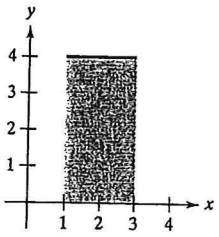
Section 5.1 Area and Estimating with Finite Sums

Find the area of the shaded region using geometry.

1.

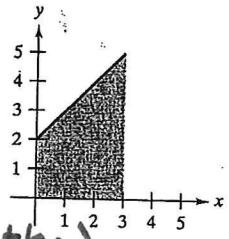
$f(x) = 4$



$A = lw = 4 \cdot 2 = 8$

2.

$f(x) = x+2$

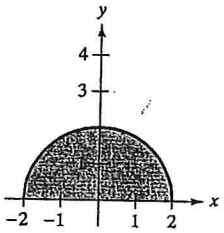


$A = \frac{1}{2} h(b_1 + b_2)$

$= \frac{1}{2} \cdot 3\,(2+5) = 21/2 = 10.5$

3.

$f(x) = \sqrt{4 - x^2}$



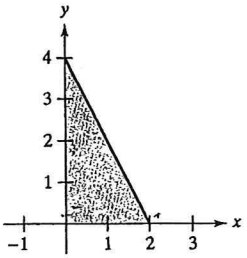
$A = \frac{1}{2}\pi r^2$

$= \frac{1}{2}\pi (2)^2$

$= 2\pi$

4.

$f(x) = 4 - 2x$



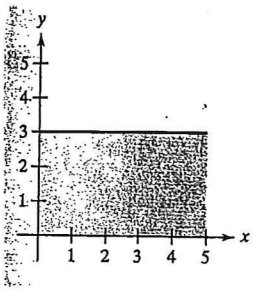
$A = \frac{1}{2} bh$

$= \frac{1}{2} \cdot 2 \cdot 4$

$= 4$

5.

$f(x) = 3$

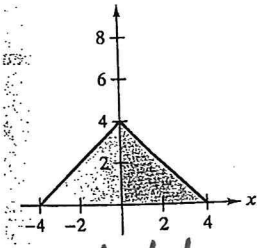


$A = lw$

$= 5 \cdot 3 = 15$

6.

$f(x) = 4 - |x|$

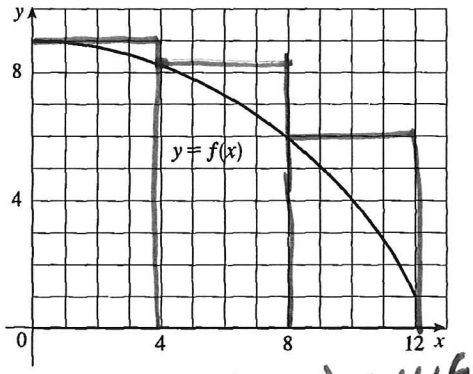


$A = \frac{1}{2} bh$

$= \frac{1}{2} \cdot 8 \cdot 4 = 16$

7. Use three rectangles to estimate the area under the curve from x=0 to x=12.

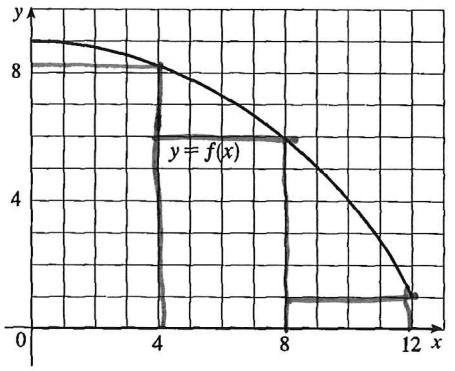
a. upper sum using the left endpoint



$A_{upper} = 4(9) + 4(8.1) + 4(6)$

$= 92.4$

b. lower sum using the right endpoint



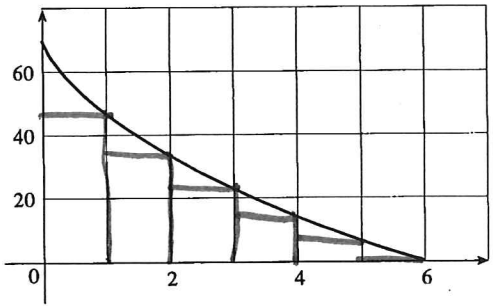
$A_{lower} = 4(8.1) + 4(6) + 4(1)$

$= 60.4$

$60.4 \leq \text{Area} \leq 92.4$

8. Use six rectangles to estimate the area under the curve from x=0 to x=6.

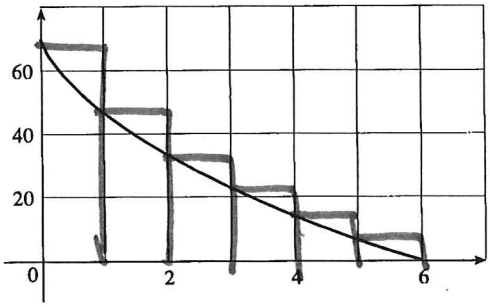
a. lower sum using the ~~left~~ right endpoint



$A_{lower} = 50 + 35 + 21 + 15 + 10 + 0$

$= 131$

b. upper sum using the ~~right~~ left endpoint



$A_{upper} = 70 + 50 + 35 + 21 + 15 + 10$

$= 201$

$131 \leq \text{Area} \leq 201$