Section 1.6
Product and Quotient Rules for Derivatives

The Product Rule for Derivatives

If $\boldsymbol{y = f(x) \cdot g(x)}$ then $\boldsymbol{y' = f(x) \cdot g'(x) + g(x)f'(x)}$.

The derivative of a product is the first factor times the derivative of the second factor, plus the second factor time the derivative of the first factor.

Example 1: Find the derivative using the Product Rule and then simplify.
$y = (x + 6)(x^2 - 2)$

Example 2: Find the derivative using the Product Rule. Don't simplify.
$y = (x^4 - 2x^3 - 7)(3x^2 - 5x)$

Example 3: Find the derivative using the Product Rule. Don't simplify.
$y = (x^2 + 4x - 11)\left(7x^3 - \sqrt{x}\right)$

Example 4: Find the derivative using the Product Rule. Don't simplify.
$y = \left(\sqrt{t} + 2\right)\left(3t - 4\sqrt{t} + 7\right)$

The Quotient Rule for Derivatives

$$\textbf{If } \boldsymbol{Q(x)} = \frac{\boldsymbol{N(x)}}{\boldsymbol{D(x)}} \textbf{ then } \boldsymbol{Q'(x)} = \frac{\boldsymbol{D(x) \cdot N'(x) - N(x) \cdot D'(x)}}{\boldsymbol{[D(x)]^2}}$$

The derivative of a quotient is the denominator times the derivative of the numerator, minus the numerator times the derivative of the denominator, all divided by the square of the denominator.

Example 5: Find the derivative using the Quotient Rule. Simplify.

$$y = \frac{1 + x^2}{x^3 + 1}$$

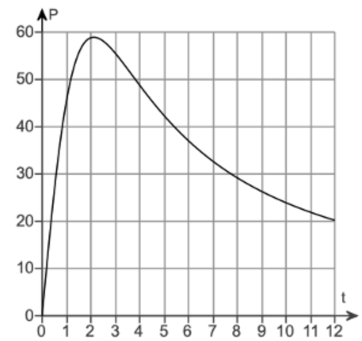
Example 6: Find the derivative using the Quotient Rule. Simplify.

$$y = \frac{t^2 - 16}{t + 4}$$

Example 7: Find the derivative using the Quotient Rule. Don't simplify.

$$y = \frac{5x^2 - 1}{2x + 3}$$

Example 8:
The population, P, in thousands, of a small city is given by the following function, where t is time in years. $P(t) = \frac{500t}{2t^2+9}$



a. Find the growth rate.
b. Find the population after 9 years.
c. Find the growth rate at t=9 year

Work the following multiple-choice problems from homework. On the test, this is a problem that I would expect to see all work.

4. Differentiate.

$$y = \frac{x^3}{x-1}$$

A. $\frac{dy}{dx} = \frac{2x^3+3x^2}{(x-1)^2}$

B. $\frac{dy}{dx} = \frac{2x^3-3x^2}{(x-1)^2}$

C. $\frac{dy}{dx} = \frac{-2x^3+3x^2}{(x-1)^2}$

D. $\frac{dy}{dx} = \frac{-2x^3-3x^2}{(x-1)^2}$

8. Differentiate.

$$f(x) = \left(5x^3+7\right)\left(4x^7-5\right)$$

A. $f'(x) = 200x^9 + 196x^6 - 75x$

B. $f'(x) = 20x^9 + 196x^6 - 75x$

C. $f'(x) = 200x^9 + 196x^6 - 75x^2$

D. $f'(x) = 20x^9 + 196x^6 - 75x^2$

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The Product Rule for Derivatives

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The derivative of a product is the first factor times the derivative of the second factor, plus the second factor time the derivative of the first factor.

Example 1: Find the derivative using the Product Rule and then simplify. Show work

$y = (x + 6)(x^2 - 2)$

$$y' = (x+6)(x^2-2)' + (x^2-2)(x+6)'$$
$$= (x+6)(2x) + (x^2-2)(1)$$
$$= 2x^2 + 12x + x^2 - 2$$
$$= 3x^2 + 12x - 2$$

Example 2: Find the derivative using the Product Rule. Don't simplify.

$y = (x^4 - 2x^3 - 7)(3x^2 - 5x)$

$$y' = (x^4-2x^3-7)(3x^2-5x)' + (3x^2-5x)(x^4-2x^3-7)'$$
$$y' = (x^4-2x^3-7)(6x-5) + (3x^2-5x)(4x^3-6x^2)$$

Example 3: Find the derivative using the Product Rule. Don't simplify.

$y = (x^2 + 4x - 11)(7x^3 - \sqrt{x})$

$$y' = (x^2+4x-11)(7x^3-\sqrt{x})' + (7x^3-\sqrt{x})(x^2+4x-11)'$$
$$y' = (x^2+4x-11)(21x^2 - \tfrac{1}{2}x^{-1/2}) + (7x^3-\sqrt{x})(2x+4)$$

Example 4: Find the derivative using the Product Rule. Don't simplify.

$y = (\sqrt{t} + 2)(3t - 4\sqrt{t} + 7)$ Rewrite

$$y' = (t^{1/2}+2)(3t-4t^{1/2}+7)' + (3t-4t^{1/2}+7)(t^{1/2}+2)'$$
$$y' = (t^{1/2}+2)(3-2t^{-1/2}) + (3t-4t^{1/2}+7)(\tfrac{1}{2}t^{-1/2})$$

The Quotient Rule for Derivatives

$$\textbf{If } Q(x) = \frac{N(x)}{D(x)} \textbf{ then } Q'(x) = \frac{D(x)\cdot N'(x) - N(x)\cdot D'(x)}{[D(x)]^2}$$

The derivative of a quotient is the denominator times the derivative of the numerator, minus the numerator times the derivative of the denominator, all divided by the square of the denominator.

Example 5: Find the derivative using the Quotient Rule. Simplify. Show work

$$y = \frac{1+x^2}{x^3+1}$$

$$y' = \frac{(x^3+1)(1+x^2)' - (1+x^2)(x^3+1)'}{(x^3+1)^2} = \frac{(x^3+1)(2x) - (1+x^2)(3x^2)}{(x^3+1)^2}$$

$$= \frac{2x^4 + 2x - 3x^2 - 3x^4}{(x^3+1)^2} = \boxed{\frac{-x^4 - 3x^2 + 2x}{(x^3+1)^2}}$$

Example 6: Find the derivative using the Quotient Rule. Simplify. Show work

$$y = \frac{t^2-16}{t+4}$$

$$y' = \frac{(t+4)(t^2-16)' - (t^2-16)(t+4)'}{(t+4)^2} = \frac{(t+4)(2t) - (t^2-16)(1)}{(t+4)^2}$$

$$= \frac{2t^2 + 8t - t^2 + 16}{(t+4)^2} = \frac{t^2 + 8t + 16}{(t+4)^2} = \frac{(t+4)^2}{(t+4)^2} = 1$$

Example 7: Find the derivative using the Quotient Rule. Don't simplify.

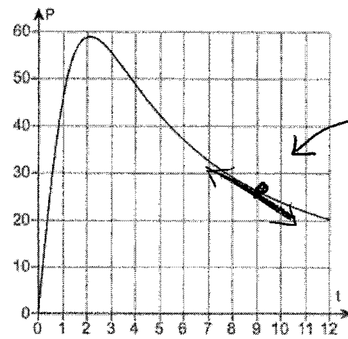
$$y = \frac{5x^2-1}{2x+3}$$

$$y' = \frac{(2x+3)(5x^2-1)' - (5x^2-1)(2x+3)'}{(2x+3)^2}$$

$$\boxed{y' = \frac{(2x+3)(10x) - (5x^2-1)(2)}{(2x+3)^2}}$$

Example 8:

The population, P, in thousands, of a small city is given by the following function, where t is time in years. $P(t) = \frac{500t}{2t^2+9}$



$(9, 26)$

$m = -3$

a. Find the growth rate.

b. Find the population after 9 years.

c. Find the growth rate at t=9 year

a) $P'(t) = \frac{(2t^2+9)(500) - (500t)(4t)}{(2t^2+9)^2}$

$P'(t) = \frac{-1000t^2 + 4500}{(2t^2+9)^2}$

b) $P(9) = \frac{500 \cdot 9}{2 \cdot 9^2 + 9}$

$= 26$

c) $P'(9) =$

$\frac{-1000 \cdot 9^2 + 4500}{(2 \cdot 9^2 + 9)^2}$

$= -3$

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○ C. $\frac{dy}{dx} = \frac{-2x^3 + 3x^2}{(x-1)^2}$

○ D. $\frac{dy}{dx} = \frac{-2x^3 - 3x^2}{(x-1)^2}$

$y' = \frac{(x-1)(x^3)' - (x^3)(x-1)'}{(x-1)^2}$

$= \frac{(x-1)(3x^2) - (x^3)(1)}{(x-1)^2}$

$= \frac{3x^3 - 3x^2 - x^3}{(x-1)^2}$

$= \frac{2x^3 - 3x^2}{(x-1)^2}$

8. Differentiate.

$f(x) = (5x^3 + 7)(4x^7 - 5)$

○ A. $f'(x) = 200x^9 + 196x^6 - 75x$

○ B. $f'(x) = 20x^9 + 196x^6 - 75x$

○ C. $f'(x) = 200x^9 + 196x^6 - 75x^2$

○ D. $f'(x) = 20x^9 + 196x^6 - 75x^2$

$y' = (5x^3+7)(4x^7-5)' + (4x^7-5)(5x^3+7)'$

$= (5x^3+7)(28x^6) + (4x^7-5)(15x^2)$

$= 140x^9 + 196x^6 + 60x^9 - 75x^2$

$= 200x^9 + 196x^6 - 75x^2$