✓ 1) $f(x) = 10x^3 + 10$ <u>average rate of change</u>

a) $[4,6]$ $[x_1, x_2]$ $\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$

$= \frac{f(6) - f(4)}{6-4} = \frac{(10(6)^3 + 10) - (10(4)^3 + 10)}{6-4}$

$= \frac{(2160 + 10) - (640 + 10)}{6-4} = 760$

✓ 1b) $[-5,5]$ $[x_1\ x_2]$

$= \frac{f(5) - f(-5)}{5-(-5)} = \frac{(10(5)^3 + 10) - (10(-5)^3 + 10)}{5-(-5)}$

$= \frac{(1250 + 10) - (-1250 + 10)}{5-(-5)} = \frac{1260 + 1240}{10} = \frac{2500}{10} = 250$

✓ 2 a) slope of $y = x^2 - 4x - 4$ at $P(3,-7)$

$\frac{f(x+h) - f(x)}{h} = \frac{f(3+h) - f(3)}{h}$

$= \frac{((3+h)^2 - 4(3+h) - 4) - (3^2 - 4(3) - 4)}{h} = \frac{(9 + 6h + h^2 - 12 - 4h - 4) + 7}{h}$

$= \frac{h^2 + 2h \cancel{-7} \cancel{+7}}{h} = \frac{h^2 + 2h}{h} = \frac{h(h+2)}{h} = \underset{h \to 0}{h+2} = 2$

√ 2b) Find equation of the tangent line.

from part a $m = 2$

$y - y_1 = m(x - x_1)$

$y - -7 = 2(x - 3)$

$y + 7 = 2x - 6$

$-7 \quad\quad -7$

$\boxed{y = 2x - 13}$

√ 3) find the limit

$$\lim_{x \to -2} (-x^2 + 5x - 4)$$

$= (-(-2)^2 + 5(-2) - 4)$

$= (-4 - 10 - 4)$

$= -18$

√ 4) find $\lim_{x \to 17} \frac{x-17}{x^2-289}$ $\quad \frac{0}{0}$

$= \lim_{x \to 17} \frac{\cancel{(x-17)}}{\cancel{(x-17)}(x+17)}$

$= \lim_{x \to 17} \frac{1}{x+17} = \frac{1}{17+17} = \frac{1}{34}$

√ 5) find $\lim_{x \to 144} \frac{\sqrt{x} - 12}{x - 144} \cdot \frac{\sqrt{x} + 12}{\sqrt{x} + 12}$

$= \lim_{x \to 144} \frac{\cancel{x - 144}}{\cancel{(x-144)}(\sqrt{x} + 12)} = \lim_{x \to 144} \frac{1}{(\sqrt{x} + 12)}$

$= \frac{1}{\sqrt{144} + 12} = \frac{1}{12 + 12} = \frac{1}{24}$

√ 6) find

$$\lim_{x \to 0} \frac{5 + 4x + \sin x}{6 \cos x}$$

$= \frac{5 + 4(0) + \sin(0)}{6 \cos 0}$

$= \frac{5 + 0 + 0}{6(1)} = \frac{5}{6}$

√ 7) $f(x) = x^2 \quad x = -8$

$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \Rightarrow \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$

$\lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{x^2}}{h} = \lim_{x \to 0} \frac{2xh + h^2}{h}$

$\lim_{h \to 0} \frac{\cancel{h}(2x + h)}{\cancel{h}} = 2x + \cancel{h}^{0} = 2x$

$2(-8) = -16$

√8) $\boxed{C}$ lim DNE

9) √a) $\lim_{x\to 3^+} f(x) = 4$

$\lim_{x\to 3^-} f(x) = 1$

√9c) $\lim_{x\to 4^+} f(x) = 5$

$\lim_{x\to 4^-} f(x) = 5$

9√b) Does $\lim_{x\to 3} f(x)$ exist?

$\boxed{\text{(A) no}}$ b/c $\lim_{x\to 3^+} f(x) \neq \lim_{x\to 3^-} f(x)$

√9d) does $\lim_{x\to 4} f(x)$ exist?

yes b/c $\lim_{x\to 4^+} f(x) = \lim_{x\to 4^-} fx$

√10) use $\lim_{x\to 0} \frac{\sin x}{x} = 1$

to find $\lim_{\theta\to 0} \frac{\sin\sqrt{5}\,\theta}{\sqrt{5}\,\theta} = 1$

Let $x = \sqrt{5}\,\theta$

√11) find $\lim_{\theta\to\frac{\pi}{2}} \theta\cos\theta$

$\frac{\pi}{2} \cdot \cos\frac{\pi}{2} = \frac{\pi}{2} \cdot 0 = 0$

√12) B

√13) not continuous at $x = 2$

14) a) is f defined at $x=2$?

yes

14 b) is f continuous at $x=2$?

yes

15) $f(x)$ at $x = 5$

(A) The function is continuous!

16) a) $\lim_{x\to\infty} \frac{19x^4}{3x^4+15x^3+14x^2} = \lim_{x\to\infty} \frac{\frac{19x^4}{x^4}}{\frac{3x^4}{x^4}+\frac{15x^3}{x^4}+\frac{14x^2}{x^4}} = \frac{19}{3+0+0}$

$= \frac{19}{3}$

√ 16b) $\lim_{x\to-\infty} \dfrac{19x^4}{3x^4+15x^3+14x^2} = \lim_{x\to-\infty} \dfrac{\frac{19x^4}{x^4}}{\frac{3x^4}{x^4}+\frac{15x^3}{x^4}+\frac{14x^2}{x^4}}$

$= \dfrac{19}{3+0+0} = \dfrac{19}{3}$

√ 17) find $\lim_{x\to\infty} \dfrac{4x^4+6x^3+9}{7x^5} = \lim_{x\to\infty} \dfrac{\frac{4x^4}{x^5}+\frac{6x^3}{x^5}+\frac{9}{x^5}}{\frac{7x^5}{x^5}}$

$= \dfrac{0+0+0}{7} = 0$

$\lim_{x\to-\infty} \dfrac{4x^4+6x^3+9}{7x^5} = \lim_{x\to-\infty} \dfrac{\frac{4x^4}{x^5}+\frac{6x^3}{x^5}+\frac{9}{x^5}}{\frac{7x^5}{x^5}}$

$= \dfrac{0+0+0}{7} = 0$

---

18) find $\lim_{x\to\infty} \sqrt[3]{\dfrac{1+8x^2}{x^2+9}} = \lim_{x\to\infty} \sqrt[3]{\dfrac{\frac{1}{x^2}+\frac{8x^2}{x^2}}{\frac{x^2}{x^2}+\frac{9}{x^2}}} = \sqrt[3]{\dfrac{0+8}{1+0}}$
√

$= \sqrt[3]{8} = 2$

---

19) find $\lim_{x\to\infty} \dfrac{\sqrt{x^4+4}}{2x^2+1} = \lim_{x\to\infty} \dfrac{\frac{\sqrt{x^4+4}}{\sqrt{x^4}}}{\frac{2x^2+1}{\sqrt{x^4}}} = \lim_{x\to\infty} \dfrac{\sqrt{\frac{x^4+4}{x^4}}}{\frac{2x^2+1}{x^2}} = \lim_{x\to\infty} \dfrac{\sqrt{1+\frac{4}{x^4}}}{2+\frac{1}{x^2}}$
√

$= \dfrac{\sqrt{1+0}}{2} = \boxed{\dfrac{1}{2}}$

√20) find $\lim_{x \to 3^+} \frac{3}{x-3} = \infty$ $\left(\frac{\text{positive}}{\text{positive}}\right)$ $\left(\frac{3}{0^+}\right)$

√21) a $\boxed{C}$ b B c) $p'(t) = 14.4(t) - 19.85$

$p'(5) = 14.4(5) - 19.85$

$= \boxed{52.15 \ \frac{\text{cells}}{\text{hour}}}$

√22) $f(t) = 3t^2$

$f'(t) = 6t$

$t = 7$ $6(7) = \boxed{42 \ \frac{\text{ft}}{\text{sec}}}$

√23) find $\frac{dy}{dx}$ if $y = -2x^{\frac{3}{2}}$

$\frac{dy}{dx} = \left(\frac{3}{2}\right) - 2x^{\frac{3}{2}-1} = \left(\frac{3}{\cancel{2}}\right) - \cancel{2}x^{\frac{3}{2}-\frac{2}{2}}$

$= \boxed{-3x^{\frac{1}{2}}}$

√24) $s = -4t^4 - 2t^3$ $t = -1$

$s'(t) = -16t^3 - 6t^2$

$s'(-1) = -16(-1)^3 - 6(-1)^2$

$s'(-1) = 16 - 6 = 10$ $\boxed{C}$

√25) $y = 14x^{-2} + 5x^3 - 2x$

find $y' = -28x^{-3} + 15x^2 - 2$ A

√26) find second derivative

$y = 4x^2 + 8x + 4x^{-3}$

$y' = 8x + 8 - 12x^{-4}$

$\boxed{y'' = 8 + 48x^{-5}}$

27) find y'

$y = (3x^3 + 3)(3x^7 - 9)$

$y' = (9x^2)(3x^7 - 9) + (3x^3 + 3)(21x^6)$

$= 27x^9 - 81x^2 + 63x^9 + 63x^6$

$= \boxed{90x^9 + 63x^6 - 81x^2}$

✓28) $y = \dfrac{x^2 - 3x + 2}{x^7 - 2}$

find $y' = \dfrac{(2x-3)(x^7-2) - (x^2-3x+2)(7x^6)}{(x^7-2)^2}$

$= \dfrac{2x^8 - 4x - 3x^7 + 6 - 7x^8 + 21x^7 + 14x^6}{(x^7-2)^2}$

$= \boxed{\dfrac{-5x^8 + 18x^7 + 14x^6 - 4x + 6}{(x^7-2)^2}}$

✓29) find w'

$w = z^{5-e}$

$\boxed{w' = (5-e)z^{4-e}}$

✓30) find y' & y''

$y = -9x^6 + 1$

$\boxed{y' = -54x^5}$

$\boxed{y'' = -270x^4}$