Section 3.10 Related Rates

Recall that f'(x)= instantaneous rate of change= slope of tangent line

Steps to Solve Related Rates

1. Draw a picture and label.
2. Identify the rates of change that are known and unknown.
   If the rate of change is increasing, it will be positive.
   If the rate of change is decreasing, it will be negative.
3. Find an equation to model the situation.
4. Differentiate both sides of the equation with respect to time and solve for the derivative that will give the unknown rate of change. Use implicit differentiation.
5. Evaluate this derivative at the appropriate point.

Example 1: **Circle** $\boldsymbol{A = \pi r^2}$

Assume that oil spilled from a ruptured tanker spreads in circular pattern whose radius increases at a constant rate of 2 ft/sec. How fast is the area of the spill increasing when the radius of the spill is 60 ft?

Example 2: **Pythagorean Theorem** $\boldsymbol{a^2 + b^2 = c^2}$

A 5 ft ladder leaning against a wall, slips in such a way that its base is moving away from the wall at a rate of 2 ft/sec at the instant when the base is 4 ft from the wall. How fast is the top of the ladder moving down the wall at that instant?

Example 3: **Trig Triangle**

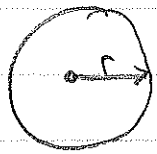
A camera is photographing a rocket launch. The camera is located 3000 ft from the launch pad. If the rocket is rising vertically at 880 ft/sec when it is 4000 ft, how fast must the camera angle change at that instant to keep the rocket in sight?

Example 4: **Cone** $\boldsymbol{V = \frac{1}{3}\pi r^2 h}$ **Similar Triangles**

Suppose that a liquid is to be cleared of sediment by pouring it through a cone shaped filter. Assume that the height of the cone is 16 in. and the radius at the base of the cone is 4 in. If the liquid is flowing out of the cone at a rate of 2 cubic inches/min when the level is 8 inches deep, how fast is the depth of the liquid changing at that instant?

Related Rates — Class Notes

Example 1



Known $\frac{dr}{dt} = 2$ ft/sec

$r = 60$ ft (instant)

Unknown $dA/dt = ?$

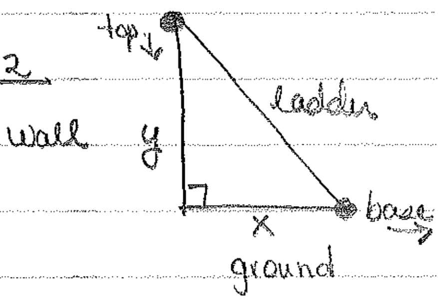
Equation $A = \pi r^2$

$$dA/dt = 2\pi r \, dr/dt$$

$$dA/dt = 2\pi(60\text{ ft})(2\text{ ft/sec})$$

$$dA/dt = 240\pi \text{ ft}^2\text{/sec}$$

Example 2



Known: ladder $= 5$ ft (constant)

$x = 4$ ft (instant)

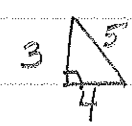
$dx/dt = 2$ ft/sec

Unknown $dy/dt = ?$

Equation $x^2 + y^2 = 5^2$ ← ladder is constant

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$2(4\text{ ft})(2\text{ ft/sec}) + 2(3\text{ ft})\frac{dy}{dt} = 0$$



$$16\text{ ft}^2\text{/sec} + 6\text{ ft}\, dy/dt = 0$$

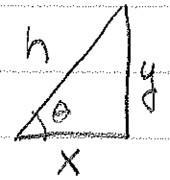
$$dy/dt = -16/6 \text{ ft/sec}$$

$$dy/dt = -8/3 \text{ ft/sec}$$

↑

negative, ladder sliding down

Example 3



Known: $x = 3000$ ft (constant)

$y = 4000$ ft (instant)

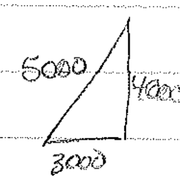
$dy/dt = 880$ ft/sec

unknown $d\theta/dt = ?$

Equation: $\tan\theta = \frac{y}{3000 \text{ ft}}$

$(3000 \text{ ft})\tan\theta = y$

$3000 \text{ ft} \cdot \sec^2\theta \; d\theta/dt = dy/dt$



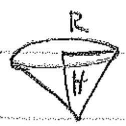
$3000 \text{ ft} \cdot \left(\frac{5000}{3000}\right)^2 \frac{d\theta}{dt} = 880$ ft/sec

$\frac{d\theta}{dt} = \frac{66}{625}$ radians/sec

radians unitless measure

$\frac{d\theta}{dt} = .11$ radians/sec

Example 4



Known $\frac{h}{r} = \frac{16}{4}$ proportion

unknown $dh/dt = ?$

$4h = 16r$

$\frac{h}{4} = r$

$dV/dt = -2$ in^3/min

$h = 8$ in (instant)

Equation $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{h}{4}\right)^2 h = \frac{1}{48}\pi h^3$

$dV/dt = 1/16\, \pi h^2\, dh/dt$

$-2 \text{ in}^3/\text{min} = 1/16\, \pi\, (8 \text{ in})^2\, dh/dt$

$-2 \text{ in}^3/\text{min} = 4\pi \text{ in}^3\, dh/dt$

$\frac{-1}{2\pi}$ in/min $= dh/dt$