Section 3.9 Inverse Trigonometric Functions

TABLE 3.1 Derivatives of the inverse trigonometric functions

1. $$\frac{d(\sin^{-1} u)}{dx} = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}, \quad |u| < 1$$

2. $$\frac{d(\cos^{-1} u)}{dx} = -\frac{1}{\sqrt{1-u^2}}\frac{du}{dx}, \quad |u| < 1$$

3. $$\frac{d(\tan^{-1} u)}{dx} = \frac{1}{1+u^2}\frac{du}{dx}$$

4. $$\frac{d(\cot^{-1} u)}{dx} = -\frac{1}{1+u^2}\frac{du}{dx}$$

5. $$\frac{d(\sec^{-1} u)}{dx} = \frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx}, \quad |u| > 1$$

6. $$\frac{d(\csc^{-1} u)}{dx} = -\frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx}, \quad |u| > 1$$

Example 1: Using the Chain Rule calculate the derivative:

$$\frac{d}{dx}(sin^{-1}x^2)$$

Example 2: Using the Chain Rule and derivative of the arcsecant function find:

$$\frac{d}{dx}sec^{-1}(5x^4)$$

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Example 1: Using the Chain Rule calculate the derivative:

$$\frac{d(\sin^{-1}u)}{dx} = \frac{1}{1-u^2}\frac{du}{dx}$$

$$\frac{d}{dx}(sin^{-1}x^2) = \frac{1}{\sqrt{1-(x^2)^2}} \cdot \frac{d}{dx}x^2 = \frac{1}{\sqrt{1-x^4}} \cdot 2x = \boxed{\frac{2x}{\sqrt{1-x^4}}}$$

$u = x^2$

Example 2: Using the Chain Rule and derivative of the arcsecant function find:

$$\frac{d}{dx}sec^{-1}(5x^4) = \frac{1}{|5x^4|\sqrt{(5x^4)^2-1}}\frac{d}{dx}5x^4 = \frac{1}{\;}$$

$u = 5x^4$

$$= \frac{1}{5x^4\sqrt{25x^8-1}}(20x^3) = \boxed{\frac{4}{x\sqrt{25x^8-1}}}$$