Section 5.3 Definite Integral

b ← upper limit

variable of integration

integral sign

$$\int_a^b f(x)\,dx$$

function is the integrand

lower limit

say "integral of f from a to b"

Rules for definite integrals

1. $\displaystyle\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$

2. $\displaystyle\int_a^a f(x)\,dx = 0$

3. $\displaystyle\int_a^b k\,f(x)\,dx = k\int_a^b f(x)\,dx$

4. $\displaystyle\int_a^b \big(f(x) \pm g(x)\big)\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

5. $\displaystyle\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$

Example Given the following

$$\int_{-1}^{1} f(x)\,dx = 5 \qquad \int_{1}^{4} f(x)\,dx = -2 \qquad \int_{-1}^{1} h(x)\,dx = 7$$

① Find $\int_{4}^{1} f(x)\,dx = -\int_{1}^{4} f(x)\,dx = -(-2) = +2$

② Find $\int_{-1}^{-1} f(x)\,dx = 0$

③ Find $\int_{-1}^{1} [2f(x) + 3h(x)]\,dx$

$$2\int_{-1}^{1} f(x)\,dx + 3\int_{-1}^{1} h(x)\,dx$$

$$2(5) + 3(7) = 31$$

④ $\int_{-1}^{4} f(x)\,dx = \int_{-1}^{1} f(x)\,dx + \int_{1}^{4} f(x)\,dx$

$$= 5 + (-2)$$

$$= 3$$

Definition If $f(x)$ is nonnegative and integrable over a closed interval $[a,b]$ then the area under the curve is the integral of f from a to b.

$$A = \int_a^b f(x)\,dx$$

Definition If f is integrable on $[a,b]$, then its average value on $[a,b]$, also called its mean is

$$av(f) = \frac{1}{b-a}\int_a^b f(x)\,dx$$