Section 4.6 Applied Optimization

1. Find two positive numbers that satisfy the given requirements. The product is 192 and the sum of the first plus three times the second is a minimum.

2. Find two positive numbers that satisfy the given requirements. The sum of the first and twice the second is 100 and the product is a maximum.

3. A rancher has 200 feet of fencing with which to enclose two adjacent rectangular corrals. What dimensions should be used so that the enclosed area will be a maximum?

4. A rectangular sheet of cardboard 8 x 15 inches is used to make an open box by cutting squares of equal size from the corners and folding up the sides. What size squares should be cut to obtain a box with largest possible volume?

5. A real estateoffice handles 50 apartment units. When the rent is \$720 per month, all units are occupied. However, on the average, for each \$40 increase in rent, one unit becomes vacant. Each occupied unit requires an average of \$48 per month for services and repairs. What rent should be charged to obtain the maximum profit?

Answers:
1. 24 and 8
2. 50 and 25
3. 25 x 100/3
4. 5/3 inches
5. rent = \$1400

1. $xy = 192$

$y = \frac{192}{x}$

$\underline{24}$ and $\underline{\ \ 8\ \ }$ are the numbers

Minimize: $S = x + 3y$

$S = x + 3\left(\frac{192}{x}\right)$

$S = x + \frac{576}{x}$

$S = x + 576x^{-1}$

$S' = 1 - 576x^{-2}$

$S' = 1 - \frac{576}{x^2}$

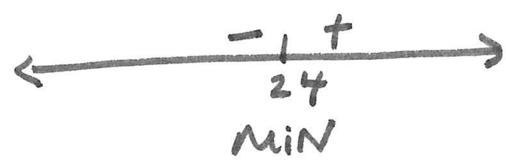
$S' = \frac{x^2}{x^2} - \frac{576}{x^2}$

$S' = \frac{x^2 - 576}{x^2}$

$x^2 - 576 = 0$ $\qquad$ $x^2 = 0$

$x^2 = 576$ $\qquad$ $x = 0$

$x = \pm 24$



$x = 24$ $\quad$ $y = \frac{192}{24} = 8$

2.

$x + 2y = 100$

$2y = 100 - x$

$y = 50 - \frac{1}{2}x$

The numbers are $\underline{50}$ and $\underline{25}$

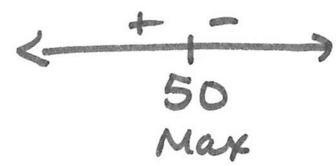
Maximize: $P = xy$

$P = x(50 - \frac{1}{2}x)$

$P = 50x - \frac{1}{2}x^2$

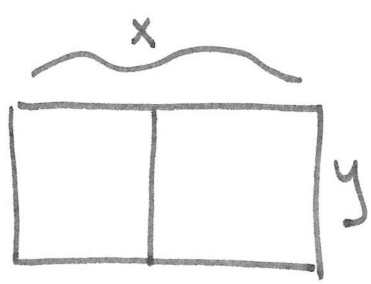
$P' = 50 - x = 0$

$50 = x$



$x = 50 \quad y = 50 - \frac{1}{2}(50)$
$= 25$

3.



Fence $= 2x + 3y$

$200 = 2x + 3y$

$200 - 2x = 3y$

$\frac{200}{3} - \frac{2}{3}x = y$

Dimensions
Large Pen $50 \times \frac{100}{3}$

The two small pens are same size
$25 \times \frac{100}{3}$

Maximize: $A = xy$

$A = x\left(\frac{200}{3} - \frac{2}{3}x\right)$

$A = \frac{200}{3}x - \frac{2}{3}x^2$

$A' = \frac{200}{3} - \frac{4}{3}x = 0$

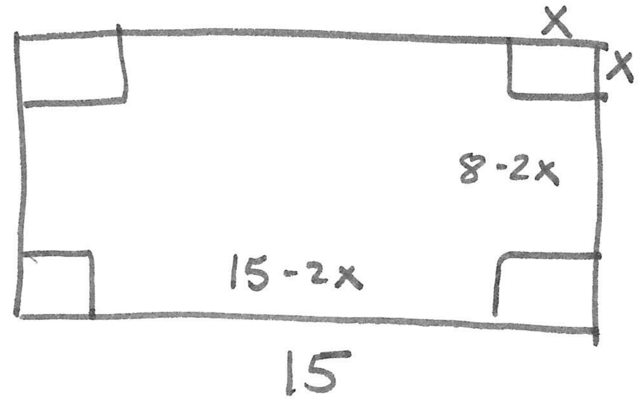
$\frac{200}{3} = \frac{4}{3}x$

$\frac{3}{4} \cdot \frac{200}{3} = x$

$50 = x$

$x = 50 \quad y = \frac{100}{3}$

4.



Maximize: $V = lwh$

$$V = (15-2x)(8-2x)(x)$$

$$V = (120 - 46x + 4x^2)x$$

$$V = 120x - 46x^2 + 4x^3$$

$$V' = 120 - 92x + 12x^2$$

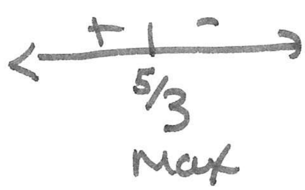
$$V' = 12x^2 - 92x + 120$$

$$0 = 4(3x^2 - 23x + 30)$$

$$0 = 4(3x-5)(x-6)$$

$$x = 5/3 \qquad \cancel{x = 6}$$

why?



Squares should be <u>5/3</u> inches.

5. x = number of vacant apartments

$$\text{Revenue} = (720+40x)(50-x)$$
$$= 36000 + 1280x - 40x^2$$

$$\text{Cost} = 48(50-x) = 2400 - 48x$$

Profit = Revenue - Cost

$$P = 36000 + 1280x - 40x^2 - (2400 - 48x)$$
$$P = 33600 + 1328x - 40x^2$$

$$P' = 1328 - 80x = 0$$
$$1328 = 80x$$
$$16.6 = x$$ ← num of apartment

so is it 16 or 17?

$P(16) = 44608$
$P(17) = 44616$ ← max

17 apartments vacant

33 apartments rented

\$1400 rent

\$44,616 max profit