Section 3.4 Derivative as a Rate of Change

DEFINITION The **instantaneous rate of change** of f with respect to x at x_0 is the derivative

$$f'(x_0) = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h},$$

provided the limit exists.

Recall that the notations $f'(x), y', \frac{dy}{dx}$ all mean derivative or instantaneous rate of change. We often omit the word instantaneous.

$\frac{dy}{dx}$ represents instantaneous rate of change of variable y with respect to the variable x.

Example 1: The area of a circle is related to its diameter $A = \frac{\pi}{4}D^2$. How fast does the area change with respect to the diameter is 10 meters?

Motion as Rate of Change:
Let $s = f(t)$ represent the position of an object on a line as a function of time.

Velocity	$v(t) = s'$
Speed	$\lvert v(t) \rvert$
Acceleration	$a(t) = v'(t) = s''$
Jerk	$j(t) = a'(t) = v''(t) = s'''$

Example 2: A dynamite blast blows a heavy rock straight up with a launch velocity of 160 ft/sec. It reaches a height of $s = 160t - 16t^2$ feet after t seconds.
a) Find $v(t)$, speed, and $a(t)$.
b) When does the rock hit the ground again?
c) How high did it go?
d) What are the velocity and speed of the rock when it is 256 ft above the ground on the way up? On the way down?

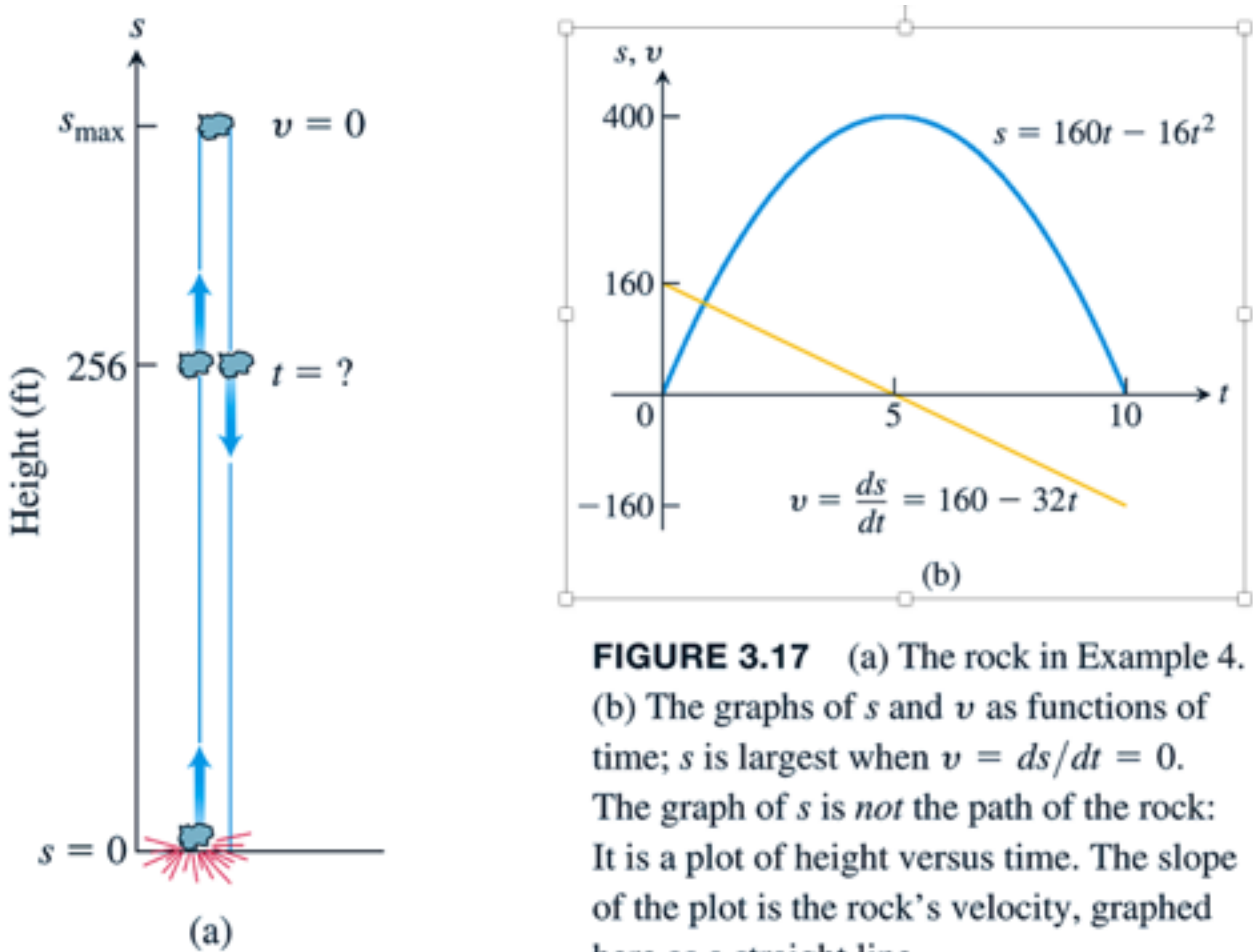

FIGURE 3.17 (a) The rock in Example 4. (b) The graphs of s and v as functions of time; s is largest when $v = ds/dt = 0$. The graph of s is *not* the path of the rock: It is a plot of height versus time. The slope of the plot is the rock's velocity, graphed here as a straight line.

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Example 1: The area of a circle is related to its diameter $A = \frac{\pi}{4}D^2$. How fast does the area change with respect to the diameter is 10 meters?

$$\frac{dA}{dD} = 2 \cdot \frac{\pi}{4} D = \frac{\pi}{2} D$$

when $D = 10$ $\rightarrow \frac{dA}{dD} = \frac{\pi}{2} \cdot 10 = 5\pi \ m^2/m$

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a) $v(t) = s' = 160 - 32t$ ft/sec

speed $= |v| = |160 - 32t|$ ft/sec

$a(t) = v' = s'' = -32$ ft/sec^2

b) rock is at ground level when $t=0$ (initial blast), goes up and then falls back to ground.

$$s = 160t - 16t^2 = 0$$ ← height of ground

$$16t(10-t) = 0$$

$t=0$ (initial) $\boxed{t=10}$ 10 seconds later

c) maxium height is achieved when velocity is zero.

$$v(t) = 160 - 32t$$
$$0 = 160 - 32t$$
$$32t = 160$$
$$t = 5 \text{ seconds}$$
$$s(t) = 160(5) - 16(5)^2 = 400 \text{ ft}$$

max height is after 5 seconds and is 400 ft.

d) $s = 160t - 16t^2 = 256$

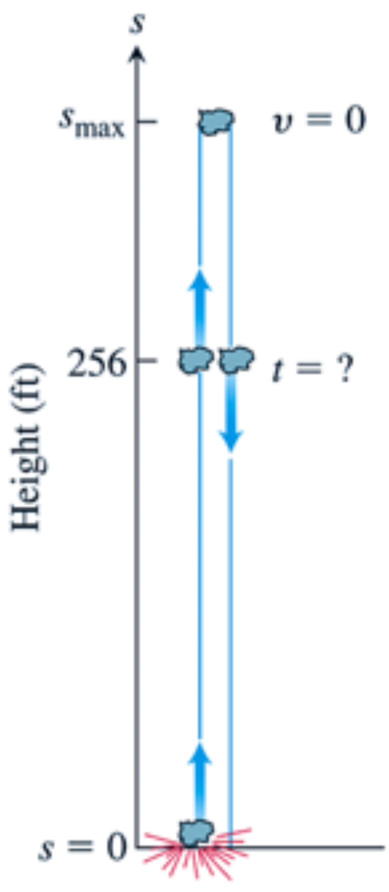
$$0 = 16t^2 - 160t + 256$$
$$0 = 16(t^2 - 10t + 16)$$
$$0 = 16(t-2)(t-8)$$

$t=2$ on way up $t=8$ on way down

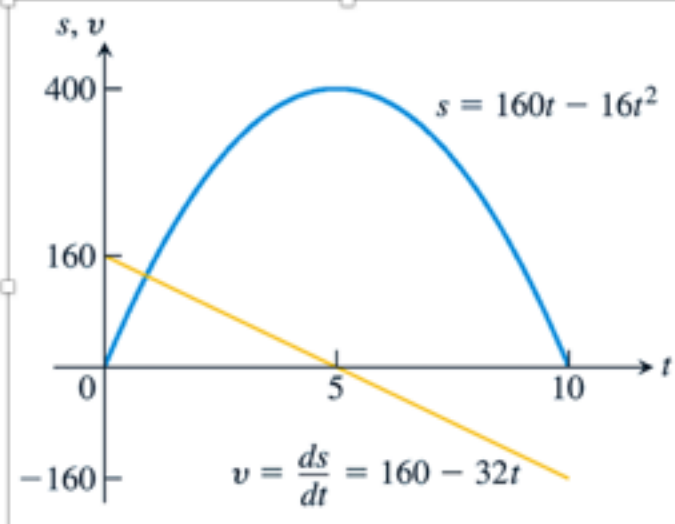
when $t=2$
$v(2) = 160 - 32(2) = 96$ ft/sec
speed $= |96| = 96$ ft/sec

when $t=8$
$v(8) = 160 - 32(8) = -96$
speed $= |-96| = 96$

* sign of velocity indicates direction



(a)



(b)

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