Section 4.3 First Derivative Test

A function is increasing when $f'(x) > 0$
A function is decreasing when $f'(x) < 0$
A function has a relative (local) maximum when $f'(x)$ changes from + to -
A function has a relative (local) minimum when $f'(x)$ changes from - to +

For the following examples, find the intervals where f(x) is increasing and decreasing. Find the relative extrema and give your answer as (x,y) points. Sketch the graph of f(x).

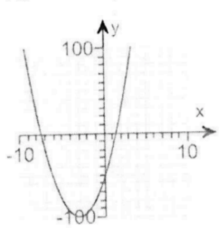
Example 1: $f(x) = -x^3 + 27x - 45$
Solve $f'(x) = 0$ to find critical values

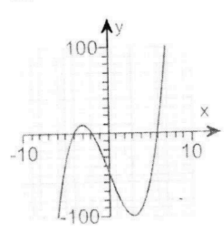
Make a first derivative number line using the critical values. Pick a test number in each interval and plug into the first derivative. Record the positive or negative sign of the results on the number line.

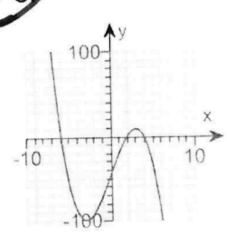
Using the number line, find the open intervals of the domain when $f(x)$ is increasing and decreasing.

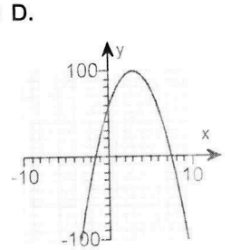
Using the number line, find the local extrema. Plug x value into $f(x)$ to find y value of points.

Graph the function. Choose the correct graph below.

A.


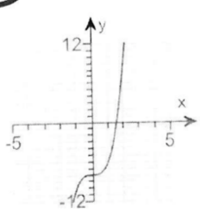
B.


C.


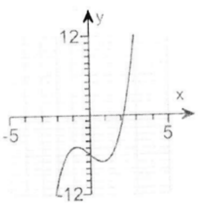
D.


Example 2: $f(x) = 2x^3 - 8$

A.



B.



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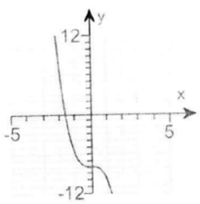

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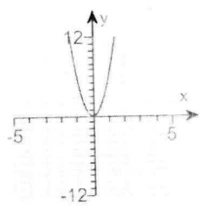
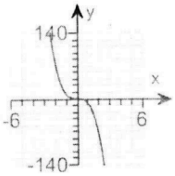
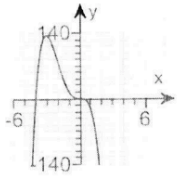


Example 3: $f(x) = 5x^4 + 20x^3$

A.



B.



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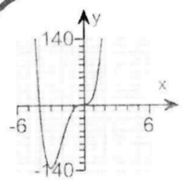
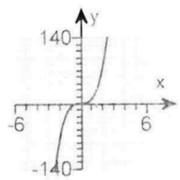
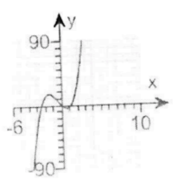


D.

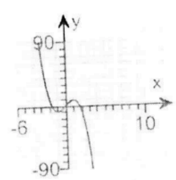


Example 4: $f(x) = 4x^3 - 12x$

A.



B.



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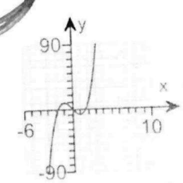
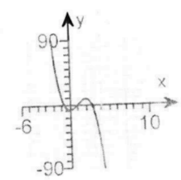
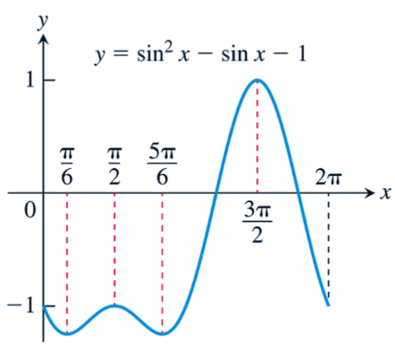


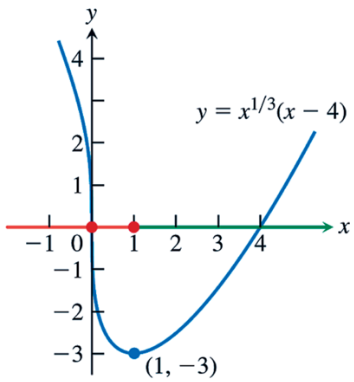
D.



Example 5: Within the interval $[0, 2\pi]$, find the critical points of $f(x) = \sin^2 x - \sin x - 1$. Identify the open intervals on which $f(x)$ is increasing and decreasing. Find the function's local extreme values.



Example 6: Find the critical points of $f(x) = \sqrt[3]{x}(x-4)$. Identify the open intervals on which $f(x)$ is increasing and decreasing. Find the function's local extreme values.



Section 4.3 First Derivative Test

A function is increasing when $f'(x) > 0$ positive
A function is decreasing when $f'(x) < 0$ negative
A function has a relative (local) maximum when $f'(x)$ changes from + to -
A function has a relative (local) minimum when $f'(x)$ changes from - to +

For the following examples, find the intervals where f(x) is increasing and decreasing. Find the relative extrema and give your answer as (x,y) points. Sketch the graph of f(x).

Example 1: $f(x) = -x^3 + 27x - 45$
Solve $f'(x) = 0$ to find critical values

$f'(x) = -3x^2 + 27 = 0$
$-3x^2 = -27$
$x^2 = 9$
$x = \pm 3$

Make a first derivative number line using the critical values. Pick a test number in each interval and plug into the first derivative. Record the positive or negative sign of the results on the number line.

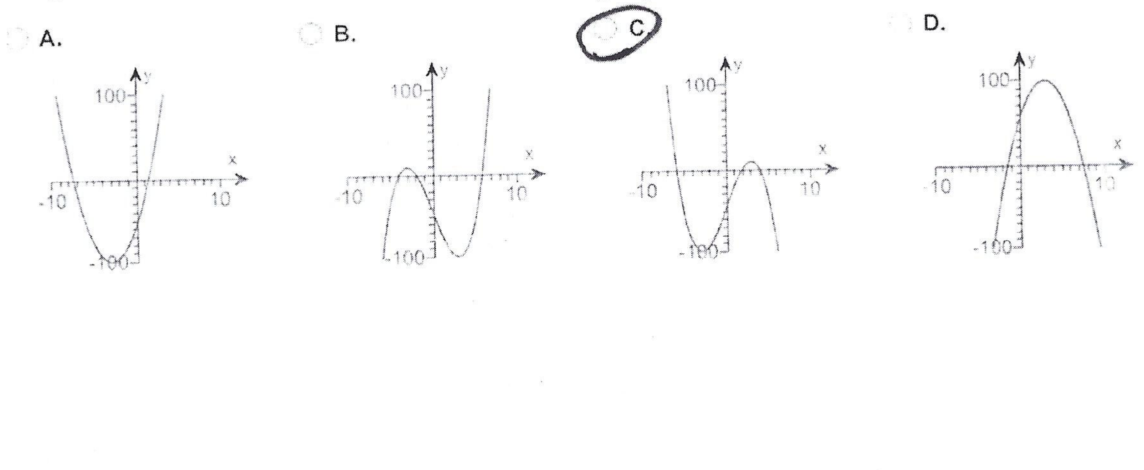
f': − (−4) | −3 | + (0) | 3 | − (4)

$f'(-4) = -3(-4)^2 + 27 = -$
$f'(0) = -3(0)^2 + 27 = +$
$f'(4) = -3(4)^2 + 27 = -$

Using the number line, find the open intervals of the domain when $f(x)$ is increasing and decreasing.
+ Increasing $(-3, 3)$
− Decreasing $(-\infty, -3) \cup (3, \infty)$

Using the number line, find the local extrema. Plug x value into $f(x)$ to find y value of points.
+ − Local Max $(3, 9)$ $f(3) = -(3)^3 + 27(3) - 45 = 9$
− + Local Min $(-3, -99)$ $f(-3) = -(-3)^3 + 27(-3) - 45 = -99$

Graph the function. Choose the correct graph below.



Example 2: $f(x) = 2x^3 - 8$

Critical values
$f'(x) = 0$
$6x^2 = 0$
$x^2 = 0$
$x = 0$

increasing $(-\infty, 0)\ (0, \infty)$
decreasing None

Local Max None
Local Min None

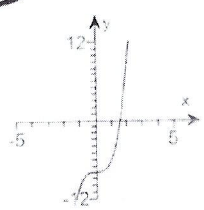
Number Line

$+ \quad + $

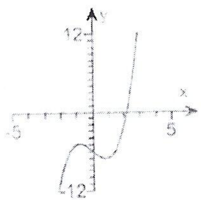
$-2 \quad 0 \quad 2$ $\quad f'$

$f'(-2) = 6(-2)^2 = +$
$f'(2) = 6(2)^2 = +$

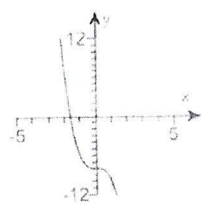
A.



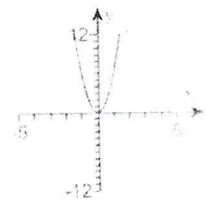
B.



C.



D.



Example 3: $f(x) = 5x^4 + 20x^3$

Critical values
$f'(x) = 20x^3 + 60x^2 = 0$
$20x^2(x+3) = 0$
$x = 0 \quad x = -3$

increasing $(-3, 0) \cup (0, \infty)$
decreasing $(-\infty, -3)$

local max None
local min $(-3, -135)$

$f(-3) = -135$

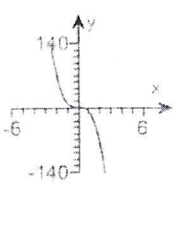
Number Line

$- \quad + \quad +$

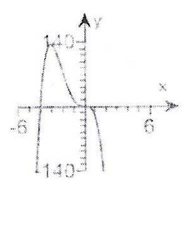
$-4 \quad -3 \quad -1 \quad 0 \quad 4$ $\quad f'$

$f'(-4) = 20(-4)^3 + 60(-4)^2 = -$
$f'(-1) = 20(-1)^3 + 60(-1)^2 = +$
$f'(4) = 20(4)^3 + 60(4)^2 = +$

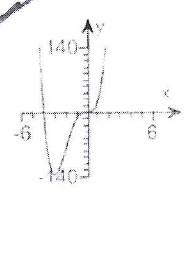
A.



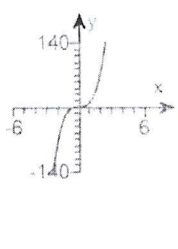
B.



C.



D.



Example 4: $f(x) = 4x^3 - 12x$

critical values $f'(x) = 12x^2 - 12 = 0$

$12(x^2 - 1) = 0$

$12(x+1)(x-1) = 0$

$x = -1 \quad x = 1$

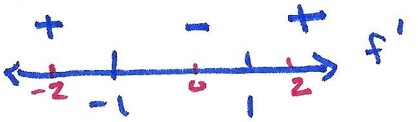
increasing $(-\infty, -1) \cup (1, \infty)$

decreasing $(-1, 1)$

local Max $(-1, 8)$

local Min $(+1, -8)$

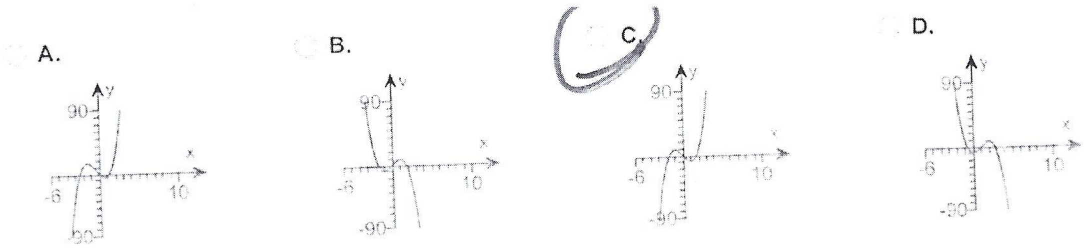
Number Line



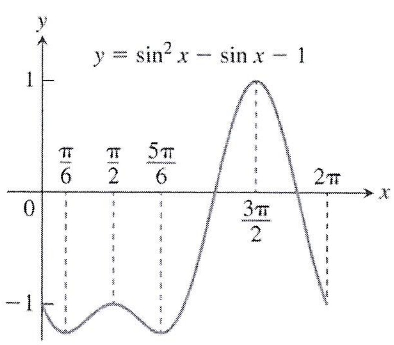
$f'(-2) = 12(-2)^2 - 12 = +$

$f'(0) = 12(0)^2 - 12 = -$

$f'(2) = 12(2)^2 - 12 = +$



Example 5: Within the interval $[0, 2\pi]$, find the critical points of $f(x) = \sin^2 x - \sin x - 1$. Identify the open intervals on which $f(x)$ is increasing and decreasing. Find the function's local extreme values.



Critical values $f'(x) = 2\sin x \cos x - \cos x = 0$

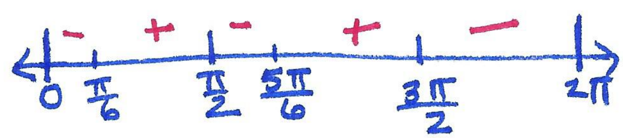
$\cos x(2\sin x - 1) = 0$

$\cos x = 0 \qquad 2\sin x - 1 = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2} \qquad \sin x = 1/2$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

number line



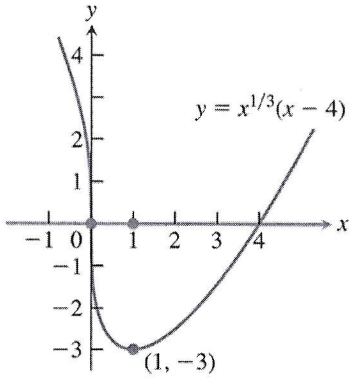
increasing $(\pi/6, \pi/2) \cup (5\pi/6, 3\pi/2)$

decreasing $(0, \pi/6) \cup (\pi/2, 5\pi/6) \cup (3\pi/2, 2\pi)$

local max $(\pi/2, -1)$ and $(\frac{3\pi}{2}, 1)$

local min $(\pi/6, -5/4)$ and $(\frac{5\pi}{6}, -5/4)$

Example 6: Find the critical points of $f(x) = \sqrt[3]{x}(x-4)$. Identify the open intervals on which $f(x)$ is increasing and decreasing. Find the function's local extreme values.



$f(x) = x^{1/3}(x-4) = x^{4/3} - 4x^{1/3}$

critical values

$f'(x) = \frac{4}{3}x^{1/3} - \frac{4}{3}x^{-2/3} = 0$

$\frac{4}{3}x^{-2/3}(x-1) = 0$

$\frac{4(x-1)}{3\sqrt[3]{x^2}} = 0$

$x = 1$ $\qquad$ $x = 0$

number line

$-$ $\quad$ $-$ $\quad$ $+$ $\quad f'$

$-1 \quad 0 \quad \frac{1}{2} \quad 1 \quad 2$

increasing $(1, \infty)$

decreasing $(-\infty, 0) \cup (0, 1/2)$

local max $\qquad$ None

local min $\qquad$ $(1, -3)$