Section 2.6 Limits Involving Infinity, Asymptotes of Graphs

<u>Limit Review</u>

$\lim_{x \to c^+} f(x)$ is the limit of f(x) as x approaches c from the right

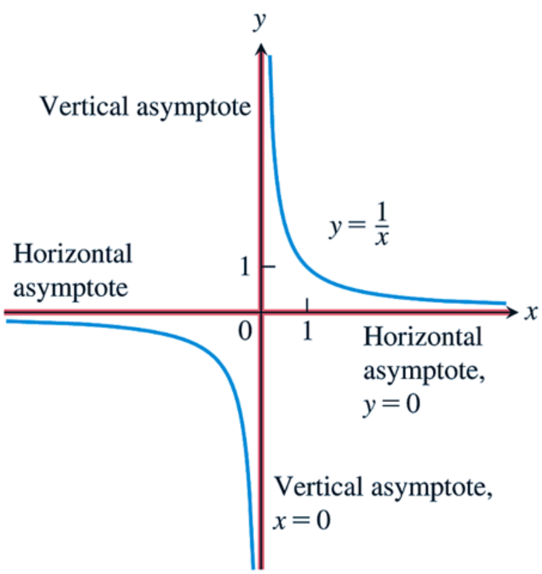
$\lim_{x \to c^-} f(x)$ is the limit of f(x) as x approaches c from the left

If f(x) is increasing without bound, the limit is +∞.

If f(x) is decreasing without bound, the limit is -∞.

Limits can also be used to describe behavior at the extreme right and left sides of the graph.

$\lim_{x \to -\infty} f(x)$ and $\lim_{x \to +\infty} f(x)$ refer to end behavior



Example 1:

Let's examine limits using the graph of $f(x) = \frac{1}{x}$.

f(0)

$\lim_{x \to 0^+} f(x)$

$\lim_{x \to 0^-} f(x)$

$\lim_{x \to 0} f(x)$

$\lim_{x \to -\infty} f(x)$

$\lim_{x \to +\infty} f(x)$

Example 2: $\lim_{x \to 0} \frac{1}{x}$

x	-.1	-.01	-.001	0	.001	.01	.1
f(x)				??			

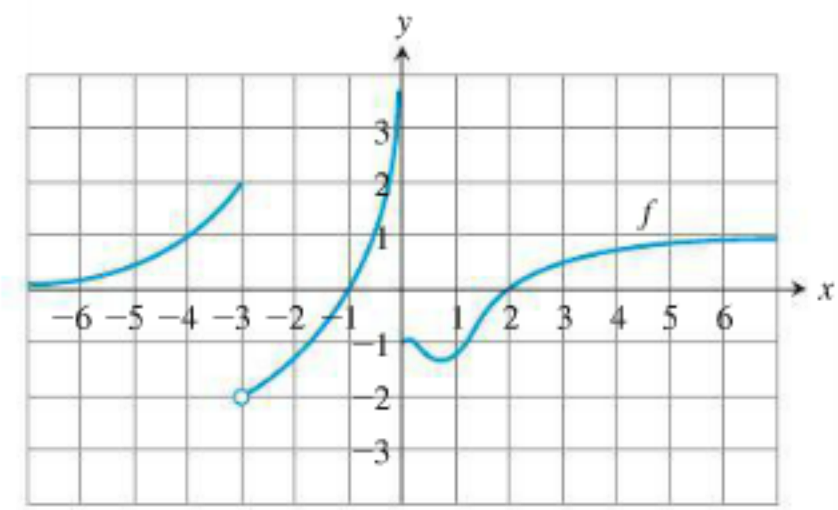
Example 3: $\lim_{x \to -\infty} \frac{1}{x}$

x	-10	-100	-1000	-∞
f(x)				??

Example 4: $\lim_{x \to +\infty} \frac{1}{x}$

x	10	100	1000	∞
f(x)				??

Example 5:



$\lim_{x\to 2} f(x)$

$\lim_{x\to -3^+} f(x)$

$\lim_{x\to -3^-} f(x)$

$\lim_{x\to -3} f(x)$

$\lim_{x\to 0^+} f(x)$

$\lim_{x\to 0^-} f(x)$

$\lim_{x\to 0} f(x)$

$\lim_{x\to -\infty} f(x)$

$\lim_{x\to +\infty} f(x)$

Example 6: Find the limit of each function as i) $x \to \infty$ and ii) $x \to -\infty$.

a) $f(x) = \frac{2}{x} - 3$

b) $f(x) = \frac{2x+3}{5x+7}$

c) $f(x) = \frac{7x^3}{x^3-3x^2+6x}$

d) $f(x) = \frac{3x^7+5x^2-1}{6x^3-7x+3}$

Example 7: Find the following limits using the Sandwich Theorem.

a) $\lim_{x\to +\infty} \frac{\sin 2x}{x}$

b) $\lim_{\theta\to -\infty} \frac{\cos\theta}{3\theta}$

Example 8: Find the following limits

a) $\lim_{x \to 0^-} \frac{1}{3x}$

b) $\lim_{x \to -8^+} \frac{2x}{x+8}$

c) $\lim_{x \to 7} \frac{14}{(x-7)^2}$

d) $\lim_{x \to \frac{\pi}{2}^-} \tan x$

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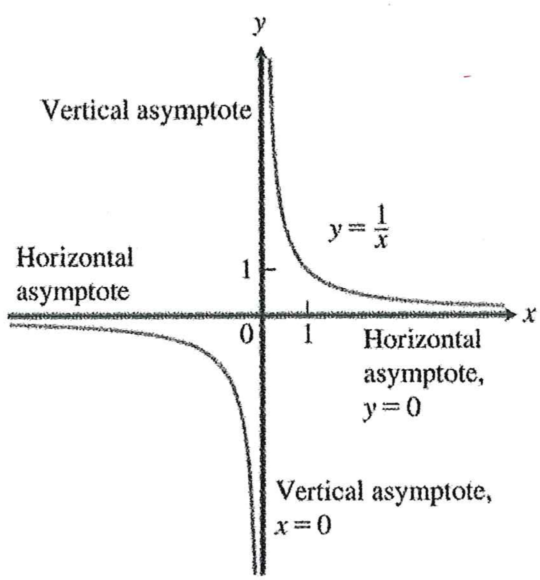


Example 1:

Let's examine limits using the graph of $f(x) = \frac{1}{x}$.

f(0) undefined

$\lim_{x \to 0^+} f(x) = +\infty$

$\lim_{x \to 0^-} f(x) = -\infty$

x=0 is vertical asymptote

$\lim_{x \to 0} f(x)$ DNE

$\lim_{x \to -\infty} f(x) = 0$

$\lim_{x \to +\infty} f(x) = 0$

y=0 is horizontal asymptote

Example 2: $\lim_{x \to 0} \frac{1}{x}$ DNE

x	-.1	-.01	-.001	0	.001	.01	.1
f(x)	-10	-100	-1000	??	1000	100	10

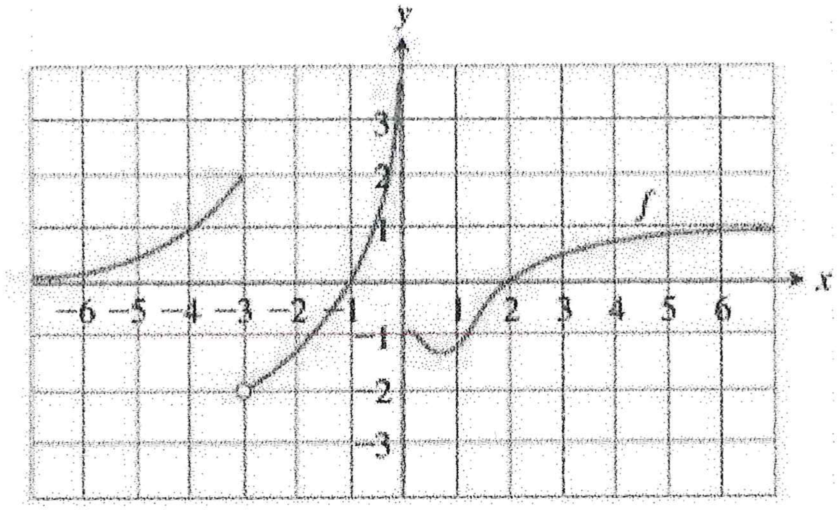
Example 3: $\lim_{x \to -\infty} \frac{1}{x} = 0$

x	-10	-100	-1000	-∞
f(x)	-.1	-.01	-.001	??

Example 4: $\lim_{x \to +\infty} \frac{1}{x} = 0$

x	10	100	1000	∞
f(x)	.1	.01	.001	??

Example 5:



$\lim_{x\to 2} f(x) = 0$

$\lim_{x\to -3^+} f(x) = -2$

$\lim_{x\to -3^-} f(x) = 2$

$\lim_{x\to -3} f(x)$ DNE

$\lim_{x\to 0^+} f(x) = -1$

$\lim_{x\to 0^-} f(x) = +\infty$

$\lim_{x\to 0} f(x)$ DNE

$\lim_{x\to -\infty} f(x) = 0$

$\lim_{x\to +\infty} f(x) = 1$

Example 6: Find the limit of each function as i) $x \to \infty$ and ii) $x \to -\infty$.

a) $f(x) = \frac{2}{x} - 3$

$\lim_{x\to\infty} \frac{2}{x} - 3 = 0 - 3 = -3$ $\lim_{x\to-\infty} f(x) = -3$

b) $f(x) = \frac{2x+3}{5x+7} = \frac{\frac{2x}{x} + \frac{3}{x}}{\frac{5x}{x} + \frac{7}{x}} = \frac{2 + \frac{3}{x}}{5 + \frac{7}{x}}$

$\lim_{x\to\infty} f(x) = \frac{2+0}{5+0} = \frac{2}{5} = \lim_{x\to-\infty}$

c) $f(x) = \frac{7x^3}{x^3-3x^2+6x} = \frac{\frac{7x^3}{x^3}}{\frac{x^3}{x^3} - \frac{3x^2}{x^3} + \frac{6x}{x^3}} = \frac{7}{1 - \frac{3}{x} + \frac{6}{x^2}}$

$\lim_{x\to\infty} = \frac{7}{1-0+0} = 7 = \lim_{x\to-\infty}$

d) $f(x) = \frac{3x^7+5x^2-1}{6x^3-7x+3}$

$= \frac{\frac{3x^7}{x^3} + \frac{5x^2}{x^3} - \frac{1}{x^3}}{\frac{6x^3}{x^3} - \frac{7x}{x^3} + \frac{3}{x^3}} = \frac{3x^4 + \frac{5}{x} - \frac{1}{x^3}}{6 - \frac{7}{x^2} + \frac{3}{x^3}}$

$\lim_{x\to\infty} \frac{3x^4+0-0}{6-0+0} = +\infty$

$\lim_{x\to-\infty} \frac{3x^4+0-0}{6-0+0} = +\infty$

Example 7: Find the following limits using the Sandwich Theorem.

a) $\lim_{x\to+\infty} \frac{\sin 2x}{x}$

$\lim_{x\to+\infty} -\frac{1}{x} \le \lim_{x\to+\infty} \frac{\sin 2x}{x} \le \lim_{x\to+\infty} \frac{1}{x}$

$0 \le \lim_{x\to+\infty} \frac{\sin 2x}{x} \le 0$

$= 0$

b) $\lim_{\theta\to-\infty} \frac{\cos\theta}{3\theta}$

$\lim_{\theta\to-\infty} \frac{-1}{3\theta} \le \lim_{\theta\to-\infty} \frac{\cos\theta}{3\theta} \le \lim_{\theta\to-\infty} \frac{1}{3\theta}$

$0 \le \lim_{\theta\to-\infty} \frac{\cos\theta}{3\theta} \le 0$

$= 0$

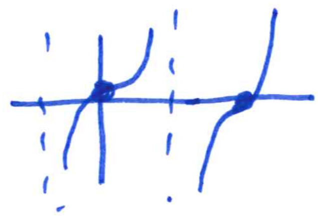
Example 8: Find the following limits

a) $\lim\limits_{x \to 0^-} \frac{1}{3x} = \frac{1}{-0} = -\infty$

b) $\lim\limits_{x \to -8^+} \frac{2x}{x+8} = \frac{-16}{+0} = -\infty$

c) $\lim\limits_{x \to 7} \frac{14}{(x-7)^2} = \frac{14}{0^2} = +\infty$

d) $\lim\limits_{x \to \frac{\pi}{2}^-} \tan x = +\infty$ recall graph



$\lim\limits_{x \to \frac{\pi}{2}^-} \frac{\sin x}{\cos x} = \frac{1}{+0}$