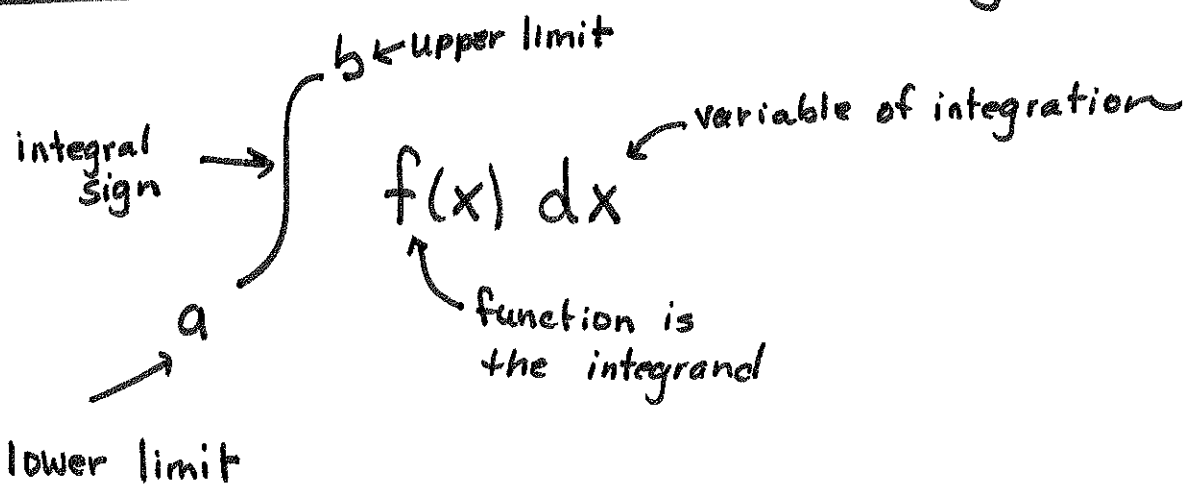


Section 5.3

Definite Integral



Say "integral of f from a to b "

Rules for definite integrals

- $\int_b^a f(x) dx = - \int_a^b f(x) dx$
- $\int_a^a f(x) dx = 0$
- $\int_a^b k f(x) dx = k \int_a^b f(x) dx$
- $\int_a^b (f(x) \pm g(x)) dx = \int_a^b f(x) dx \pm \int_a^b g(x) dx$
- $\int_a^b f(x) dx + \int_b^c f(x) dx = \int_a^c f(x) dx$

Example Given the following

$$\int_{-1}^1 f(x) dx = 5 \quad \int_{-1}^4 f(x) dx = -2 \quad \int_{-1}^1 h(x) dx = 7$$

① Find $\int_4^1 f(x) dx = -\int_{-1}^4 f(x) dx = -(-2) = +2$

② Find $\int_{-1}^{-1} f(x) dx = 0$

③ Find $\int_{-1}^1 [2f(x) + 3h(x)] dx$
 $2 \int_{-1}^1 f(x) dx + 3 \int_{-1}^1 h(x) dx$

$$2(5) + 3(7) = 31$$

④ $\int_{-1}^4 f(x) dx = \int_{-1}^1 f(x) dx + \int_1^4 f(x) dx$

$$= 5 + (-2)$$

$$= 3$$

Definition If $f(x)$ is nonnegative and integrable over a closed interval $[a, b]$ then the area under the curve is the integral of f from a to b .

$$A = \int_a^b f(x) dx$$

Definition If f is integrable on $[a, b]$, then its average value on $[a, b]$, also called its mean is

$$av(f) = \frac{1}{b-a} \int_a^b f(x) dx$$