Section 3.5 Derivatives of Trigonometric Functions

The derivative of the sine function is the cosine function:

$$\frac{d}{dx}(\sin x) = \cos x.$$

The derivative of the cosine function is the negative of the sine function:

$$\frac{d}{dx}(\cos x) = -\sin x$$

Example 1: Using the rules above, find $\frac{d}{dx}(\tan x)$.

Example 2: Using the rules above, find $\frac{d}{dx}(\cot x)$.

The derivatives of the other trigonometric functions:

$$\frac{d}{dx}(\tan x) = \sec^2 x \qquad \frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x \qquad \frac{d}{dx}(\csc x) = -\csc x \cot x$$

Example 3: Find the fourth derivative of $y = \sin x$.

Example 4: Find $f'(x)$ when $f(x) = 3\cos x - 4\sec x$.

Example 5: Find $f'(x)$ when $f(x) = x^2 \sec x$.

Example 6: Find $f'(x)$ when $f(x) = \sin^2 x + \cos^2 x$.

Example 7: Find $f'(x)$ when $f(x) = \sqrt{x}\sec x + 3$.

Example 8: Find $f'(x)$ when $f(x) = x^2 \cot x - \frac{1}{x^2}$.

Example 9: Find $f'(x)$ when $f(x) = \frac{4}{\cos x} + \frac{1}{\tan x}$.

Example 10: Find $f'(x)$ when $f(x) = (\sec x + \tan x)(\sec x - \tan x)$.

Example 11: Find the x-values in $0 \leq x < 2\pi$ where the tangent line is horizontal to the curve $f(x) = x + \sin x$.

Example 12: Find the x-values in $0 \leq x < 2\pi$ where the tangent line is horizontal to the curve $f(x) = x - \cot x$.

Example 13: Find the x-values in $0 \leq x < 2\pi$ where the tangent line is horizontal to the curve $f(x) = x + 2\cos x$.

Example 14: Find the equation of the tangent line to the curve $f(x) = \sin x$ at $x = \frac{3\pi}{2}$.

Example 15: Find the equation of the tangent line to the curve $f(x) = \sec x$ at $x = \frac{\pi}{4}$.

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The derivative of the sine function is the cosine function:

$$\frac{d}{dx}(\sin x) = \cos x.$$

The derivative of the cosine function is the negative of the sine function:

$$\frac{d}{dx}(\cos x) = -\sin x$$

Example 1: Using the rules above, find $\frac{d}{dx}(\tan x)$.

$y = \tan x = \frac{\sin x}{\cos x}$ quotient rule

$y' = \frac{\cos x(\sin x)' - \sin x(\cos x)'}{(\cos x)^2} = \frac{\overbrace{\cos x \cos x - \sin x(-\sin x)}^{\cos^2 x + \sin^2 x}}{(\cos x)^2}$

$= \frac{1}{\cos^2 x} = \sec^2 x$

Example 2: Using the rules above, find $\frac{d}{dx}(\cot x)$.

$y = \cot x = \frac{\cos x}{\sin x}$

$y' = \frac{\sin x(-\sin x) - \cos x(\cos x)}{(\sin x)^2} = \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x} = \frac{-1}{\sin^2 x}$

$= -\csc^2 x$

The derivatives of the other trigonometric functions:

$$\frac{d}{dx}(\tan x) = \sec^2 x \qquad \frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x \qquad \frac{d}{dx}(\csc x) = -\csc x \cot x$$

Example 3: Find the fourth derivative of $y = \sin x$.

$y' = \cos x$

$y'' = -\sin x$

$y''' = -\cos x$

$y^{(4)} = -(-\sin x) = \sin x$

Example 4: Find $f'(x)$ when $f(x) = 3\cos x - 4\sec x$.

$f'(x) = -3\sin x - 4\sec x \tan x$

Example 5: Find $f'(x)$ when $f(x) = x^2 \sec x$. product rule

$f'(x) = x^2 \cdot \sec x \tan x + (\sec x)(2x)$

$= x \sec x \,(x \tan x + 2)$

Example 6: Find $f'(x)$ when $f(x) = \sin^2 x + \cos^2 x$. $= 1$

$f'(x) = 0$

product

Example 7: Find $f'(x)$ when $f(x) = \sqrt{x}\sec x + 3$.

$f'(x) = \sqrt{x}\sec x \tan x + \sec x \left(\frac{1}{2} x^{-1/2}\right) + 0$

Example 8: Find $f'(x)$ when $f(x) = x^2 \cot x - \frac{1}{x^2}$. $= x^2 \cot x - x^{-2}$

$f'(x) = x^2(-\csc^2 x) + \cot x\,(2x) + 2x^{-3}$

Example 9: Find $f'(x)$ when $f(x) = \frac{4}{\cos x} + \frac{1}{\tan x}$. $= 4\sec x + \cot x$

$f'(x) = 4\sec x \tan x - \csc^2 x$

Example 10: Find $f'(x)$ when $f(x) = (\sec x + \tan x)(\sec x - \tan x)$. $= \sec^2 x - \tan^2 x = 1$

$f(x) = 1$

$f'(x) = 0$

$m = f' = 0$

Example 11: Find the x-values in $0 \leq x < 2\pi$ where the tangent line is horizontal to the curve $f(x) = x + \sin x$.

$f'(x) = 1 + \cos x = 0$

$\cos x = -1$

$x = \pi$

Example 12: Find the x-values in $0 \leq x < 2\pi$ where the tangent line is horizontal to the curve $f(x) = x - \cot x$.

$f'(x) = 1 + \csc^2 x = 0$

$\csc^2 x = -1$

No Solution

Example 13: Find the x-values in $0 \leq x < 2\pi$ where the tangent line is horizontal to the curve $f(x) = x + 2\cos x$.

$f'(x) = 1 - 2\sin x = 0$

$1 = 2\sin x$

$\frac{1}{2} = \sin x$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

Example 14: Find the equation of the tangent line to the curve $f(x) = \sin x$ at $x = \frac{3\pi}{2}$.

$m = y' = \cos x$

$m = \cos \frac{3\pi}{2} = 0$

pt $\left(\frac{3\pi}{2}, -1\right)$

$y = \sin \frac{3\pi}{2} = -1$

$y - y_1 = m(x - x_1)$

$y + 1 = 0\left(x - \frac{3\pi}{2}\right)$

$y = -1$

Example 15: Find the equation of the tangent line to the curve $f(x) = \sec x$ at $x = \frac{\pi}{4}$.

$m = f'(x) = \sec x \tan x$

$m = f'(\pi/4) = \sec \pi/4 \tan \pi/4$

$= (\sqrt{2})(1)$

$= \sqrt{2}$

point $(\pi/4, \sqrt{2})$

$y = \sec \pi/4 = \sqrt{2}$

$y - y_1 = m(x - x_1)$

$y - \sqrt{2} = \sqrt{2}\left(x - \frac{\pi}{4}\right)$