Section 4.8 Antiderivatives

DEFINITION A function F is an **antiderivative** of f on an interval I if $f'(x) = f(x)$ for all x in I.

Big Idea: Suppose we know that $f'(x) = 3x^2$. What is $f(x)$? $f(x) = x^3$
Is this the only solution? No. $f(x) = x^3 + 2$ $\quad f(x) = x^3 - 10$ and so on.
In general, $f(x) = x^3 + C$ where C is a constant.

THEOREM 8 If F is an antiderivative of f on an interval I, then the most general antiderivative of f on I is

$$F(x) + C$$

where C is an arbitrary constant.

Example 1: Find antiderivatives for the following.

a) $f'(x) = 2x$ $\quad f(x) = x^2 + C$

b) $f'(x) = 5x^4$ $\quad f(x) = x^5 + C$

c) $f'(x) = \cos x$ $\quad f(x) = \sin x + C$

d) $f'(x) = 3$ $\quad f(x) = 3x + C$

e) $f'(x) = x$ $\quad f(x) = \frac{1}{2}x^2 + C$

TABLE 4.2 Antiderivative formulas, k a nonzero constant

Function	General antiderivative
1. x^n	$\frac{1}{n+1}x^{n+1} + C, \quad n \neq -1$
2. $\sin kx$	$-\frac{1}{k}\cos kx + C$
3. $\cos kx$	$\frac{1}{k}\sin kx + C$
4. $\sec^2 kx$	$\frac{1}{k}\tan kx + C$
5. $\csc^2 kx$	$-\frac{1}{k}\cot kx + C$
6. $\sec kx \tan kx$	$\frac{1}{k}\sec kx + C$
7. $\csc kx \cot kx$	$-\frac{1}{k}\csc kx + C$

TABLE 4.3 Antiderivative linearity rules

	Function	General antiderivative
1. *Constant Multiple Rule:*	$kf(x)$	$kF(x) + C, \quad k$ a constan
2. *Sum or Difference Rule:*	$f(x) \pm g(x)$	$F(x) \pm G(x) + C$

> **DEFINITION** The collection of all antiderivatives of f is called the **indefinite integral** of f with respect to x, and is denoted by
>
> $$\int f(x)\,dx.$$
>
> The symbol $\int$ is an **integral sign**. The function f is the **integrand** of the integral, and x is the **variable of integration**.

Example 2: Find the following.

a) $\int 3x\,dx = 3\int x\,dx = 3\cdot\frac{1}{2}x^2 + C = \frac{3}{2}x^2 + C$

b) $\int x^5\,dx = \frac{1}{5+1}x^{5+1} = \frac{1}{6}x^6 + C$

c) $\int \sqrt{x}\,dx = \frac{1}{\frac{1}{2}+1}x^{\frac{1}{2}+1} = \frac{2}{3}x^{3/2} + C$

d) $\int \frac{1}{x^3}\,dx = \int x^{-3}\,dx = \frac{1}{-3+1}x^{-3+1} = -\frac{1}{2}x^{-2} + C$

e) $\int \sqrt[3]{x^2}\,dx = \int x^{2/3}\,dx = \frac{3}{5}x^{5/3} + C$

Example 3: Find the following.

a) $\int 2\sin x\,dx = 2\int \sin x\,dx = -2\cos x + C$

b) $\int \cos 4x\,dx = \frac{1}{4}\sin 4x + C$

c) $\int \sec^2 x\,dx = \tan x + C$

d) $\int \frac{\sin x}{\cos^2 x}\,dx = \int \frac{\sin x\cdot 1}{\cos x\cdot\cos x}\,dx = \int \tan x\sec x\,dx$

$= \int \sec x\tan x\,dx = \sec x + C$

Example 4: Find the following.

a) $\int dx = \int 1\,dx = x + C$

b) $\int (x+2)\,dx = \int x\,dx + \int 2\,dx = \frac{1}{2}x^2 + 2x + C$

c) $\int (3x^4 - 5x^2 + x)\,dx = \frac{3}{5}x^5 - \frac{5}{3}x^3 + \frac{1}{2}x^2 + C$

d) $\int \frac{x+1}{\sqrt{x}}\,dx = \int \frac{x}{\sqrt{x}} + \frac{1}{\sqrt{x}}\,dx = \int x^{1/2} + x^{-1/2}\,dx$

$= \frac{2}{3}x^{3/2} + 2x^{1/2} + C$

Example 5: Solve the initial value problem given that $F'(x) = 3x^2 - 1, F(2) = 4$, find $F(x)$

$\int 3x^2 - 1\,dx = x^3 - x + c$ general solution

$F(x) = x^3 - x + c$

$F(2) = 2^3 - 2 + c = 4$

so $c = -2$

$F(x) = x^3 - x - 2$ particular solution

Example 6: Solve the initial value problem given that $\frac{dy}{dx} = \frac{1}{x^2}, y(2) = 0$, find y

$\int \frac{1}{x^2}\,dx = \int x^{-2}\,dx = -x^{-1} + C$

$y = \frac{-1}{x} + C$ general solution

$y(2) = \frac{-1}{2} + C = 0$

so $C = 1/2$

$y = -\frac{1}{x} + \frac{1}{2}$ particular solution