5.2 Sigma Notation + Area

The capital greek letter sigma, Σ, denotes a sum.

$$\sum_{i=1}^{5} i = 1+2+3+4+5 = 15$$

upper limit of summation

lower limit of summation

index of summation

Ex. Evaluate $\sum_{i=1}^{4} 2^i$

$$2^1+2^2+2^3+2^4 = 2+4+8+16 = 30$$

Ex Evaluate $\sum_{j=3}^{5} \frac{1}{j}$

$$\frac{1}{3}+\frac{1}{4}+\frac{1}{5} = \frac{20}{60}+\frac{15}{60}+\frac{12}{60} = \frac{47}{60}$$

Ex Write in sigma notation

$$(2+1)+(3+1)+(4+1)+\cdots(9+1) = \sum_{j=2}^{9}(j+1)$$

Ex Write in sigma notation

$$\frac{1}{3+1}+\frac{1}{3+2}+\frac{1}{3+3}+\frac{1}{3+4} = \sum_{j=1}^{4}\frac{1}{3+j}$$

What if your upper limit is larger than you want to write out by hand? Tedious!

<u>Summation Formulas</u> will be given on test pg 253

$$\sum_{i=1}^{n} c = cn \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

<u>Summation Properties</u>

can factor out constant $\quad \Sigma c a_i = c \Sigma a_i$

$$\Sigma a_i + b_i = \Sigma a_i + \Sigma b_i$$

$$\Sigma a_i - b_i = \Sigma a_i - \Sigma b_i$$

<u>Examples</u> Evaluate $\displaystyle\sum_{i=1}^{40} 3i = 3\sum_{i=1}^{40} i = 3 \cdot \frac{40(40+1)}{2} =$

$$= 3 \cdot 20 \cdot 41 = 2460$$

Evaluate $\displaystyle\sum_{i=1}^{10} i^2 - 1 = \sum_{i=1}^{10} i^2 - \sum_{i=1}^{10} 1$

$$= \frac{10(10+1)(20+1)}{6} - 1(10)$$

$$= \frac{\cancel{10}^{5} \cdot 11 \cdot \cancel{21}^{7}}{\cancel{6}_{\cancel{2}}} - 10$$

$$= 385 - 10$$

$$= 375$$

What if we want to find the limit of an infinite sum?

Ex Find the $\lim_{n\to\infty} \frac{4}{3n^3}(2n^3+3n^2+n) = \frac{8n^3}{3n^3} = \frac{8}{3}$

Ex Find $\lim_{n\to\infty} \frac{1}{n^2}\left[\frac{n(n+1)}{2}\right] = \frac{n^2+n}{2n^2} = \frac{n^2}{2n^2} = 1/2$

Ex $\lim_{n\to\infty} \sum_{i=1}^{n}\left(\frac{2i}{n}\right)\left(\frac{2}{n}\right) = \sum_{i=1}^{n} \frac{4i}{n^2} = \frac{4}{n^2}\sum_{i=1}^{n} i = \frac{4}{n^2}\,\frac{n(n+1)}{2}$

$\frac{4n^2+4n}{2n^2} = \frac{4n^2}{2n^2} = 2$

Ex $\lim_{n\to\infty} \sum_{i=1}^{n}\left(1+\frac{2i}{n}\right)\left(\frac{3}{n}\right) = \lim_{n\to\infty} \sum \frac{3}{n} + \frac{6i}{n^2}$

$\lim_{n\to\infty} \frac{3}{n}\sum 1 + \frac{6}{n^2}\sum i \quad = \frac{3}{n}\,n + \frac{6}{n^2}\,\frac{n(n+1)}{2}$

$= 3 + 3$

$= 6$