Section 3.6 Chain Rule of Derivatives

Review function composition from algebra. Review outer and inner functions.

Chain Rule
$(f \circ g)' = f'(g(x)) \cdot g'(x)$
Derivative of a composite function= (derv. of outer function) times (derv. of inner function)

THEOREM 2—The Chain Rule If $f(u)$ is differentiable at the point $u = g(x)$ and $g(x)$ is differentiable at x, then the composite function $(f \circ g)(x) = f(g(x))$ is differentiable at x, and

$$(f \circ g)'(x) = f'(g(x)) \cdot g'(x).$$

In Leibniz's notation, if $y = f(u)$ and $u = g(x)$, then

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx},$$

where dy/du is evaluated at $u = g(x)$.

Example 1: Using the rules above, find the derivative of the following. Do not simplify.

a) $y = \sin(x^2 - 5x)$

b) $y = \tan(x^3)$

c) $y = (x^2 - 3x + 5)^{10}$

d) $y = \sqrt{5 - x^2}$

e) $y = \sqrt[3]{x^3 - 7x^2}$

Example 2: Using the rules above, find the derivative of the following. Do not simplify.

a) $y = \sin^2 5x$

b) $y = \cot^6 3\,x^9$

Example 3: Find the derivative using the product and chain rule. Do not simplify.

$y = (x - 3)^5(x^2 + 1)^4$

Example 4: Find the derivative using the quotient and chain rule. Do not simplify.

$y = \left(\frac{\cos x}{\sqrt{x}}\right)^3$

Example 5: Find the derivative using the product and chain rule. Simplify.

$y = (4x + 3)^4(x + 1)^{-3}$

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> **THEOREM 2—The Chain Rule** If $f(u)$ is differentiable at the point $u = g(x)$ and $g(x)$ is differentiable at x, then the composite function $(f \circ g)(x) = f(g(x))$ is differentiable at x, and
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> $$(f \circ g)'(x) = f'(g(x)) \cdot g'(x).$$
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> where dy/du is evaluated at $u = g(x)$.

Example 1: Using the rules above, find the derivative of the following. Do not simplify.

a) $y = \sin(x^2 - 5x)$

$y' = \cos(x^2 - 5x) \cdot (2x - 5)$

b) $y = \tan(x^3)$

$y' = \sec^2(x^3) \cdot 3x^2$

c) $y = (x^2 - 3x + 5)^{10}$

$y' = 10(x^2 - 3x + 5)^9 \cdot (2x - 3)$

d) $y = \sqrt{5 - x^2} = (5 - x^2)^{1/2}$

$y' = \frac{1}{2}(5 - x^2)^{-1/2} \cdot (-2x)$

e) $y = \sqrt[3]{x^3 - 7x^2} = (x^3 - 7x^2)^{1/3}$

$y' = 1/3\,(x^3 - 7x^2)^{-2/3} \cdot (3x^2 - 14x)$

outer, middle, inner

Example 2: Using the rules above, find the derivative of the following. Do not simplify.

a) $y = \sin^2 5x \Rightarrow y = (\sin(5x))^2$

$y' = 2(\sin 5x)^1(\cos 5x)\cdot(5)$

b) $y = \cot^6 3x^9 \Rightarrow y = (\cot(3x^9))^6$

$y' = 6(\cot 3x^9)^5(-\csc^2 3x^9)\cdot(27x^8)$

Example 3: Find the derivative using the product and chain rule. Do not simplify.

$y = (x-3)^5(x^2+1)^4$

$y' = (x-3)^5(4(x^2+1)^3(2x)) + (x^2+1)^4(5(x-3)^4(1))$

Example 4: Find the derivative using the quotient and chain rule. Do not simplify.

$y = \left(\frac{\cos x}{\sqrt{x}}\right)^3 = \left(\frac{\cos x}{x^{1/2}}\right)^3$

$y' = 3\left(\frac{\cos x}{x^{1/2}}\right)^2 \cdot \left(\frac{x^{1/2}(-\sin x) - \cos x\,(\frac{1}{2}x^{-1/2})}{(x^{1/2})^2}\right)$

Example 5: Find the derivative using the product and chain rule. Simplify.

$y = (4x+3)^4(x+1)^{-3}$

$y' = (4x+3)^4(-3(x+1)^{-4}(1)) + (x+1)^{-3}(4(4x+3)^3(4))$

$y' = -3(4x+3)^4(x+1)^{-4} + 16(4x+3)^3(x+1)^{-3}$

$y' = (4x+3)^3(x+1)^{-4}(-3(4x+3) + 16(x+1))$

$y' = (4x+3)^3(x+1)^{-4}(-12x-9+16x+16)$

$y' = (4x+3)^3(x+1)^{-4}(4x+7)$

$y' = \frac{(4x+3)^3(4x+7)}{(x+1)^4}$