Section 1.5
Derivative Rules

Recall that the derivative is $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ provided that the limit exists.
The derivative is the slope of the tangent line. It is also the instantaneous rate of change.

The notation of the derivative is as follows.

Good news! There are short cut rules for derivatives.

Derivative of a Constant When $f(x) = c$ then $f'(x) = 0$

Example 1: Find the derivative of the following.

a. $f(x) = 10$

b. $y = 2$

Power Rule When $f(x) = x^n$ then $f'(x) = nx^{n-1}$
To find the derivative of a power function, bring the exponent to the front of the variable as a coefficient and make the new exponent 1 less than the original exponent.

Example 2: Find the derivative of the following.

a. $f(x) = x^2$

b. $f(x) = x^3$

c. $f(x) = x^{100}$

d. $f(x) = x$

Example 3: Find the derivative of the following. Your final answer should be in radical form. Hint: Rewrite each function with rational exponent, then take the derivative.

a. $f(x) = \sqrt{x}$

b. $f(x) = \sqrt[3]{x}$

c. $f(x) = \sqrt[5]{x^4}$

d. $f(x) = \sqrt[4]{x^3}$

Example 4: Find the derivative of the following. Your final answer should have positive exponents.
Hint: Rewrite each function with a negative exponent, then take the derivative.

a. $f(x) = \frac{1}{x^2}$

b. $f(x) = \frac{1}{x^3}$

c. $f(x) = \frac{1}{x}$

d. $f(x) = \frac{1}{\sqrt{x}}$

Constant times a Function When $f(x) = c \cdot x^n$ then $f'(x) = c \cdot nx^{n-1}$
The derivative of a constant times a function is the constant times the derivative of the function.

Example 5: Find the derivative of the following.

a. $f(x) = 5x^2$

b. $f(x) = -6x^4$

c. $f(x) = 3x$

d. $f(x) = \frac{5}{x^2}$

Sum or Difference of Functions Take the derivative of each term
The derivative of a sum is the sum of the derivatives.
The derivate of a difference is the difference of the derivatives.

Example 6: Find the derivative of the following.

a. $f(x) = x^4 + x^3 - x^2 + 10$

b. $f(x) = 0.03x^2 - 6x - 1$

c. $f(x) = 4x^{\frac{1}{2}} - 5x^{-4} + 6$

d. $f(x) = x^5 - x^7 - \sqrt[3]{x} + \frac{1}{x^3}$

Example 7: Find the derivative of the following. Rewrite first

a. $f(x) = (x+4)(x^2-1)$

b. $f(x) = \frac{3x^2+x-2}{x}$

Example 8:

Find the equation of the tangent line to the graph of $f(x) = x^3 - 2x + 1$ when x=2.

Example 9:

Given that $f(x) = 4 - x^2$ find the equation of the tangent line to the curve $f(x)$ at the point (1, 3).

Example 10:

Given that $y = x + \frac{4}{x}$ find the equation of the tangent line to the curve at the point (4, 5)

Example 11:

Find the points on the graph of $y = x^3 - 3x^2$ at which the tangent line is horizontal.

Example 12:
Find the points on the graph of $f(x) = -x^3 + x^2 + 5x - 1$ at which the tangent line is horizontal.

Example 13. In a certain memory experiment, a person is able to memorize M words after t minutes, where $M(t) = -0.001t^3 + 0.1t^2$.

a. Find the rate of change of the number of words memorized with respect to time.
b. How many words are memorized during the first 10 minutes?
c. At what rate are words being memorized after 10 minutes?

Example 14: The function $R(v) = \frac{5800}{v}$ can be used to determine the heart rate, R, of a person whose heart pumps 5800 milliliters (mL) of blood per minute and v milliliters of blood per beat.

a. Find the rate of change of heart rate with respect to v, the output per beat.
b. Find the heart rate at $v = 60$ mL per beat.
c. Find the rate of change at $v = 60$ mL per beat.

Section 1.5
Derivative Rules

Recall that the derivative is $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ provided that the limit exists.
The derivative is the slope of the tangent line. It is also the instantaneous rate of change.

The notation of the derivative is as follows. $f'(x)$, f', y', $\frac{dy}{dx}$

Good news! There are short cut rules for derivatives.

Derivative of a Constant When $f(x) = c$ then $f'(x) = 0$

special case
$f(x) = x$
$f'(x) = 1$

Example 1: Find the derivative of the following.

a. $f(x) = 10$

$f'(x) = 0$

b. $y = 2$

$y' = 0$

Power Rule When $f(x) = x^n$ then $f'(x) = nx^{n-1}$
To find the derivative of a power function, bring the exponent to the front of the variable as a coefficient and make the new exponent 1 less than the original exponent.

Example 2: Find the derivative of the following.

a. $f(x) = x^2$

$f'(x) = 2x^{2-1} = 2x$

b. $f(x) = x^3$

$f'(x) = 3x^{3-1} = 3x^2$

c. $f(x) = x^{100}$

$f'(x) = 100x^{100-1} = 100x^{99}$

d. $f(x) = x = x^1$

$f'(x) = 1x^{1-1} = 1x^0 = 1 \cdot 1 = 1$

Example 3: Find the derivative of the following. Your final answer should be in radical form. Hint: Rewrite each function with rational exponent, then take the derivative.

a. $f(x) = \sqrt{x} = x^{\frac{1}{2}}$

$f'(x) = \frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2}x^{-1/2}$

$= \frac{1}{2\sqrt{x}}$

b. $f(x) = \sqrt[3]{x} = x^{\frac{1}{3}}$

$f'(x) = \frac{1}{3}x^{\frac{1}{3}-1} = \frac{1}{3}x^{-\frac{2}{3}}$

$= \frac{1}{3\sqrt[3]{x^2}}$

c. $f(x) = \sqrt[5]{x^4} = x^{\frac{4}{5}}$

$f'(x) = \frac{4}{5}x^{\frac{4}{5}-1} = \frac{4}{5}x^{-1/5}$

$= \frac{4}{5\sqrt[5]{x}}$

d. $f(x) = \sqrt[4]{x^3} = x^{\frac{3}{4}}$

$f'(x) = \frac{3}{4}x^{\frac{3}{4}-1} = \frac{3}{4}x^{-1/4}$

$= \frac{3}{4\sqrt[4]{x}}$

Example 4: Find the derivative of the following. Your final answer should have positive exponents.
Hint: Rewrite each function with a negative exponent, then take the derivative.

a. $f(x) = \frac{1}{x^2} = x^{-2}$

$f'(x) = -2x^{-2-1} = -2x^{-3}$

$= \frac{-2}{x^3}$

b. $f(x) = \frac{1}{x^3} = x^{-3}$

$f'(x) = -3x^{-3-1} = -3x^{-4}$

$= \frac{-3}{x^4}$

c. $f(x) = \frac{1}{x} = x^{-1}$

$f'(x) = -x^{-1-1} = -x^{-2}$

$= \frac{-1}{x^2}$

d. $f(x) = \frac{1}{\sqrt{x}} = x^{-1/2}$

$f'(x) = -\frac{1}{2}x^{-1/2-1} = -\frac{1}{2}x^{-3/2}$

$= \frac{-1}{2\sqrt[3]{x^3}}$

Constant times a Function When $f(x) = c \cdot x^n$ then $f'(x) = c \cdot nx^{n-1}$
The derivative of a constant times a function is the constant times the derivative of the function.

Example 5: Find the derivative of the following.

a. $f(x) = 5x^2$

$f'(x) = 5 \cdot 2x = 10x$

b. $f(x) = -6x^4$

$f'(x) = -6 \cdot 4x^3 = -24x^3$

c. $f(x) = 3x$

$f'(x) = 3 \cdot 1 = 3$

d. $f(x) = \frac{5}{x^2} = 5x^{-2}$

$f'(x) = 5 \cdot -2x^{-3} = -10x^{-3}$

Sum or Difference of Functions Take the derivative of each term
The derivative of a sum is the sum of the derivatives.
The derivate of a difference is the difference of the derivatives.

Example 6: Find the derivative of the following.

a. $f(x) = x^4 + x^3 - x^2 + 10$

$f'(x) = 4x^3 + 3x^2 - 2x + 0$

$= 4x^3 + 3x^2 - 2x$

b. $f(x) = 0.03x^2 - 6x - 1$

$f'(x) = 0.06x - 6 - 0$

$= 0.06x - 6$

c. $f(x) = 4x^{\frac{1}{2}} - 5x^{-4} + 6$

$f'(x) = 2x^{-1/2} + 20x^{-5} + 0$

$= 2x^{-1/2} + 20x^{-5}$

d. $f(x) = x^5 - x^7 - \sqrt[3]{x} + \frac{1}{x^3}$ rewrite

$f(x) = x^5 - x^7 - x^{1/3} + x^{-3}$

$f'(x) = 5x^4 - 7x^6 - \frac{1}{3}x^{-\frac{2}{3}} - 3x^{-4}$

Example 7: Find the derivative of the following. Rewrite first

a. $f(x) = (x+4)(x^2-1)$

$f(x) = x^3 - x + 4x^2 - 4$

$f'(x) = 3x^2 - 1 + 8x - 0$

$f'(x) = 3x^2 + 8x - 1$

b. $f(x) = \frac{3x^2+x-2}{x} = \frac{3x^2}{x} + \frac{x}{x} - \frac{2}{x}$

$f(x) = 3x + 1 - 2x^{-1}$

$f'(x) = 3 + 0 + 2x^{-2}$

$= 3 + 2x^{-2}$

Example 8:
Find the equation of the tangent line to the graph of $f(x) = x^3 - 2x + 1$ when x=2.

point
$x = 2 \quad y = 2^3 - 2 \cdot 2 + 1 = 5$
$(2, 5)$

slope
$f'(x) = 3x^2 - 2$
$m = f'(2) = 3 \cdot 2^2 - 2 = 10$

equation
$y - y_1 = m(x - x_1)$
$y - 5 = 10(x - 2)$
$y - 5 = 10x - 20$
$y = 10x - 15$

Example 9:
Given that $f(x) = 4 - x^2$ find the equation of the tangent line to the curve $f(x)$ at the point (1, 3).

point
$(1, 3)$

slope
$f'(x) = -2x$
$m = f'(1) = -2(1) = -2$

equation
$y - y_1 = m(x - x_1)$
$y - 3 = -2(x - 1)$
$y - 3 = -2x + 2$
$y = -2x + 5$

Example 10:
Given that $y = x + \frac{4}{x}$ find the equation of the tangent line to the curve at the point (4, 5)

point $(4, 5)$

slope $y = x + 4x^{-1}$
$y' = 1 - 4x^{-2}$
$m = 1 - 4 \cdot 4^{-2} = \frac{3}{4}$

equation
$y - y_1 = m(x - x_1)$
$y - 5 = \frac{3}{4}(x - 4)$
$y - 5 = \frac{3}{4}x - 3$
$y = \frac{3}{4}x + 2$

Example 11:
Find the points on the graph of $y = x^3 - 3x^2$ at which the tangent line is horizontal.
means $f'(x) = 0$

$y' = 3x^2 - 6x = 0$

$3x \cdot x - 3x \cdot 2 = 0$

$3x(x - 2) = 0$

$3x = 0 \qquad x - 2 = 0$
$x = 0 \qquad x = 2$

points
$(0, 0)$
$(2, -4)$

$y = 0^3 - 3 \cdot 0^2 = 0$
$y = 2^3 - 3 \cdot 2^2 = 8 - 12$

Example 12:

means $f'(x) = 0$

Find the points on the graph of $f(x) = -x^3 + x^2 + 5x - 1$ at which the tangent line is horizontal.

$f'(x) = -3x^2 - 2x + 5 = 0$

$3x^2 + 2x - 5 = 0$ divide by -1

$(3x - 5)(x + 1) = 0$

$3x - 5 = 0$ $\quad$ $x + 1 = 0$

$3x = 5$ $\quad$ $x = -1$

$x = \frac{5}{3}$

points

$\left(\frac{5}{3}, \frac{148}{27}\right)$

$(-1, -4)$

$y = -(5/3)^3 + \left(\frac{5}{3}\right)^2 + 5\left(\frac{5}{3}\right) - 1$

Example 13. In a certain memory experiment, a person is able to memorize M words after t minutes, where $M(t) = -0.001t^3 + 0.1t^2$.

a. Find the rate of change of the number of words memorized with respect to time.

b. How many words are memorized during the first 10 minutes?

c. At what rate are words being memorized after 10 minutes?

a) $M'(t) = -0.003t^2 + 0.2t$

b) $M(10) = -0.001 \cdot 10^3 + 0.1 \cdot 10^2 = 9$ words

c) $M'(10) = -0.003 \cdot 10^2 + 0.2 \cdot 10 = +1.7$ words per minute

Example 14: The function $R(v) = \frac{5800}{v}$ can be used to determine the heart rate, R, of a person whose heart pumps 5800 milliliters (mL) of blood per minute and v milliliters of blood per beat.

a. Find the rate of change of heart rate with respect to v, the output per beat.

b. Find the heart rate at $v = 60$ mL per beat.

c. Find the rate of change at $v = 60$ mL per beat.

a) $R(v) = 5800v^{-1}$

$R'(v) = -5800v^{-2} = \frac{-5800}{v^2}$

b) $R(60) = \frac{5800}{60} = 96.67$ beats/min

c) $R'(60) = \frac{-5800}{60^2} = -1.61$ beat/min per mL