Math 1232 Review Material

Scientific Notation- Numbers that are very large or very small are often expressed in scientific notation.

Standard Form	Scientific Notation	Application	Calculator Display
93,000,000 mi	9.3×10^7mi	Distance to Sun	9.3E7
256,000	2.56×10^5	Number of Cell Towers in 2010	2.56E5
9,000,000,000	9×10^9	Estimated Population in 2050	9E9
0.00000538 sec	5.38×10^{-6} sec	Time for Light to Travel 1 mile	5.38E-6
0.000005 cm	5×10^{-6} cm	Size of a Typical Virus	5E-6

Example 1: Write 0.000578 in scientific notation.

Example 2: Write 636,900 in scientific notation.

Example 3: Write 2.2×10^{-3} in standard decimal form.

Exponent Rules

1. $a^n = a \cdot a \cdot a \cdot \cdots \cdot a$ (n factors of a)
2. $a^0 = 1$
3. $a^1 = a$
4. $a^m \cdot a^n = a^{m+n}$
5. $\frac{a^m}{a^n} = a^{m-n}$
6. $(a^m)^n = a^{mn}$
7. $\frac{1}{a^n} = a^{-n}$
8. $\sqrt[n]{a^m} = a^{\frac{m}{n}}$

Example 4:

a. 4^3

b. 2^0

c. 3^1

d. $x^2 \cdot x^3$

e. $\frac{x^{10}}{x^6}$

f. $(x^2)^3$

g. $\frac{1}{x^2}$

h. $\frac{1}{x}$

i. $\sqrt{x}$

j. $\sqrt[3]{x}$

k. $\sqrt[4]{x^3}$

Math 1232 Review Material

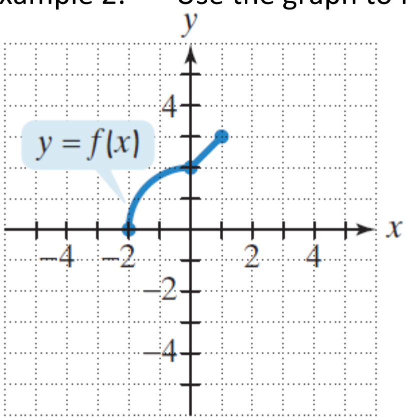
Function Notation: f(x) is read "f of x " or "the function f evaluated at x".
Remember that f(x) is just another name for y.

Example 1: Given that $f(x) = x^2 - 2x$, find the following.

a) f(-2) b) f(4) c) f(-1)

d) f(x + h)

Example 2: Use the graph to find the indicated values.



a. $f(0)$

b. $f(1)$

c. $f(-2)$

Example 3: Given that $f(x) = 9x - 4$, find $\frac{f(x+h)-f(x)}{h}$.

Example 4: Given that $f(x) = x^2 - 3x$, find $\frac{f(x+h)-f(x)}{h}$.

The slope m of the line through the points (x_1, y_1) and (x_2, y_2) is $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{rise}{run}$.
Slope is also the average rate of change.

A line with positive slope ($m > 0$) rises from left to right.

A line with negative slope ($m < 0$) falls from left to right.

A line with zero slope ($m = 0$) is horizontal.

A line with undefined slope is vertical.

The point-slope form of the equation of a line that passes through the point (x_1, y_1) and has slope of m is $y - y_1 = m(x - x_1)$

Example 5: Find the equation of the line through (-2, -3) having slope $m = 4$.

Math 1232 Review Material

Example 1: Solve the following linear equations.

a. $3(x-4)=2x-1$

b. $2x-5=3-(1+2x)$

The product of two polynomials may be found by multiplying every term in the first polynomial by every term in the second polynomial and then combining like terms. When multiplying two binomials, we often call the process FOIL to remind us to multiply the first terms (F), outside terms (O), inside terms (I), and last terms (L).

Example 2: Multiply $(x+1)(x+3)$

When factoring a polynomial, we first look for factors that are common to each term in an expression. By applying a distribute property, we can write a polynomial as two factors.

Example 3: Factor by finding the greatest common factor.

a. $2x^2+4x$

b. $9x^4-6x^3-3x^2$

To factor the trinomial x^2+bx+c, find integers m and n that satisfy $m\cdot n=c$ and $m+n=b$.
Then $x^2+bx+c=(x+m)(x+n)$. Think of this as FOIL in reverse.

Example 4: Factor the following.

a. $x^2+10x+16$

b. $x^2+7x-30$

Example 5: Factor the following.

a. $3x^2+7x+2$

b. $6x^2-x-2$

Example 6: Factor the difference of squares.

a. $x^2 - 9$

b. $x^2 - 25$

The zero-product property says that if $ab = 0$ then either $a = 0$ or $b = 0$ or both. This only works for 0.

Example 7: Solve the following using factoring and the zero-product property.

a. $x^2 - 7x = 0$

b. $x^2 = 3x$

c. $x^2 - 2x + 1 = 0$

d. $2x^2 + 2x - 11 = 1$

The square root property says that if k is a nonnegative number, then the solutions to the equation $x^2 = k$ are given by $x = \pm\sqrt{k}$

Example 8: Solve the following using the square root property.

a. $x^2 = 16$

b. $x^2 - 1 = 8$

Quadratic Formula: The solutions to $ax^2 + bx + c = 0$ are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Example 9: Solve $6x^2 - x - 2 = 0$ using the quadratic formula.

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Scientific Notation- Numbers that are very large or very small are often expressed in scientific notation.

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Example 1: Write 0.000578 in scientific notation. 5.78×10^{-4}

Example 2: Write 636,900 in scientific notation. 6.369×10^{5}

Example 3: Write 2.2×10^{-3} in standard decimal form. .0022

Exponent Rules

1. $a^n = a \cdot a \cdot a \cdot \cdots \cdot a$ (n factors of a)
2. $a^0 = 1$
3. $a^1 = a$
4. $a^m \cdot a^n = a^{m+n}$
5. $\frac{a^m}{a^n} = a^{m-n}$
6. $(a^m)^n = a^{mn}$
7. $\frac{1}{a^n} = a^{-n}$
8. $\sqrt[n]{a^m} = a^{\frac{m}{n}}$

Example 4:

a. $4^3 = 4 \cdot 4 \cdot 4 = 64$

b. $2^0 = 1$

c. $3^1 = 3$

d. $x^2 \cdot x^3 = x^{2+3} = x^5$

e. $\frac{x^{10}}{x^6} = x^{10-6} = x^4$

f. $(x^2)^3 = x^{2 \cdot 3} = x^6$

g. $\frac{1}{x^2} = x^{-2}$

h. $\frac{1}{x} = x^{-1}$

i. $\sqrt{x} = x^{1/2}$

j. $\sqrt[3]{x} = x^{1/3}$

k. $\sqrt[4]{x^3} = x^{3/4}$

Math 1232 Review Material

Function Notation: f(x) is read "f of x " or "the function f evaluated at x".
Remember that f(x) is just another name for y.

Example 1: Given that $f(x) = x^2 - 2x$, find the following.

a) $f(-2) = (-2)^2 - 2(-2)$
$4 + 4 = 8$

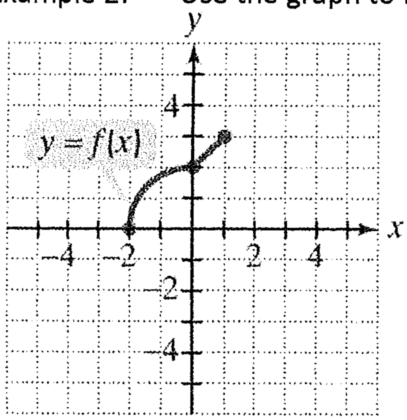
b) $f(4) = 4^2 - 2(4)$
$= 16 - 8 = 8$

c) $f(-1) = (-1)^2 - 2(-1)$
$= 1 + 2 = 3$

d) $f(x + h) =$

$(x+h)^2 - 2(x+h) = x^2 + 2xh + h^2 - 2x - 2h$
FOIL

Example 2: Use the graph to find the indicated values.



a. $f(0) = 2$

b. $f(1) = 3$

c. $f(-2) = 0$

Example 3: Given that $f(x) = 9x - 4$, find $\frac{f(x+h)-f(x)}{h}$.

$$\frac{f(x+h)-f(x)}{h} = \frac{9(x+h)-4-(9x-4)}{h} = \frac{9x+9h-4-9x+4}{h} = \frac{9h}{h} = 9$$

Example 4: Given that $f(x) = x^2 - 3x$, find $\frac{f(x+h)-f(x)}{h}$.

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^2 - 3(x+h) - (x^2 - 3x)}{h}$$

$$= \frac{x^2 + 2xh + h^2 - 3x - 3h - x^2 + 3x}{h}$$

$$= \frac{2xh + h^2 - 3h}{h}$$

$$= \frac{h(2x + h - 3)}{h} = 2x + h - 3$$

The slope m of the line through the points (x_1, y_1) and (x_2, y_2) is $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{rise}{run}$.
Slope is also the average rate of change.

A line with positive slope ($m > 0$) rises from left to right. $\swarrow\!\nearrow$ increasing

A line with negative slope ($m < 0$) falls from left to right. decreasing

A line with zero slope ($m = 0$) is horizontal. $\longleftrightarrow$ constant

A line with undefined slope is vertical. $\updownarrow$

The point-slope form of the equation of a line that passes through the point (x_1, y_1) and has slope of m is $y - y_1 = m(x - x_1)$

Example 5: Find the equation of the line through (-2, -3) having slope $m = 4$.

$$y - y_1 = m(x - x_1)$$

$$y - (-3) = 4(x - (-2))$$

$$y + 3 = 4(x + 2)$$

$$y + 3 = 4x + 8$$

$$y = 4x + 5$$

Math 1232 Review Material

Example 1: Solve the following linear equations.

a. $3(x-4) = 2x - 1$ distribute

$3x - 12 = 2x - 1$ subtract $2x$

$x - 12 = -1$ add 12

$x = 11$

b. $2x - 5 = 3 - (1 + 2x)$ distribute

$2x - 5 = 3 - 1 - 2x$ combine like terms

$2x - 5 = 2 - 2x$ add $2x$

$4x - 5 = 2$ add 5

$4x = 7$ divide by 4

$x = 7/4$

The product of two polynomials may be found by multiplying every term in the first polynomial by every term in the second polynomial and then combining like terms. When multiplying two binomials, we often call the process FOIL to remind us to multiply the first terms (F), outside terms (O), inside terms (I), and last terms (L).

Example 2: Multiply $(x+1)(x+3) = x^2 + 3x + 1x + 3 = x^2 + 4x + 3$

When factoring a polynomial, we first look for factors that are common to each term in an expression. By applying a distribute property, we can write a polynomial as two factors.

Example 3: Factor by finding the greatest common factor.

a. $2x^2 + 4x$

$2x \cdot x + 2x \cdot 2$

$2x(x + 2)$

b. $9x^4 - 6x^3 - 3x^2$

$3x^2 \cdot 3x^2 - 3x^2 \cdot 2x - 3x^2 \cdot 1$

$3x^2(3x^2 - 2x - 1)$

To factor the trinomial $x^2 + bx + c$, find integers m and n that satisfy $m \cdot n = c$ and $m + n = b$. Then $x^2 + bx + c = (x+m)(x+n)$. Think of this as FOIL in reverse.

Example 4: Factor the following.

a. $x^2 + 10x + 16$

$\cancel{(x + 1)(x + 16)}$

$(x + 2)(x + 8)$ ← $2 \cdot 8 = 16$, $2 + 8 = 10$

$(x + 4)(x + 4)$

b. $x^2 + 7x - 30$

$(x \quad 1)(x \quad 30)$

$(x \quad 2)(x \quad 15)$

$\cancel{(x \quad 5)(x \quad 6)}$

$(x - 3)(x + 10)$ ← $-3 \cdot 10 = -30$, $-3 + 10 = 7$

Example 5: Factor[redacted] the following.

a. $3x^2 + 7x + 2$

$(3x + 2)(x + 1)$

$(3x + 1)(x + 2)$

because

$3x^2 + 6x + 1x + 2$

$3x^2 + 7x + 2$ ✓

b. $6x^2 - x - 2$

$(6x \quad 1)(x \quad 2)$

$(6x \quad 2)(x \quad 1)$

$(3x \quad 1)(2x \quad 2)$

$(3x - 2)(2x + 1)$

because $6x^2 + 3x - 4x - 2$

$6x^2 - x - 2$ ✓

Example 6: Factor the difference of squares.

a. $x^2 - 9$

$(x-3)(x+3)$

b. $x^2 - 25$

$(x-5)(x+5)$

The zero-product property says that if $ab = 0$ then either $a = 0$ or $b = 0$ or both. This only works for 0.

Example 7: Solve the following using factoring and the zero-product property.

a. $x^2 - 7x = 0$

$x(x-7)=0$

$x=0 \quad x-7=0$

$x=7$

Solution $\{0, 7\}$

b. $x^2 = 3x$

$x^2 - 3x = 0$

$x(x-3)=0$

$x=0 \quad x-3=0$

$x=3$

Solution $\{0, 3\}$

c. $x^2 - 2x + 1 = 0$

$(x-1)(x-1)=0$

$x-1=0 \quad x-1=0$

$x=1 \quad x=1$

Solution $\{1\}$

d. $2x^2 + 2x - 11 = 1$

$2x^2+2x-12=0$

$x^2+x-6=0$

$(x+3)(x-2)=0$

$x+3=0 \quad x-2=0$

$x=-3 \quad x=2$

Solution $\{-3, 2\}$

The square root property says that if k is a nonnegative number, then the solutions to the equation $x^2 = k$ are given by $x = \pm\sqrt{k}$

Example 8: Solve the following using the square root property.

a. $x^2 = 16$

$x = \pm 4$

b. $x^2 - 1 = 8$

$x^2 = 9$

$x = \pm 3$

Quadratic Formula: The solutions to $ax^2 + bx + c = 0$ are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Example 9: Solve $6x^2 - x - 2 = 0$ using the quadratic formula.

$a=6 \quad b=-1 \quad c=-2$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(6)(-2)}}{2(6)} = \frac{1 \pm \sqrt{1+48}}{12} = \frac{1 \pm \sqrt{49}}{12} = \frac{1 \pm 7}{12}$$

$$x = \frac{1+7}{12} = \frac{8}{12} = \frac{2}{3} \qquad x = \frac{1-7}{12} = \frac{-6}{12} = \frac{-1}{2}$$

Solution $\{-\frac{1}{2}, \frac{2}{3}\}$