Student: ____________________	**Instructor:** Keith Barrs	**Assignment:** Sample Test 3
Date: ______________________	**Course:** Math 1113	
Time: ______________________	**Book:** Sullivan: Precalculus, 8e	

1. Find the exact value of the following expression within the interval $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$.

$$\sin^{-1}\frac{\sqrt{3}}{2}$$

$\sin^{-1}\frac{\sqrt{3}}{2} = \square$

(Simplify your answer. Type an exact answer, using π as needed. Use integers or fractions for any numbers in the expression. Type N if there is no solution.)

---

2. Use a calculator to find the value of the following expression rounded to two decimal places.

$$\cos^{-1}\frac{2}{3}$$

$\cos^{-1}\frac{2}{3} = \square$

(Type your answer in radians. Round to the nearest hundredth as needed.)

---

3. Find the exact value of the following expression.

$$\sin^{-1}\left(\sin\frac{5\pi}{4}\right)$$

$\sin^{-1}\left(\sin\frac{5\pi}{4}\right) = \square$

(Simplify your answer. Type an exact answer, using π as needed. Use integers or fractions for any numbers in the expression. Type N if there is no solution.)

---

4. Find the exact value, if any, of the composite function. Do not use a calculator.

$$\cos\left(\cos^{-1}\frac{7}{9}\right)$$

$\cos\left(\cos^{-1}\frac{7}{9}\right) = \square$ (Type N if there is no solution.)

5. Find the exact value of the expression.

$$\csc\left[\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)\right]$$

$\csc\left[\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)\right] = \square$

(Simplify your answer. Type an exact answer, using radicals as needed. Rationalize all denominators.)

---

6. Find the exact value of the expression.

$$\cos^{-1}\left(\cos\frac{11\pi}{6}\right)$$

$\cos^{-1}\left(\cos\frac{11\pi}{6}\right) = \square$

(Simplify your answer. Type an exact answer, using π as needed. Use integers or fractions for any numbers in the expression. Type N if there is no solution.)

---

7. Find the exact value of the expression.

$$\tan\left[\sin^{-1}\left(-\frac{1}{2}\right)\right]$$

$\tan\left[\sin^{-1}\left(-\frac{1}{2}\right)\right] = \square$

(Simplify your answer. Type an exact answer, using radicals as needed. Rationalize all denominators.)

---

8. Find the exact value of the expression.

$$\cot\left[\cos^{-1}\left(\frac{\sqrt{5}}{6}\right)\right]$$

$\cot\left[\cos^{-1}\left(\frac{\sqrt{5}}{6}\right)\right] = \square$

(Simplify your answer. Type an exact answer, using radicals as needed. Rationalize all denominators.)

9. Multiply $\frac{\sin\theta}{1-\cos\theta}$ by $\frac{1+\cos\theta}{1+\cos\theta}$. Type your answer in terms of sine and/or cosine.

$\frac{\sin\theta}{1-\cos\theta}\cdot\frac{1+\cos\theta}{1+\cos\theta}=\square$ (Simplify your answer.)

10. Multiply and simplify $\frac{(\sin\theta+\cos\theta)(\sin\theta+\cos\theta)-1}{\sin\theta\cos\theta}$.

$\frac{(\sin\theta+\cos\theta)(\sin\theta+\cos\theta)-1}{\sin\theta\cos\theta}=\square$

(Use integers or fractions for any numbers in the expression.)

11. Establish the identity.

$(\tan\theta+\cot\theta)\sin\theta=\sec\theta$

Which of the following shows the key steps in establishing the identity?

- A. $(\tan\theta+\cot\theta)\sin\theta=\left(\frac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}\right)\sin\theta=\cot\theta+1=\sec\theta$
- B. $(\tan\theta+\cot\theta)\sin\theta=\sin\theta\tan\theta+\sin\theta\left(\frac{\sec\theta}{\sin\theta}\right)=\cot\theta+1=\sec\theta$
- C. $(\tan\theta+\cot\theta)\sin\theta=\left(\frac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}\right)\sin\theta=\left(\frac{1}{\cos\theta\sin\theta}\right)\sin\theta=\sec\theta$
- D. $(\tan\theta+\cot\theta)\sin\theta=\left(\frac{\sin^2\theta\cos^2\theta}{\cos\theta\sin\theta}\right)\sin\theta=\left(\frac{1}{\cos\theta\sin\theta}\right)\sin\theta=\sec\theta$

12. Establish the identity.

$$(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$$

Which of the following statements establishes the identity?

A. $(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = \tan^2\theta - \sec^2\theta = 1$

B. $(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = \left(\dfrac{1}{\cos\theta} + \dfrac{1}{\cot\theta}\right)\left(\dfrac{1}{\cos\theta} - \dfrac{1}{\cot\theta}\right)$
$= \left(\dfrac{1}{\cos\theta\cot\theta}\right)\left(\dfrac{1}{\cos\theta\cot\theta}\right) = 1$

C. $(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = \sec^2\theta - 2\sec\theta\tan\theta - \tan^2\theta = 1$

D. $(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = \sec^2\theta - \tan^2\theta = 1$

---

13. Establish the identity.

$$\frac{2\csc\theta}{\sec\theta} + \frac{3\cos\theta}{\sin\theta} = 5\cot\theta$$

Which of the following statements establishes the identity?

A. $\dfrac{2\csc\theta}{\sec\theta} + \dfrac{3\cos\theta}{\sin\theta} = \dfrac{2/\cos\theta}{1/\sin\theta} + \dfrac{3\cos\theta}{\sin\theta} = \dfrac{2\sin\theta}{\cos\theta} + \dfrac{3\cos\theta}{\sin\theta} = 2\cot\theta + 3\cot\theta = 5\cot\theta$

B. $\dfrac{2\csc\theta}{\sec\theta} + \dfrac{3\cos\theta}{\sin\theta} = \dfrac{2/\sin\theta}{1/\cos\theta} + \dfrac{3\cos\theta}{\sin\theta} = \dfrac{2\sin\theta}{\cos\theta} + \dfrac{3\cos\theta}{\sin\theta} = 2\cot\theta + 3\cot\theta = 5\cot\theta$

C. $\dfrac{2\csc\theta}{\sec\theta} + \dfrac{3\cos\theta}{\sin\theta} = \dfrac{2/\sin\theta}{1/\cos\theta} + \dfrac{3\cos\theta}{\sin\theta} = \dfrac{2\cos\theta}{\sin\theta} + \dfrac{3\cos\theta}{\sin\theta} = 2\cot\theta + 3\cot\theta = 5\cot\theta$

D. $\dfrac{2\csc\theta}{\sec\theta} + \dfrac{3\cos\theta}{\sin\theta} = \dfrac{2/\cos\theta}{1/\sin\theta} + \dfrac{3\cos\theta}{\sin\theta} = \dfrac{2\cos\theta}{\sin\theta} + \dfrac{3\cos\theta}{\sin\theta} = 2\cot\theta + 3\cot\theta = 5\cot\theta$

14. Establish the identity.

$$\frac{\csc\theta - \sin\theta}{\csc\theta + \sin\theta} = \frac{\cos^2\theta}{1 + \sin^2\theta}$$

Which of the following statements establishes the identity? (A, B, C, or D)

A. $$\frac{\csc\theta - \sin\theta}{\csc\theta + \sin\theta} = \frac{\frac{1}{\sin\theta} - \sin\theta}{\frac{1}{\sin\theta} + \sin\theta} = \frac{\frac{1 - \cos^2\theta}{\cos\theta}}{\frac{1 + \cos^2\theta}{\cos\theta}} = \frac{1 - \sin^2\theta}{1 + \sin^2\theta} = \frac{\cos^2\theta}{1 + \sin^2\theta}$$

B. $$\frac{\csc\theta - \sin\theta}{\csc\theta + \sin\theta} = \frac{\frac{1}{\cos\theta} - \cos\theta}{\frac{1}{\cos\theta} + \cos\theta} = \frac{\frac{1 - \sin^2\theta}{\sin\theta}}{\frac{1 + \sin^2\theta}{\sin\theta}} = \frac{1 - \sin^2\theta}{1 + \sin^2\theta} = \frac{\cos^2\theta}{1 + \sin^2\theta}$$

C. $$\frac{\csc\theta - \sin\theta}{\csc\theta + \sin\theta} = \frac{\frac{1}{\cos\theta} - \cos\theta}{\frac{1}{\cos\theta} + \cos\theta} = \frac{\frac{1 - \cos^2\theta}{\cos\theta}}{\frac{1 + \cos^2\theta}{\cos\theta}} = \frac{1 - \sin^2\theta}{1 + \sin^2\theta} = \frac{\cos^2\theta}{1 + \sin^2\theta}$$

D. $$\frac{\csc\theta - \sin\theta}{\csc\theta + \sin\theta} = \frac{\frac{1}{\sin\theta} - \sin\theta}{\frac{1}{\sin\theta} + \sin\theta} = \frac{\frac{1 - \sin^2\theta}{\sin\theta}}{\frac{1 + \sin^2\theta}{\sin\theta}} = \frac{1 - \sin^2\theta}{1 + \sin^2\theta} = \frac{\cos^2\theta}{1 + \sin^2\theta}$$

---

15. Use a sum or difference formula to find the exact value of the trigonometric function.

$\sin 75°$

$\sin 75° = \square$

(Simplify your answer, including any radicals. Use integers or fractions for any numbers in the expression.)

---

16. Find the exact value of the expression.

$\sin 40° \cos 5° + \cos 40° \sin 5°$

$\sin 40° \cos 5° + \cos 40° \sin 5° = \square$

(Type an exact answer, using radicals as needed.)

17. Find the exact value of each of the following under the given conditions:

$$\tan\alpha = -\frac{4}{3},\ \frac{\pi}{2} < \alpha < \pi;\quad \cos\beta = \frac{3}{4},\ 0 < \beta < \frac{\pi}{2}$$

(a) $\sin(\alpha+\beta)$ (b) $\cos(\alpha+\beta)$ (c) $\sin(\alpha-\beta)$ (d) $\tan(\alpha-\beta)$

(a) $\sin(\alpha+\beta) = \square$

(Type an exact answer using radicals as needed. Use integers or fractions for any numbers in the expression. Simplify your answer.)

(b) $\cos(\alpha+\beta) = \square$

(Type an exact answer using radicals as needed. Use integers or fractions for any numbers in the expression. Simplify your answer.)

(c) $\sin(\alpha-\beta) = \square$

(Type an exact answer using radicals as needed. Use integers or fractions for any numbers in the expression. Simplify your answer.)

(d) $\tan(\alpha-\beta) = \square$

(Type an exact answer using radicals as needed. Use integers or fractions for any numbers in the expression. Do not rationalize the denominator. Simplify your answer.)

18. Establish the identity.

$$\cos\left(\frac{3\pi}{2}+\theta\right) = \sin\theta$$

Choose the sequence of steps below that verifies the identity.

A. $\cos\left(\frac{3\pi}{2}+\theta\right) = \cos\frac{\pi}{2}\cos\theta + \sin\frac{\pi}{2}\sin\theta = (0)\cos\theta + (1)\sin\theta = \sin\theta$

B. $\cos\left(\frac{3\pi}{2}+\theta\right) = \cos\frac{\pi}{2}\cos\theta - \sin\frac{\pi}{2}\sin\theta = (0)\cos\theta - (1)\sin\theta = \sin\theta$

C. $\cos\left(\frac{3\pi}{2}+\theta\right) = \cos\frac{3\pi}{2}\cos\theta + \sin\frac{3\pi}{2}\sin\theta = (0)\cos\theta + (-1)\sin\theta = \sin\theta$

D. $\cos\left(\frac{3\pi}{2}+\theta\right) = \cos\frac{3\pi}{2}\cos\theta - \sin\frac{3\pi}{2}\sin\theta = (0)\cos\theta - (-1)\sin\theta = \sin\theta$

19.

Use the information given about the angle θ, $\cos\theta = -\frac{\sqrt{3}}{4}$, $\frac{\pi}{2} < \theta < \pi$, to find the exact values of the following.

(a) $\sin(2\theta)$ (b) $\cos(2\theta)$ (c) $\sin\frac{\theta}{2}$ (d) $\cos\frac{\theta}{2}$

(a) $\sin(2\theta) = \square$ (Type an exact answer, using radicals as needed.)

(b) $\cos(2\theta) = \square$ (Type an exact answer, using radicals as needed.)

(c) $\sin\frac{\theta}{2} = \square$ (Type an exact answer, using radicals as needed.)

(d) $\cos\frac{\theta}{2} = \square$ (Type an exact answer, using radicals as needed.)

20.

Use the half-angle formulas to find the exact value of the trigonometric function $\sin 112.5°$.

$\sin 112.5° = \square$

(Simplify your answer, including any radicals. Use integers or fractions for any numbers in the expression.)

21.

Use the half-angle formulas to find the exact value of the trigonometric function $\tan\left(-\frac{7\pi}{8}\right)$.

$\tan\left(-\frac{7\pi}{8}\right) = \square$

(Simplify your answer, including any radicals. Use integers or fractions for any numbers in the expression.)

22. Establish the identity.

$$2\sin^3\theta\cos\theta + 2\sin\theta\cos^3\theta = \sin(2\theta)$$

Choose the sequence of steps below that verifies the identity.

- A. $2\sin^3\theta\cos\theta + 2\sin\theta\cos^3\theta = (2\cos^2\theta + \sin^2\theta)(\sin\theta\cos\theta) = 1\cdot\sin(2\theta) = \sin(2\theta)$
- B. $2\sin^3\theta\cos\theta + 2\sin\theta\cos^3\theta = (\cos^2\theta + \sin^2\theta)(2\sin\theta\cos\theta) = 1\cdot\sin(2\theta) = \sin(2\theta)$
- C. $2\sin^3\theta\cos\theta + 2\sin\theta\cos^3\theta = (\cos^2\theta + \sin^2\theta)(2\sin^2\theta + 1) = 1\cdot\sin(2\theta) = \sin(2\theta)$
- D. $2\sin^3\theta\cos\theta + 2\sin\theta\cos^3\theta = (\cos^2\theta - \sin^2\theta)(2\sin\theta\cos\theta) = 1\cdot\sin(2\theta) = \sin(2\theta)$

---

23. Express the given product as a sum containing only sines or cosines.

$$\cos(7\theta)\cos(9\theta)$$

$\cos(7\theta)\cos(9\theta) = \square$

(Simplify your answer, including any radicals. Use integers or fractions for any numbers in the expression.)

---

24. Express the given sum or difference as a product of sines and/or cosines.

$$\cos\frac{\theta}{2} - \cos\frac{7\theta}{2}$$

$\cos\frac{\theta}{2} - \cos\frac{7\theta}{2} = \square$

(Simplify your answer, including any radicals. Use integers or fractions for any numbers in the expression.)

25\.

Establish the identity.

$$\frac{\cos\theta + \cos(3\theta)}{2\cos(2\theta)} = \cos\theta$$

Choose the correct sequence of steps to establish the identity.

A. $\frac{\cos\theta + \cos(3\theta)}{2\cos(2\theta)} = \frac{2\sin(2\theta)\sin\theta}{2\cos(2\theta)} = \cos\theta$

B. $\frac{\cos\theta + \cos(3\theta)}{2\cos(2\theta)} = \frac{2\sin(2\theta)\cos\theta}{2\cos(2\theta)} = \cos\theta$

C. $\frac{\cos\theta + \cos(3\theta)}{2\cos(2\theta)} = \frac{2\cos(2\theta)\cos\theta}{2\cos(2\theta)} = \cos\theta$

D. $\frac{\cos\theta + \cos(3\theta)}{2\cos(2\theta)} = \frac{-2\sin\theta\cos(2\theta)}{2\cos(2\theta)} = \cos\theta$

26\.

Establish the identity $\frac{\sin(8\theta) + \sin(4\theta)}{\cos(8\theta) + \cos(4\theta)} = \tan(6\theta)$.

Which of the following statements establishes the identity?

A. $$\frac{\sin(8\theta) + \sin(4\theta)}{\cos(8\theta) + \cos(4\theta)} = \frac{2\sin\frac{8\theta+4\theta}{2}\cos\frac{8\theta-4\theta}{2}}{2\cos\frac{8\theta+4\theta}{2}\cos\frac{8\theta-4\theta}{2}} = \frac{\sin\frac{8\theta+4\theta}{2}}{\cos\frac{8\theta+4\theta}{2}} = \tan(6\theta)$$

B. $$\frac{\sin(8\theta) + \sin(4\theta)}{\cos(8\theta) + \cos(4\theta)} = \frac{2\sin\frac{8\theta-4\theta}{2}\cos\frac{8\theta+4\theta}{2}}{2\cos\frac{8\theta-4\theta}{2}\cos\frac{8\theta+4\theta}{2}} = \frac{\sin\frac{8\theta-4\theta}{2}}{\cos\frac{8\theta-4\theta}{2}} = \tan(6\theta)$$

C. $$\frac{\sin(8\theta) + \sin(4\theta)}{\cos(8\theta) + \cos(4\theta)} = \frac{2\sin\frac{8\theta-4\theta}{2}\cos\frac{8\theta+4\theta}{2}}{2\cos\frac{8\theta+4\theta}{2}\cos\frac{8\theta-4\theta}{2}} = \frac{\sin\frac{8\theta+4\theta}{2}}{\cos\frac{8\theta+4\theta}{2}} = \tan(6\theta)$$

Student: ____________________	**Instructor:** Keith Barrs	**Assignment:** Sample Test 3
Date: ____________________	**Course:** Math 1113	
Time: ____________________	**Book:** Sullivan: Precalculus, 8e	

1. $\frac{\pi}{3}$

2. 0.84

3. $-\frac{\pi}{4}$

4. $\frac{7}{9}$

5. $\sqrt{2}$

6. $\frac{\pi}{6}$

7. $-\frac{\sqrt{3}}{3}$

8. $\frac{\sqrt{155}}{31}$

9. $\frac{1+\cos\theta}{\sin\theta}$

10. 2

11. C

12. D

13. C

14. D

15. $\frac{1}{4}(\sqrt{6}+\sqrt{2})$

16. $\frac{\sqrt{2}}{2}$

17. $\frac{12-3\sqrt{7}}{20}$
$\frac{-9-4\sqrt{7}}{20}$
$\frac{12+3\sqrt{7}}{20}$
$-\frac{12+3\sqrt{7}}{9-4\sqrt{7}}$

18. D

19. $-\frac{\sqrt{39}}{8}$
$-\frac{5}{8}$
$\frac{1}{2}\sqrt{\frac{4+\sqrt{3}}{2}}$
$\frac{1}{2}\sqrt{\frac{4-\sqrt{3}}{2}}$

20. $\frac{\sqrt{2+\sqrt{2}}}{2}$

21. $\sqrt{2}-1$

22. B

23. $\frac{1}{2}[\cos(2\theta)+\cos(16\theta)]$

24. $2\sin(2\theta)\sin\frac{3\theta}{2}$

25. C

26. A