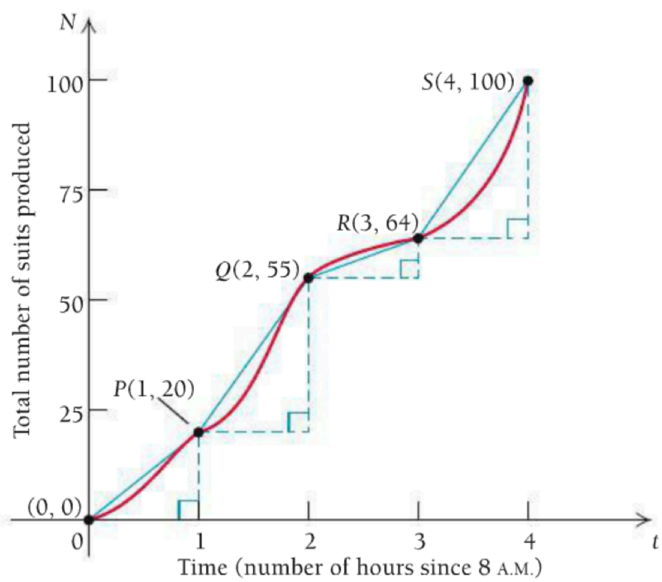
Section 1.3
Average Rate of Change

If a car travels 110 miles in 2 hours, its average rate of change is ______________ . Suppose you accelerate on the highway and glancing at the speedometer, you see that at that instant your ________________rate of change is 55 mph. These are two quite different concepts. Average rate of change is something that we are familiar with. Instantaneous rate of change involves ideas of limits and calculus.

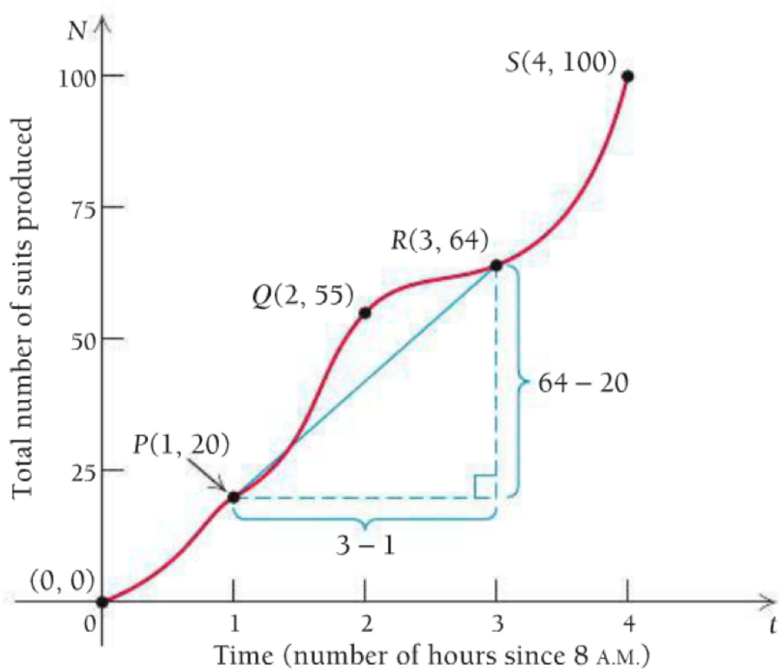


Example 1:

How many suits were produced from 9 AM to 10 AM?

How many suits were produced from 10 AM to 11 AM?

What was the hourly rate of production from 8 AM to 9 AM?



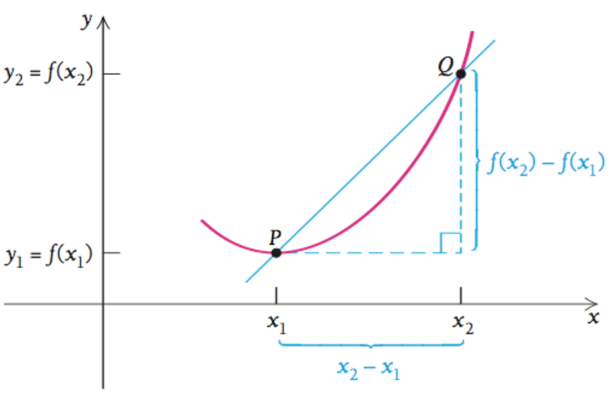
Example 2:

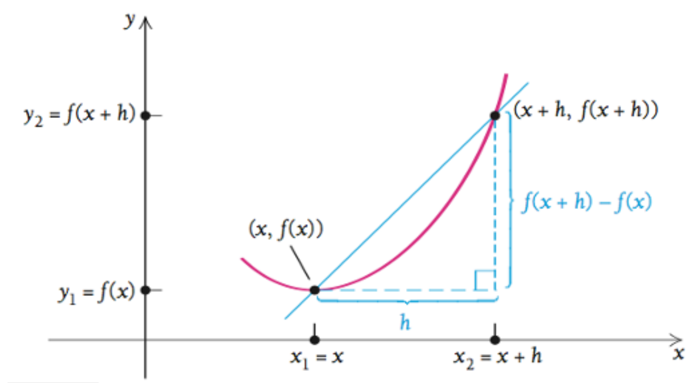
What was the average number of suits produced per hour from 9 AM to 11 AM?

The average rate of change of y with respect to x, as x changes from x_1 to x_2, is

Example 3: For $f(x) = x^2 + 1$, find the average rate of change as x changes from 1 to 3.

Example 4: For $f(x) = x^2 - 5$, find the average rate of change as x changes from 0 to 4.





Use the diagrams to show average rate of change as the different quotient.

Example 5: For $f(x) = x^2$, find the simplified form of the difference quotient. Complete the chart.

x	h	$\frac{f(x+h) - f(x)}{h}$
5	2	
5	1	
5	0.1	
5	0.01	

Section 1.3
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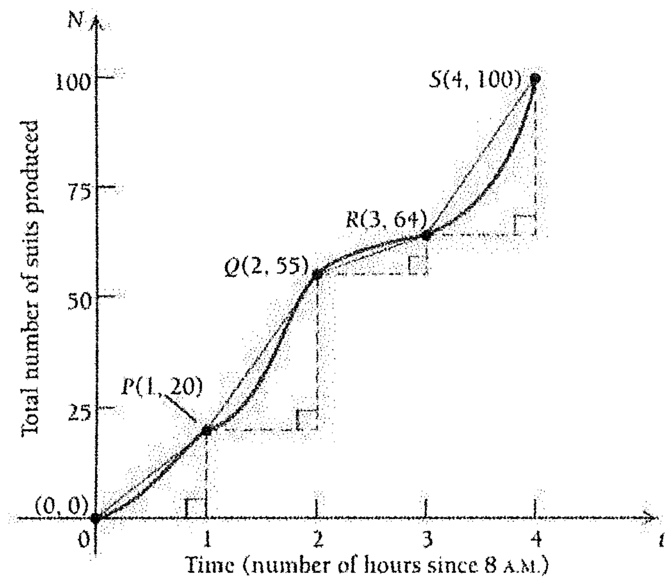
If a car travels 110 miles in 2 hours, its average rate of change is 55 mph. Suppose you accelerate on the highway and glancing at the speedometer, you see that at that instant your instantaneous rate of change is 55 mph. These are two quite different concepts. Average rate of change is something that we are familiar with. Instantaneous rate of change involves ideas of limits and calculus.



Example 1:

How many suits were produced from 9 AM to 10 AM?

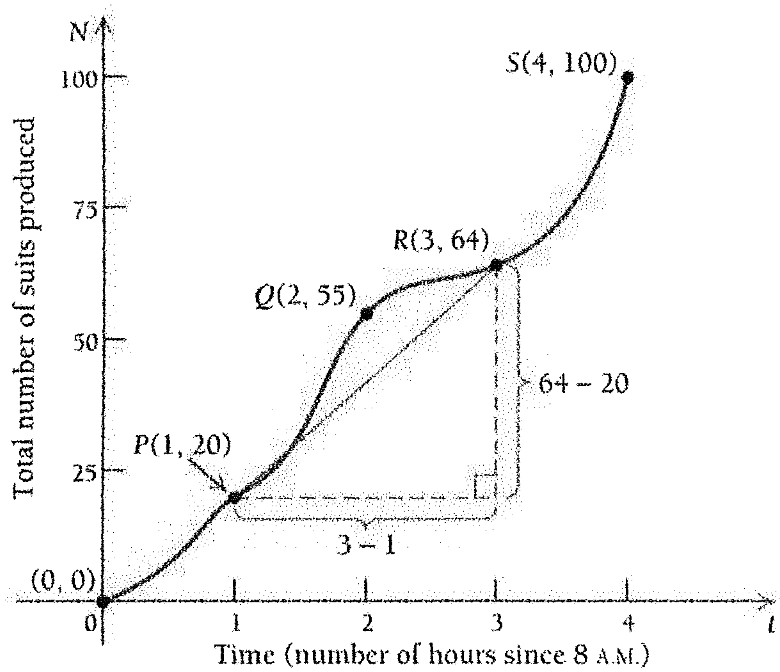
55 − 20 = 35 suits

How many suits were produced from 10 AM to 11 AM?

64 − 55 = 9 suits

What was the hourly rate of production from 8 AM to 9 AM?

20 suits per hour



Example 2:

What was the average number of suits produced per hour from 9 AM to 11 AM?

$$\frac{64-20}{3-1} = \frac{44}{2} = 22$$

22 suits per hour

average rate of change is slope!

The average rate of change of y with respect to x, as x changes from x_1 to x_2, is $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Example 3: For $f(x) = x^2 + 1$, find the average rate of change as x changes from 1 to 3.

when $x_1 = 1$, $y_1 = 1^2 + 1 = 2$

when $x_2 = 3$, $y_2 = 3^2 + 1 = 10$

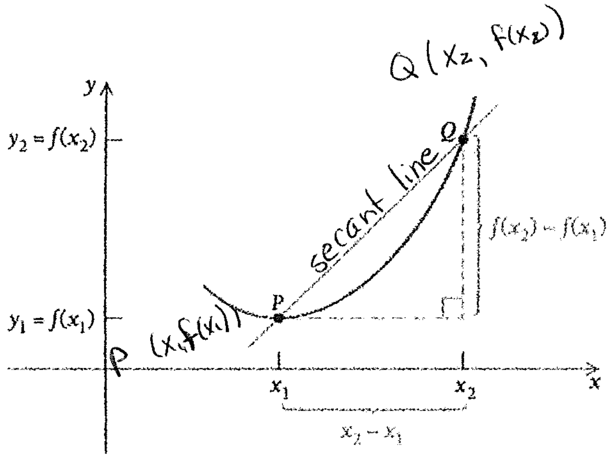
average rate of change $= m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{10-2}{3-1} = \dfrac{8}{2} = 4$

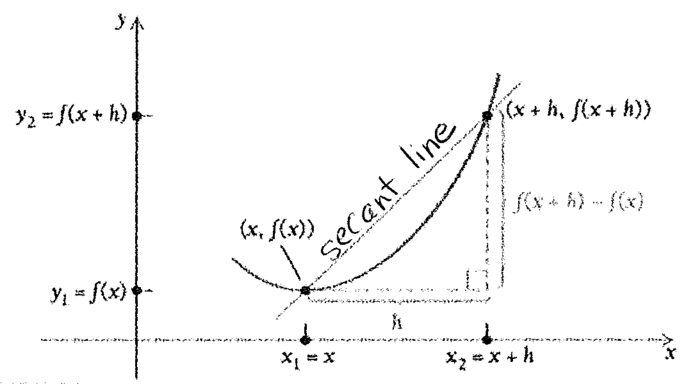
Example 4: For $f(x) = x^2 - 5$, find the average rate of change as x changes from 0 to 4.

when $x_1 = 0$, $y_1 = 0^2 - 5 = -5$

when $x_2 = 4$, $y_2 = 4^2 - 5 = 11$

average rate of change $= m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{11-(-5)}{4-0} = \dfrac{16}{4} = 4$





Use the diagrams to show average rate of change as the different quotient.

average rate of change $= m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{f(x_2) - f(x_1)}{x_2 - x_1} = \dfrac{f(x+h) - f(x)}{(x+h) - x} = \dfrac{f(x+h) - f(x)}{h}$

Example 5: For $f(x) = x^2$, find the simplified form of the difference quotient. Complete the chart.

$2x+h$

x	**h**	$\mathbf{\dfrac{f(x+h) - f(x)}{h}}$
5	2	$2(5) + 2 = 12$
5	1	$2(5) + 1 = 11$
5	0.1	$2(5) + 0.1 = 10.1$
5	0.01	$2(5) + 0.01 = 10.01$
5	0.001	$= 10.001$

see pattern? limit

$$\frac{f(x+h) - f(h)}{h} = \frac{(x+h)^2 - x^2}{h}$$

$$= \frac{x^2 + 2xh + h^2 - x^2}{h}$$

$$= \frac{2xh + h^2}{h}$$

$$= \frac{h(2x+h)}{h}$$

$$= 2x + h$$