Section 2.1 Rates of Change and Tangent Lines to Curves

Big Ideas-- Average Rate of Change
Instantaneous Rate of Change
Slope of Tangent Line

DEFINITION The **average rate of change** of $y = f(x)$ with respect to x over the interval $[x_1, x_2]$ is

$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(x_1 + h) - f(x_1)}{h}, \qquad h \neq 0.$$

Example 1:
Find the average rate of change for $f(x) = x^2 - 2x$ over the interval $[1, 3]$.

Example 2:
Find the average rate of change for $f(t) = 3 + \cos t$ over the interval $\left[\frac{\pi}{2}, \frac{4\pi}{3}\right]$.

Example 3:
Let y denote the distance fallen in feet after t seconds, $y = f(t) = 16t^2$. Find the average speed of a rock dropped from a tall cliff during the first 2 seconds of falling.

How could you find the "instantaneous" rate of change or speed at the time t=1 second? The big idea is to find the average rate of change with increasingly smaller intervals. Recall that $y = f(t) = 16t^2$. Complete the following table.

Interval	Average Rate of Change (in ft/sec)
[1, 2]	$\frac{f(2)-f(1)}{2-1} = \frac{64-48}{1} = 48$
[1, 1.5]	
[1, 1.1]	
[1, 1.01]	
[1, 1.001]	
[1, 1.0001]	

As you can see, as the interval gets smaller, the average rate of speed gets closer to ______, the instantaneous speed at t=1 second.

This is the same idea as the slope of the secant line to a curve (the slope between any two points on the curve, average rate of change) and the slope of the tangent line to a curve (only one point on curve, instantaneous rate of change).

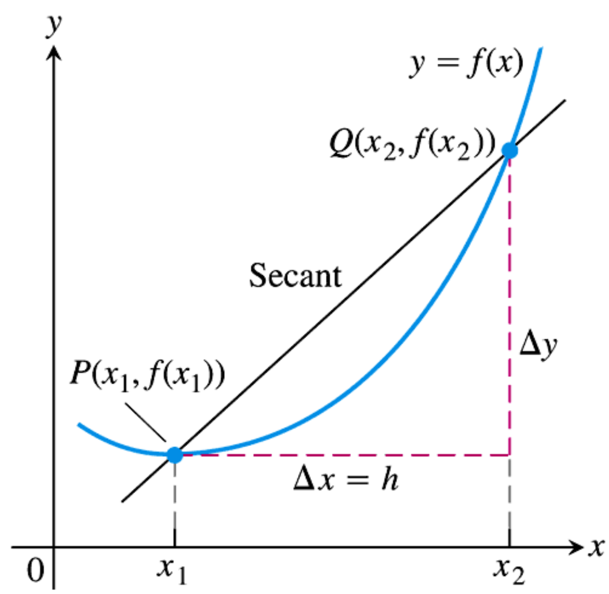

FIGURE 2.1 A secant to the graph $y = f(x)$. Its slope is $\Delta y/\Delta x$, the average rate of change of f over the interval $[x_1, x_2]$.

There is a nice animation of the slope of the tangent line at https://www.geogebra.org/m/sVCRDDmA

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Example 1:
Find the average rate of change for $f(x) = x^2 - 2x$ over the interval $[1, 3]$.

$$\frac{\Delta y}{\Delta x} = \frac{f(3) - f(1)}{3 - 1} = \frac{(3^2 - 2\cdot 3) - (1^2 - 2\cdot 1)}{3 - 1} = \frac{4}{2} = 2$$

Example 2:
Find the average rate of change for $f(t) = 3 + \cos t$ over the interval $\left[\frac{\pi}{2}, \frac{4\pi}{3}\right]$.

$$\frac{\Delta y}{\Delta x} = \frac{f\left(\frac{4\pi}{3}\right) - f\left(\frac{\pi}{2}\right)}{\frac{4\pi}{3} - \frac{\pi}{2}} = \frac{\left(3 + \cos\frac{4\pi}{3}\right) - \left(3 + \cos\frac{\pi}{2}\right)}{\frac{4\pi}{3} - \frac{\pi}{2}}$$

$$= \frac{(3 - 1/2) - (3 + 0)}{\frac{8\pi}{6} - \frac{3\pi}{6}} = \frac{-1/2}{\frac{5\pi}{6}} = -1/2 \cdot \frac{6}{5\pi} = \frac{-3}{5\pi}$$

Example 3:
Let y denote the distance fallen in feet after t seconds, $y = f(t) = 16t^2$. Find the average speed of a rock dropped from a tall cliff during the first 2 seconds of falling. $[0, 2]$

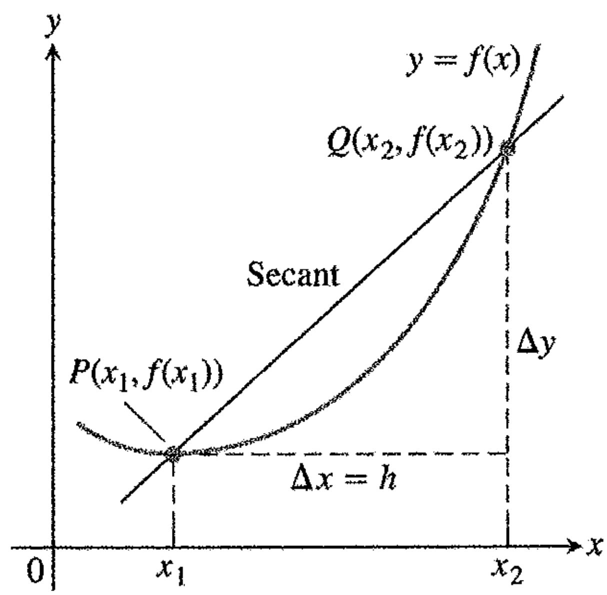
$$\frac{\Delta y}{\Delta x} = \frac{f(2) - f(0)}{2 - 0} = \frac{64 - 0}{2 - 0} = 32 \text{ ft/sec}$$

How could you find the "instantaneous" rate of change or speed at the time t=1 second? The big idea is to find the average rate of change with increasingly smaller intervals. Recall that $y = f(t) = 16t^2$. Complete the following table.

Interval	Average Rate of Change (in ft/sec)
$[1, 2]$	$\frac{f(2)-f(1)}{2-1} = \frac{64-48}{1} = 48$
$[1, 1.5]$	$= 40$
$[1, 1.1]$	$= 33.6$
$[1, 1.01]$	$= 32.16$
$[1, 1.001]$	$= 32.016$
$[1, 1.0001]$	$= 32.0016$

As you can see, as the interval gets smaller, the average rate of speed gets closer to 32 ft/sec, the instantaneous speed at t=1 second.

This is the same idea as the slope of the secant line to a curve (the slope between any two points on the curve, average rate of change) and the slope of the tangent line to a curve (only one point on curve, instantaneous rate of change).



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