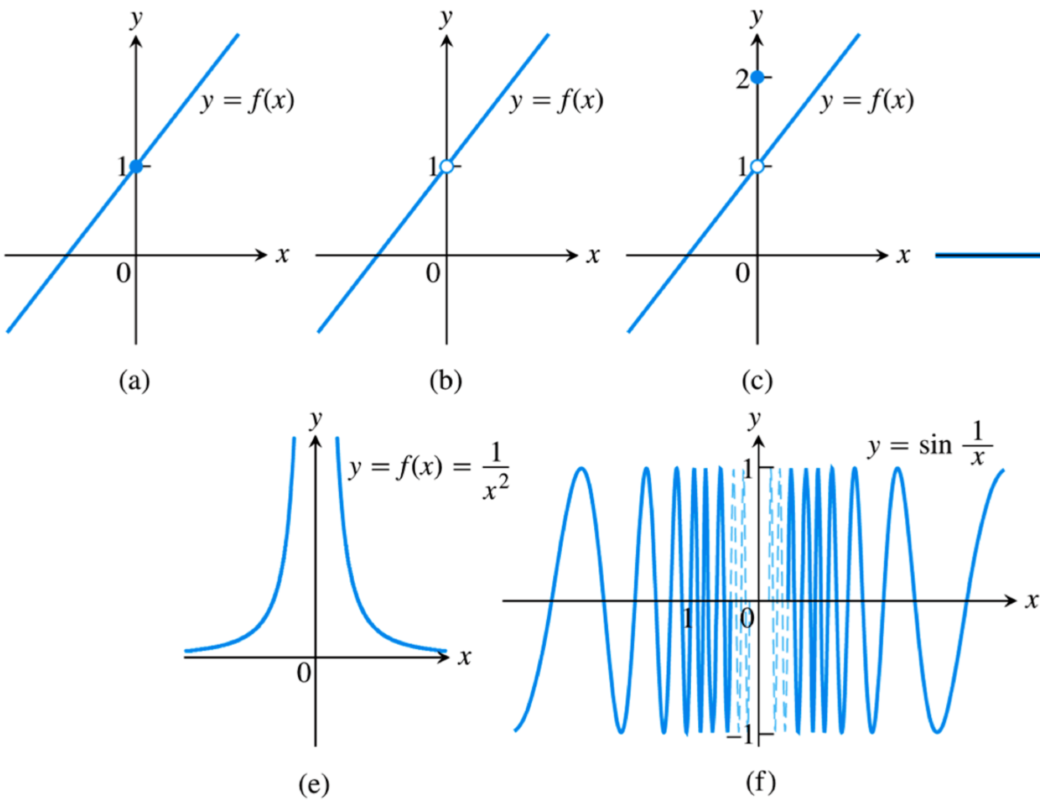
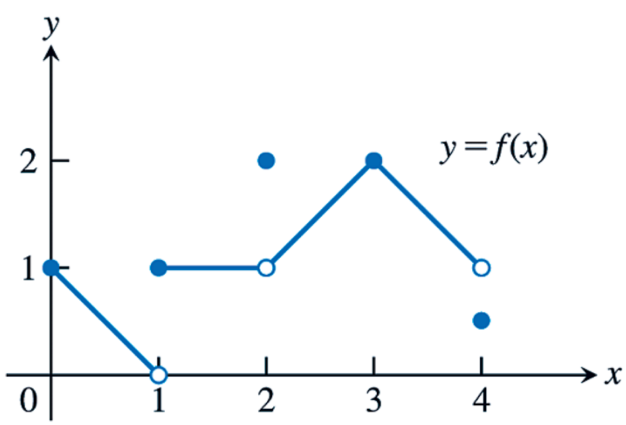
Section 2.5 Continuity and Intermediate Value Theorem

Big Idea: A function is <u>continuous</u> if you can draw it without lifting your pencil, no breaks, jumps, or holes.

Look at the functions below, only (a) is continuous at x=0. The other functions, (b)-(f) are discontinuous at x=0.



Example 1: Consider the function below. Where is the function discontinuous?



Continuity Test

A function $f(x)$ is continuous at a point $x = c$ if and only if it meets the following three conditions.

1. $f(c)$ exists (c lies in the domain of f).
2. $\lim_{x\to c} f(x)$ exists (f has a limit as $x \to c$).
3. $\lim_{x\to c} f(x) = f(c)$ (the limit equals the function value).

Example 2: At what points are the following functions continuous?

a) $f(x) = \frac{1}{x^2-9}$

b) $f(x) = 3x + 5$

c) $f(x) = \sin x$

d) $f(x) = \tan x$

e) $f(x) = \begin{cases} \frac{x^2-x-6}{x-3}, x \neq 3 \\ 5, \quad x = 3 \end{cases}$

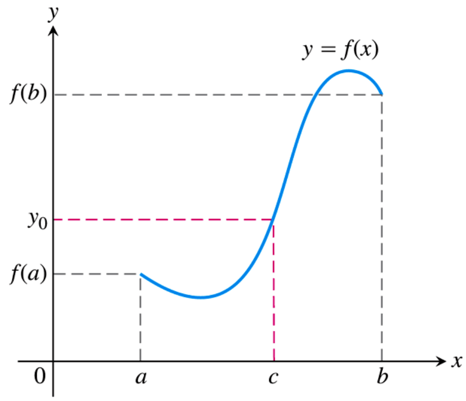
Example 3:

For what value of a is $f(x) = \begin{cases} x^2 - 1, x < 3 \\ 2ax, \quad x \geq 3 \end{cases}$ continuous at every x?

Example 4:

Define f(2) in such a way that extends $f(x) = \frac{x^2+3x-10}{x-2}$ to be continuous at x=2.

THEOREM 11—The Intermediate Value Theorem for Continuous Functions If f is a continuous function on a closed interval $[a, b]$, and if y_0 is any value between $f(a)$ and $f(b)$, then $y_0 = f(c)$ for some c in $[a, b]$.



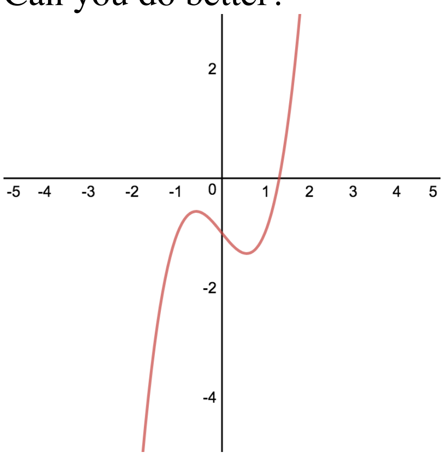
Example 5:

Show that $f(x) = x^3 - x - 1$ has a zero between x=1 and x=2.

f(1)=
f(2)=

So by the intermediate value theorem, f(c)=0 for some c in the interval $[1, 2]$.

Can you do better?



Section 2.5 Continuity and Intermediate Value Theorem

Big Idea: A function is <u>continuous</u> if you can draw it without lifting your pencil, no breaks, jumps, or holes.

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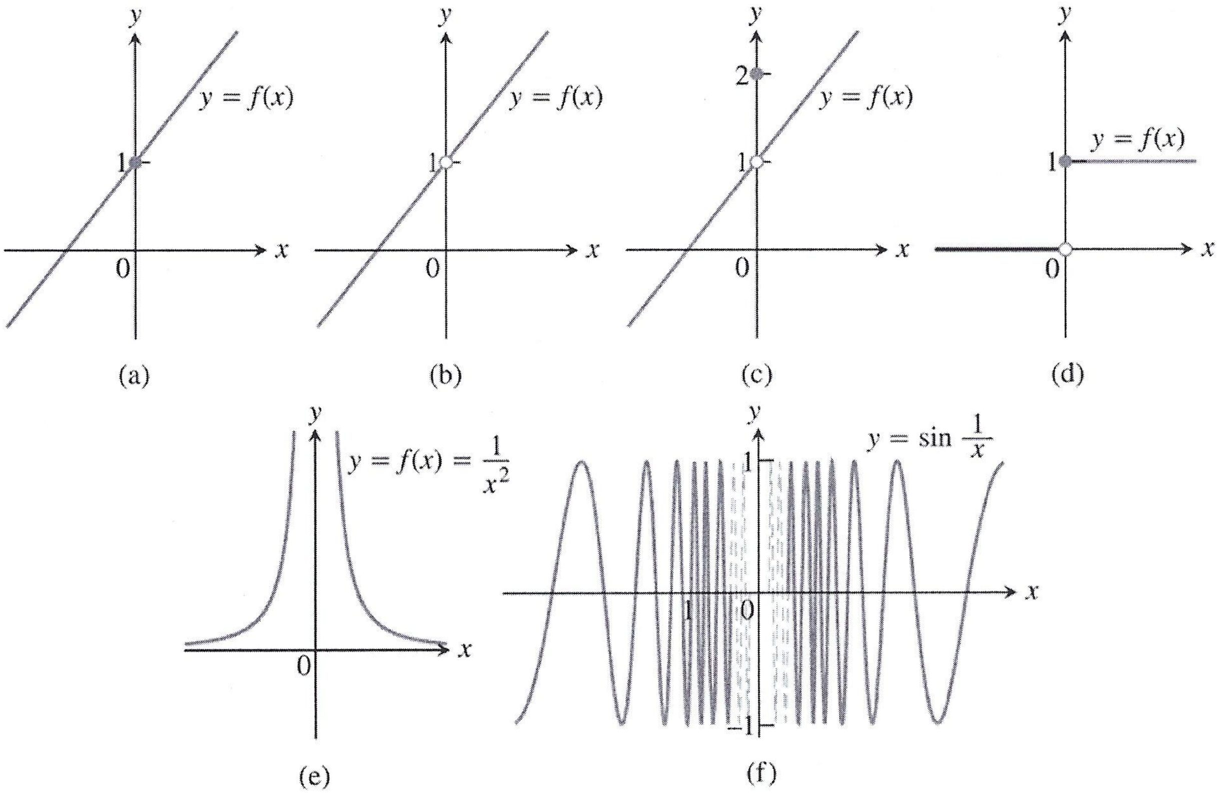
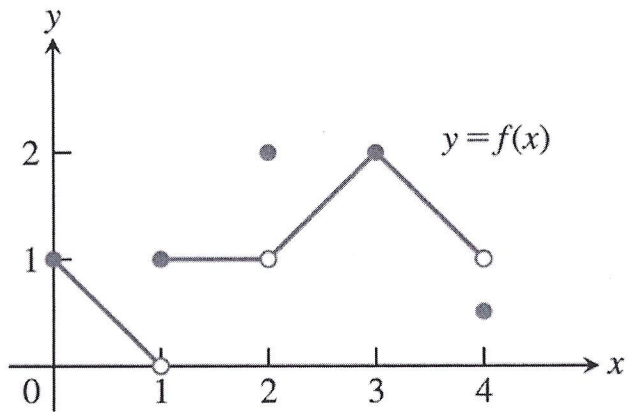


Example 1: Consider the function below. Where is the function discontinuous?



f(x) is dis continous at
x = 1, x = 2, x = 4

<u>Note</u> x = 1 is jump discontinuity

x = 2 and x = 4 are point discontinuities (removable)

Continuity Test

A function $f(x)$ is continuous at a point $x = c$ if and only if it meets the following three conditions.

1. $f(c)$ exists (c lies in the domain of f).
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3. $\lim_{x \to c} f(x) = f(c)$ (the limit equals the function value).

Example 2: At what points are the following functions continuous?

a) $f(x) = \frac{1}{x^2-9}$

discont. on $x = 3$, $x = -3$

cont on $(-\infty, -3), (-3, 3), (3, \infty)$

b) $f(x) = 3x + 5$

continuous $(-\infty, \infty)$

c) $f(x) = \sin x$

continuous $(-\infty, \infty)$

d) $f(x) = \tan x$

discontinuous $x = \pi/2, \frac{3\pi}{2}$, etc $(2n+1)\frac{\pi}{2}$

e) $f(x) = \begin{cases} \frac{x^2-x-6}{x-3}, & x \neq 3 \\ 5, & x = 3 \end{cases}$

$f(3) = 5$

$\lim_{x \to 3} f(x) = 5$

continuous $(-\infty, \infty)$

Example 3:

For what value of a is $f(x) = \begin{cases} x^2 - 1, x < 3 \\ 2ax, \quad x \geq 3 \end{cases}$ continuous at every x? $a = 4/3$

$3^2 - 1 = 2a(3)$

$8 = 6a$

$4/3 = a$

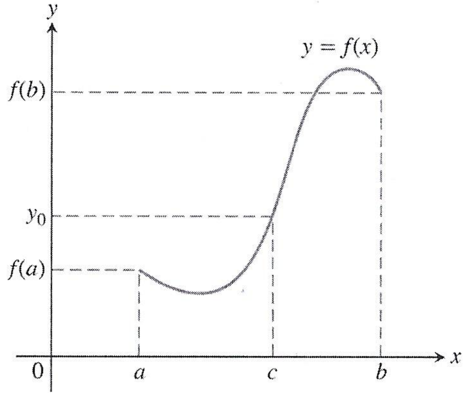
Example 4:

Define f(2) in such a way that extends $f(x) = \frac{x^2+3x-10}{x-2}$ to be continuous at x=2.

$\lim_{x \to 2} \frac{(x-2)(x+5)}{x-2} = \lim_{x \to 2} x + 5 = 7$

So $f(2) = 7$

THEOREM 11—The Intermediate Value Theorem for Continuous Functions If f is a continuous function on a closed interval $[a, b]$, and if y_0 is any value between $f(a)$ and $f(b)$, then $y_0 = f(c)$ for some c in $[a, b]$.



Example 5:
Show that $f(x) = x^3 - x - 1$ has a zero between x=1 and x=2.

f(1)= 1 − 1 − 1 = −1
f(2)= 8 − 2 − 1 = 5

So by the intermediate value theorem, f(c)=0 for some c in the interval [1, 2].

Can you do better?