Section 3.7 Implicit Differentiation

When you want to find the derivative of an equation that is not an explicit function of x (or some other single variable), you cannot use the rules for differentiation that we have learned thus far.

Examples:

To find derivatives for equations which cannot be expressed as explicit functions of one variable, we must use a method called implicit differentiation.

Explicit variables match	Implicit variables don't match
$\frac{d}{dx}(x) = 1$	$\frac{d}{dx}(y) = \frac{dy}{dx}$
$\frac{d}{dt}(t) = 1$	$\frac{d}{dt}(x) = \frac{dx}{dt}$
$\frac{d}{dx}(x^3) = 3x^2$	$\frac{d}{dx}(y^3) = 3y^2\frac{dy}{dx}$
$\frac{d}{dx}(sinx) = cosx$	$\frac{d}{dx}(siny) = cosy\frac{dy}{dx}$

Using the Method of Implicit Differentiation

1. Find $\frac{dy}{dx}$ for $x^2 + y^2 = 1$.

   Take the derivative of both sides of the equation with respect to "x"

   Algebraically solve for $\frac{dy}{dx}$

2. Find $\frac{d^2y}{dx^2}$ for $x^2 + y^2 = 1$. Refer back to #1

3. a) Find $\frac{dy}{dx}$ for $y = 3x^5 + x^3y^2$

b) Find slope of the tangent line to the graph of for $y = 3x^5 + x^3y^2$ at the point (1, -2)

4. Find $\frac{dy}{dx}$ for $\cos(xy) = y$.

3.7

Section [redacted] Implicit Differentiation

When you want to find the derivative of an equation that is not an explicit function of x (or some other single variable), you cannot use the rules for differentiation that we have learned thus far.

$7x^4 + x^3y + y = 4$

Examples: $xy = y^3$

$5y^2 + \sin y = x^2$

To find derivatives for equations which cannot be expressed as explicit functions of one variable, we must use a method called implicit differentiation.

Explicit variables match	Implicit variables don't match
$\frac{d}{dx}(x) = 1$	$\frac{d}{dx}(y) = \frac{dy}{dx}$
$\frac{d}{dt}(t) = 1$	$\frac{d}{dt}(x) = \frac{dx}{dt}$
$\frac{d}{dx}(x^3) = 3x^2$	$\frac{d}{dx}(y^3) = 3y^2\frac{dy}{dx}$
$\frac{d}{dx}(sinx) = cosx$	$\frac{d}{dx}(siny) = cosy\frac{dy}{dx}$

Using the Method of Implicit Differentiation

1. Find $\frac{dy}{dx}$ for $x^2 + y^2 = 1$.

Take the derivative of both sides of the equation with respect to "x"

$$2x + 2y\frac{dy}{dx} = 0$$

Algebraically solve for $\frac{dy}{dx}$

$$2x + 2y\frac{dy}{dx} = 0$$

$$2y\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{2y} = \frac{-x}{y}$$

2. Find $\frac{d^2y}{dx^2}$ for $x^2 + y^2 = 1$. Refer back to #1

$$\frac{dy}{dx} = \frac{-x}{y}$$

$$\frac{d^2y}{dx^2} = \frac{y(-1) - (-x)\frac{dy}{dx}}{y^2}$$

$$\frac{d^2y}{dx^2} = \frac{-y + x\left(\frac{-x}{y}\right)}{y^2} \cdot \frac{y}{y} = \frac{-y^2 - x^2}{y^3} = \frac{-1(x^2+y^2)}{y^3} = \frac{-1}{y^3}$$

3. a) Find $\frac{dy}{dx}$ for $y = 3x^5 + x^3y^2$

$$\frac{dy}{dx} = 15x^4 + x^3 2y\frac{dy}{dx} + y^2 \cdot 3x^2$$

$$\frac{dy}{dx} - 2x^3y\frac{dy}{dx} = 15x^4 + 3x^2y^2$$

$$\frac{dy}{dx}(1 - 2x^3y) = 15x^4 + 3x^2y^2$$

$$\frac{dy}{dx} = \frac{15x^4 + 3x^2y^2}{1 - 2x^3y}$$

b) Find slope of the tangent line to the graph of for $y = 3x^5 + x^3y^2$ at the point (1, -2)

$$m = \frac{dy}{dx} = \frac{15(1)^4 + 3(1)^2(-2)^2}{1 - 2(1)^3(2)} = \frac{27}{5}$$

4. Find $\frac{dy}{dx}$ for $\cos(xy) = y$.

$$-\sin(xy)\left(x \cdot \frac{dy}{dx} + y(1)\right) = \frac{dy}{dx}$$

$$-x\sin(xy)\frac{dy}{dx} + -y\sin(xy) = \frac{dy}{dx}$$

$$-y\sin(xy) = \frac{dy}{dx} + x\sin(xy)\frac{dy}{dx}$$

$$-y\sin(xy) = \frac{dy}{dx}(1 + x\sin(xy))$$

$$\frac{-y\sin(xy)}{1 + x\sin(xy)} = \frac{dy}{dx}$$