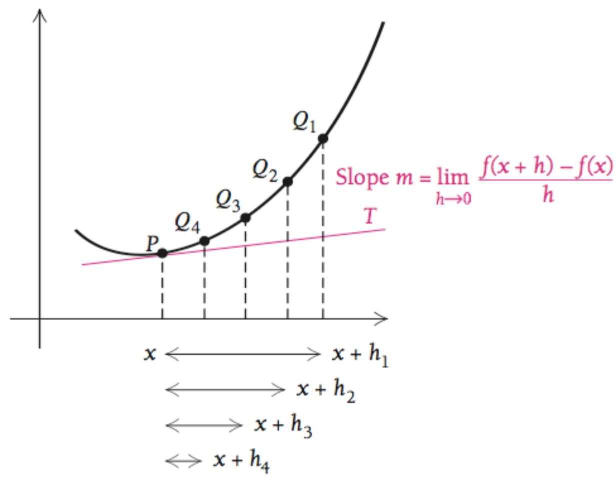
Section 1.4
Differentiation Using Limits of Difference Quotient

The **slope of the tangent line** at (*x*, *f* (*x*)) is

$$m = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

This limit is also the **instantaneous rate of change** of *f* (*x*) at *x*.

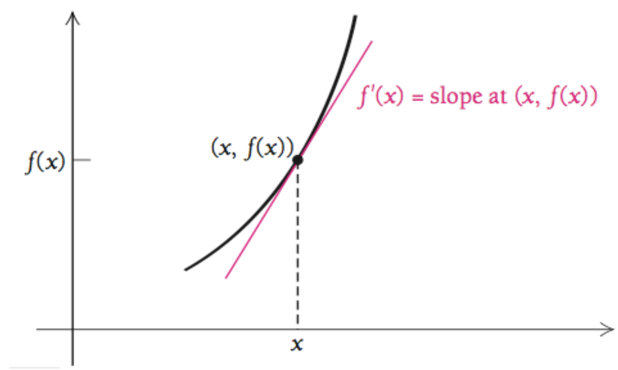


For a function $y = f(x)$, its **derivative** at *x* is the function f'defined by

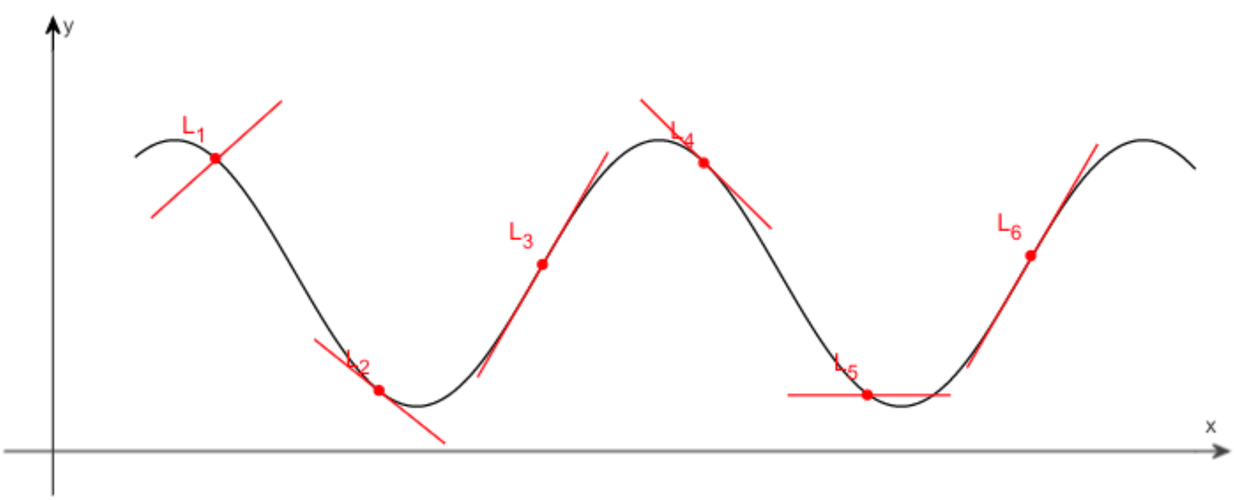
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

provided the limit exists.

If $f'(x)$ exists, then we say that f is **differentiable** at *x*.



Example 1: Which of the lines in the following graph appear to be tangent lines?



Example 2: Given the function $f(x) = x^2$, find the following.

a. $f'(x)$

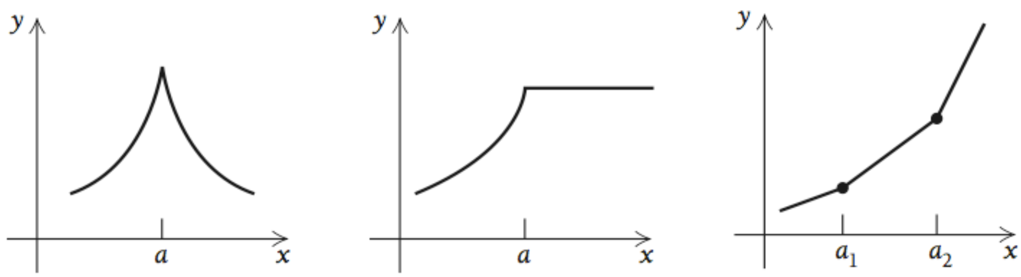
b. $f'(-1)$

c. $f'(2)$

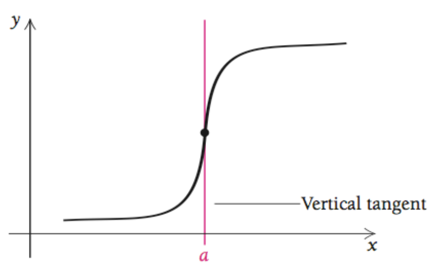
d. The equation of the tangent line at x=2

If $f'(x)$ exists, then we say that f is **differentiable** at *x*. When is f **not differentiable**?

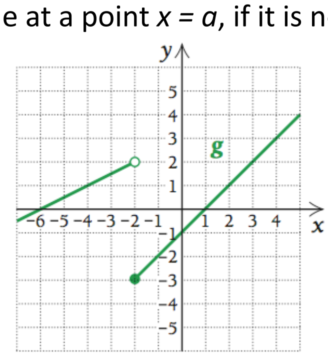
I. A function *f*(*x*) is not differentiable at a point *x* = *a*, if there is a “corner” at *a*.



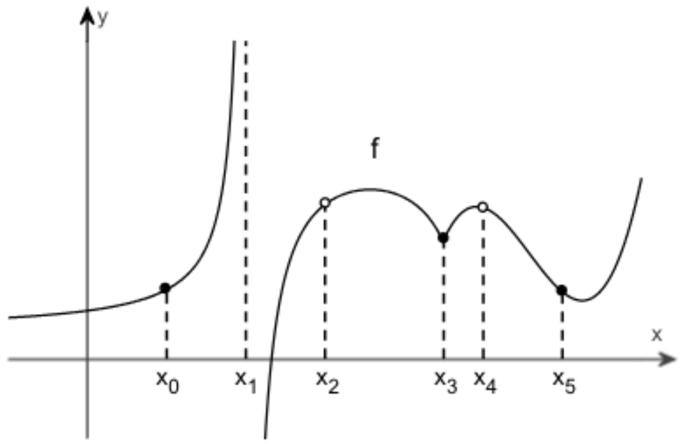
II. A function *f* (*x*) is not differentiable at a point *x* = *a*, if there is a vertical tangent at *a*.



III. A function *f* (*x*) is not differentiable at a point *x* = *a*, if it is not continuous at *a*.



Example 4: List the points in the graph at which the function is not differentiable.

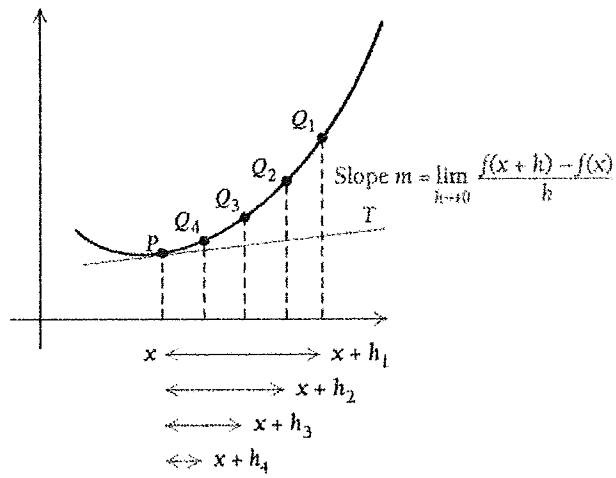


Section 1.4
Differentiation Using Limits of Difference Quotient

The **slope of the tangent line** at $(x, f(x))$ is

$$m = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

This limit is also the **instantaneous rate of change** of $f(x)$ at x.

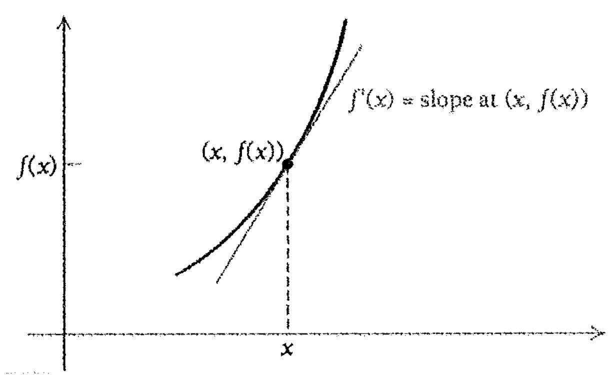


For a function $y = f(x)$, its **derivative** at x is the function f' defined by

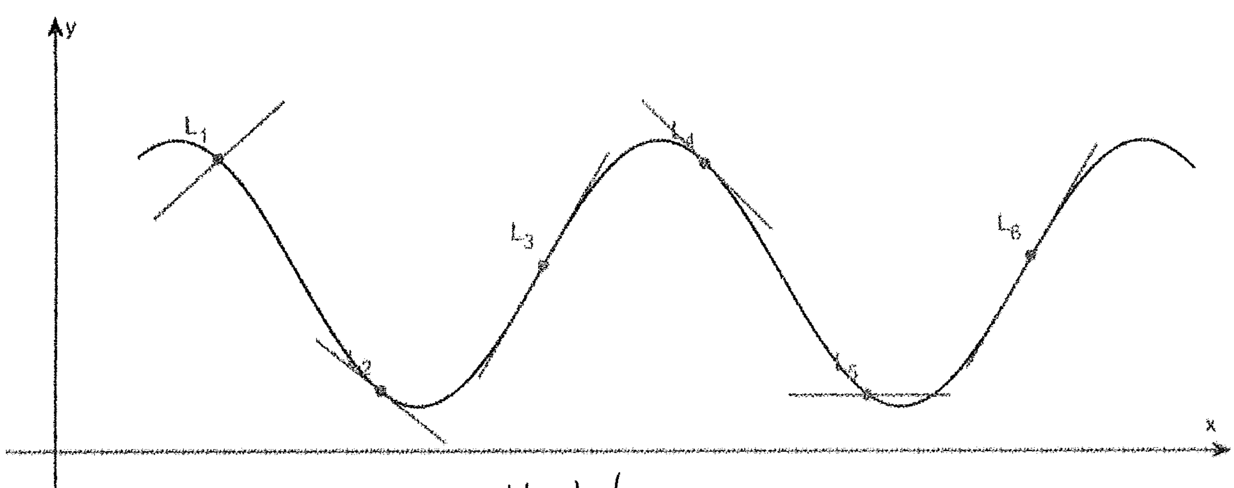
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

provided the limit exists.

If $f'(x)$ exists, then we say that f is **differentiable** at x.



Example 1: Which of the lines in the following graph appear to be tangent lines?



Lines L2, L3, L4, L6

Example 2: Given the function $f(x) = x^2$, find the following.

a. $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h}$

$= \lim_{h \to 0} \frac{2xh + h^2}{h} = \lim_{h \to 0} \frac{h(2x+h)}{h} = \lim_{h \to 0} 2x + h = 2x + 0 = 2x$

b. $f'(-1) = 2(-1) = -2$ slope of tangent line when $x = -1$

c. $f'(2) = 2(2) = 4$ slope of tangent line when $x = 2$

d. The equation of the tangent line at x=2. Remember $y = f(x) = x^2$

$x_1 = 2$, $y_1 = (2)^2 = 4$

$m = f'(2) = 4$

Equation $y - y_1 = m(x - x_1)$

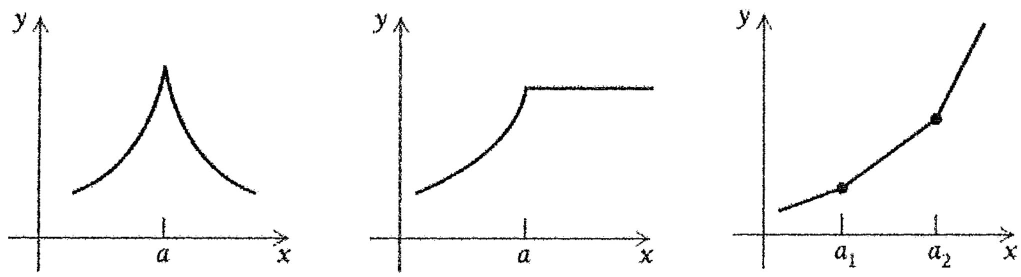
$y - 4 = 4(x - 2)$

$y - 4 = 4x - 8$

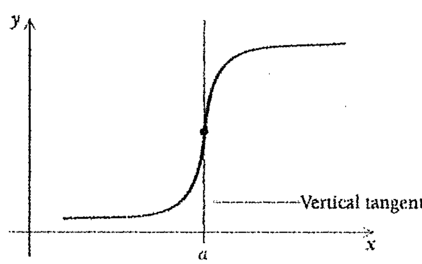
$y = 4x - 4$

If $f'(x)$ exists, then we say that f is **differentiable** at x. When is f **not differentiable**?

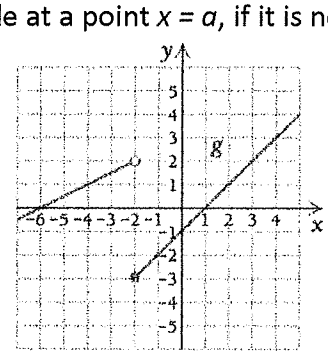
I. A function $f(x)$ is not differentiable at a point $x = a$, if there is a "corner" at a.



II. A function $f(x)$ is not differentiable at a point $x = a$, if there is a vertical tangent at a.

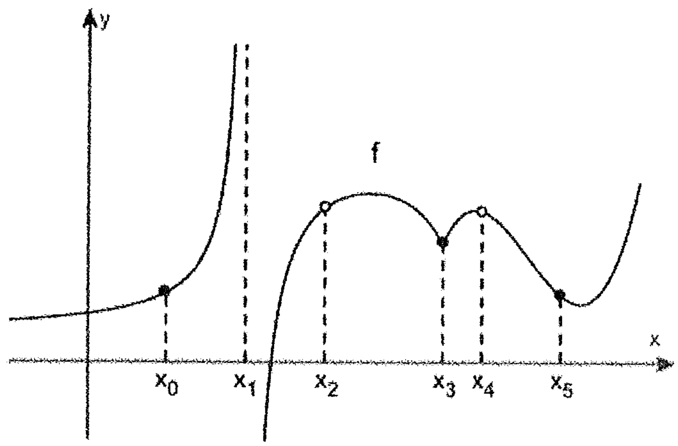


III. A function $f(x)$ is not differentiable at a point $x = a$, if it is not continuous at a.



not differentiable at $x = -2$

Example 4: List the points in the graph at which the function is not differentiable.



not differentiable at x_1, x_2, x_3, x_4