2}\cos\theta-\sin\frac{3\pi}{2}\sin\theta$$

$$=(0)(\cos\theta)-(-1)\sin\theta$$

$$=0+\sin\theta$$

$$=\sin\theta$$

choice D

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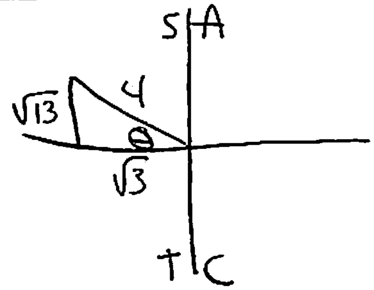
19) $\cos\theta=\frac{-\sqrt{3}}{4}$ $\quad \frac{\pi}{2}<\theta<\pi$

$\frac{\pi}{4}<\frac{\theta}{2}<\frac{\pi}{2}$



$4^2=(\sqrt{3})^2+x^2$

$x^2=13$

$x=\sqrt{13}$

$\sin\theta=\frac{\sqrt{13}}{4}$

$\tan\theta=\frac{-\sqrt{13}}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{-\sqrt{39}}{3}$

a) $\sin(2\theta)=2\sin\theta\cos\theta$

$$=2\left(\frac{\sqrt{13}}{4}\right)\left(-\frac{\sqrt{3}}{4}\right)$$

$$=\frac{2}{1}\cdot\frac{-\sqrt{39}}{16}=\frac{-\sqrt{39}}{8}$$

b) $\cos(2\theta)=2\cos^2\theta-1$

$$=2\left(\frac{\sqrt{3}}{4}\right)^2-1$$

$$=2\left(\frac{3}{16}\right)-1$$

$$=\frac{3}{8}-\frac{8}{8}=\frac{-5}{8}$$

c) $\sin\left(\frac{\theta}{2}\right)=\pm\sqrt{\frac{1-\cos\theta}{2}}=+\sqrt{\frac{1-\frac{-\sqrt{3}}{4}}{2}}$

$$=+\sqrt{\frac{\frac{4}{4}+\frac{\sqrt{3}}{4}}{2}}=+\sqrt{\frac{4+\sqrt{3}}{4}\cdot\frac{1}{2}}=+\sqrt{\frac{4+\sqrt{3}}{2}\cdot\frac{1}{4}}$$

$$=\boxed{\frac{1}{2}\,{}^{+}\!\sqrt{\frac{4+\sqrt{3}}{2}}}$$

d) $\cos\frac{\theta}{2}=\pm\sqrt{\frac{1+\cos\alpha}{2}}$

$$=+\sqrt{\frac{1-\frac{\sqrt{3}}{4}}{2}}=+\sqrt{\frac{\frac{4-\sqrt{3}}{4}}{2}}$$

$$=+\sqrt{\frac{4-\sqrt{3}}{2}\cdot\frac{1}{4}}=\boxed{\frac{1}{2}\,{}^{+}\!\sqrt{\frac{4-\sqrt{3}}{2}}}$$

20) use half angle formula to find

$\sin 112.5^\circ \qquad \frac{\alpha}{2} = 112.5 \quad \text{so} \quad \alpha = 225^\circ$

$$\sin \frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$$

$$= \pm\sqrt{\frac{1-\cos 225^\circ}{2}} = +\sqrt{\frac{1-\frac{-\sqrt{2}}{2}}{2}}$$

$$= \sqrt{\frac{\frac{2}{2}+\frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{2+\sqrt{2}}{2}\cdot\frac{1}{2}} = \sqrt{\frac{2+\sqrt{2}}{4}} = \frac{\sqrt{2+\sqrt{2}}}{2}$$

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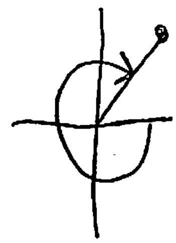
21) Use half angle to find

$\tan\left(-\frac{7\pi}{8}\right) \qquad \frac{\alpha}{2} = -\frac{7\pi}{8} \Rightarrow \alpha = -\frac{7\pi}{4} = -315^\circ$



$$\tan\left(\frac{\alpha}{2}\right) = \frac{1-\cos\alpha}{\sin\alpha} = \frac{1-\cos(-315^\circ)}{\sin(-315^\circ)} =$$

$$= \frac{1-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \frac{\frac{2-\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \frac{2-\sqrt{2}}{\cancel{2}}\cdot\frac{\cancel{2}}{\sqrt{2}} = \frac{2-\sqrt{2}}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}-2}{2}$$

$$= \frac{\cancel{2}(\sqrt{2}-1)}{\cancel{2}} = \boxed{\sqrt{2}-1}$$

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22) Prove!

$$2\sin^3\theta\cos\theta + 2\sin\theta\cos^3\theta = \sin 2\theta$$

GCF $2\sin\theta\cos\theta\left(\sin^2\theta + \cos^2\theta\right) =$

$2\sin\theta\cos\theta\,(1) = \sin(2\theta)$ FORMULA

Choice B

23) Express as a sum

$\cos(7\theta)\cos(9\theta) = \frac{1}{2}[\cos(7\theta+9\theta) + \cos(7\theta-9\theta)]$

$\cos\alpha \cos\beta = \frac{1}{2}[\cos(\alpha+\beta) + \cos(\alpha-\beta)]$

$\alpha = 7\theta$
$\beta = 9\theta$

$= \frac{1}{2}[\cos(16\theta) + \cos(-2\theta)]$

$= \frac{1}{2}[\cos(16\theta) + \cos(2\theta)]$

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24) Express as a product

$\cos\left(\frac{\theta}{2}\right) - \cos\frac{7\theta}{2} = -2\sin\left(\frac{\frac{\theta}{2}+\frac{7\theta}{2}}{2}\right)\sin\left(\frac{\frac{\theta}{2}-\frac{7\theta}{2}}{2}\right)$

Use $\cos X - \cos Y = -2\sin\left(\frac{X+Y}{2}\right)\sin\left(\frac{X-Y}{2}\right)$

$X = \frac{\theta}{2}$
$Y = \frac{7\theta}{2}$

$= -2\sin\left(\frac{\frac{8\theta}{2}}{2}\right)\sin\left(\frac{\frac{-6\theta}{2}}{2}\right)$

$= -2\sin\left(\frac{4\theta}{2}\right)\sin\left(\frac{-3\theta}{2}\right)$

note $\sin\left(\frac{-3\theta}{2}\right) = -\sin\frac{3\theta}{2}$

$= +2\sin(2\theta)\sin\left(\frac{3\theta}{2}\right)$

25) Prove

$$\frac{\cos(\theta)+\cos(3\theta)}{2\cos(2\theta)} = \cos\theta$$

numerator use $\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$

$x=\theta$

$y=3\theta$

$$\frac{2\cos\left(\frac{\theta+3\theta}{2}\right)\cos\left(\frac{\theta-3\theta}{2}\right)}{2\cos(2\theta)} = \frac{2\cos\left(\frac{4\theta}{2}\right)\cos\left(\frac{-2\theta}{2}\right)}{2\cos(2\theta)} = \frac{2\cancel{\cos}(2\theta)\cos(-\theta)}{2\cancel{\cos}2\theta}$$

$$= \cos(-\theta) = \boxed{\cos\theta}$$ $\boxed{\text{choice C}}$

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26) Prove! $\dfrac{\sin(8\theta)+\sin(4\theta)}{\cos(8\theta)+\cos(4\theta)} = \tan(6\theta)$

use $\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$

& $\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$

$$\frac{2\sin\left(\frac{8\theta+4\theta}{2}\right)\cos\left(\frac{8\theta-4\theta}{2}\right)}{2\cos\left(\frac{8\theta+4\theta}{2}\right)\cos\left(\frac{8\theta-4\theta}{2}\right)} = \frac{2\sin\left(\frac{12\theta}{2}\right)\cos\left(\frac{4\theta}{2}\right)}{2\cos\left(\frac{12\theta}{2}\right)\cos\frac{4\theta}{2}}$$

$$= \frac{\cancel{2}\sin(6\theta)\cancel{\cos(2\theta)}}{\cancel{2}\cos(6\theta)\cancel{\cos(2\theta)}} = \frac{\sin(6\theta)}{\cos(6\theta)} = \tan(6\theta)$$

choice A