Section 5.5 Indefinite Integrals and the Substitution Method

U-substitution is backward to chain rule

could multiply out + take integral → too long

Examples

✓ What is inside function? compare to chain rule

1. $\int (x^3+x)^5(3x^2+1)dx$

2. $\int \sqrt{2x+1}dx$

3. $\int sec^2(5t+1)\cdot 5dt$

4. $\int \cos(7\theta+3)\,d\theta$

5. $\int x^2 sin(x^3)\,dx$

6. $\int x\sqrt{2x+1}dx$

7. $\int \frac{2x}{\sqrt[3]{x^2+1}}dx$

8. $\int sin^2 xdx$

8. $\int cos^2 xdx$

1. $\int \boxed{(x^3+x)^5}\boxed{(3x^2+1)\,dx}$

$u = x^3+x$ inside

$\frac{du}{dx} = 3x^2+1$

$du = (3x^2+1)\,dx$

$\int u^5\,du$

$\frac{1}{6}u^6 + C$

$\frac{1}{6}(x^3+x)^6 + C$

2. $\int \sqrt{2x+1}\, dx = \int \boxed{(2x+1)^{1/2}}\, \boxed{dx}$

$u = 2x+1$

$\frac{du}{dx} = 2$

$du = 2\,dx$

$\frac{1}{2}\,du = dx$

$= \int u^{1/2}\, \frac{1}{2}\, du$

$= \frac{1}{2} \int u^{1/2}\, du$

$= \frac{1}{2}\, \frac{2}{3}\, u^{3/2} + C$

$= \frac{1}{3}(2x+1)^{3/2} + C$

3. $\int \boxed{\sec^2(5t+1)}\, \boxed{\cdot 5\,dt}$

$u = 5t+1$

$\frac{du}{dt} = 5$

$du = 5\,dt$

$\int \sec^2 u \cdot du$

$\tan u + C$

$\tan(5t+1) + C$

4. $\int \boxed{\cos(7\theta+3)}\, \boxed{d\theta}$

$u = 7\theta + 3$

$\frac{du}{d\theta} = 7$

$du = 7\,d\theta$

$\frac{1}{7}\,du = d\theta$

$\int \cos u \cdot \frac{1}{7}\, du$

$\frac{1}{7} \int \cos u\, du$

$\frac{1}{7} \sin u + C$

$\frac{1}{7} \sin(7\theta+3) + C$

5. $\int x^2 \sin(x^3)\,dx = \int \boxed{\sin(x^3)} \cdot \boxed{x^2\,dx}$

$u = x^3$

$\frac{du}{dx} = 3x^2$

$du = 3x^2\,dx$

$\frac{1}{3}\,du = x^2\,dx$

$= \int \sin u \cdot \frac{1}{3}\,du$

$= \frac{1}{3} \int \sin u\,du$

$= -\frac{1}{3} \cos u + C$

$= -\frac{1}{3} \cos(x^3) + C$

6. $\int x \boxed{\sqrt{2x+1}} \boxed{dx}$

$\uparrow$

compare to #2
what is extra piece?

$u = 2x+1 \longrightarrow u-1 = 2x$

$\frac{u-1}{2} = x$

$\frac{1}{2}(u-1) = x$

$\frac{du}{dx} = 2$

$du = 2\,dx$

$\frac{1}{2}\,du = dx$

$\int \frac{1}{2}(u-1)\sqrt{u} \cdot \frac{1}{2}\,du$

$\frac{1}{4} \int (u-1)u^{1/2}\,du$

$\frac{1}{4} \int (u^{3/2} - u^{1/2})\,du$

$\frac{1}{4}\left(\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right) + C$

$\frac{1}{10}u^{5/2} - \frac{1}{6}u^{3/2} + C$

$\frac{1}{10}(2x+1)^{5/2} - \frac{1}{6}(2x+1)^{3/2} + C$

7. $\int \frac{2x}{\sqrt[3]{x^2+1}} = \int \boxed{(x^2+1)^{-1/3}}\boxed{(2x)\,dx}$

$u = x^2+1$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$= \int u^{-1/3}\,du$

$= \frac{3}{2}u^{2/3} + C$

$= 3/2\,(x^2+1)^{2/3} + C$

8. $\int \sin^2 x\,dx$

must use ~~double angle~~ power reducing

$\sin^2 x = \frac{1-\cos 2x}{2}$

$\int \frac{1}{2}(1-\cos 2x)\,dx$

$\frac{1}{2}\int 1\,dx - \frac{1}{2}\int \cos 2x\,dx$

$u = 2x$

$\frac{du}{dx} = 2$

$du = 2dx$

$\frac{1}{2}du = dx$

$\frac{1}{2}\int 1\,dx - \frac{1}{2}\int \cos u \cdot \frac{1}{2}\,du$

$\frac{1}{2}x - \frac{1}{4}\sin u + C$

$\frac{1}{2}x - \frac{1}{4}\sin 2x + C$

9. $\int \cos^2 x\,dx$

$\cos^2 x = \frac{1}{2}(1+\cos 2x)$

$\frac{1}{2}\left(\int 1\,dx + \int \cos 2x\,dx\right)$ etc