Section 1.7 Chain Rule

The Chain Rule for Derivatives $\frac{d}{dx}[f(g(x)] = f'(g(x)) \cdot g'(x)$

The derivative of the outer function with inner function unchanged times the derivative of the inner function.

The Extended Power Rule $\frac{d}{dx}[g(x)]^k = k[g(x)]^{k-1} \cdot g'(x)$

Keep the inside, take the derivative of the outside, multiply by the derivative of the inside.

Example 2: Find the derivatives of the following.

a. $y = (2x^2 + 5x)^{10}$

b. $y = (1 + x^3)^3$

c. $y = (5 - 2x)^{\frac{3}{2}}$

d. $y = \sqrt{1 - x}$

e. $y = \frac{1}{(3x-16)^2}$

Example 3: Differentiate and simplify. $y = (x^2 - 7x)^4(3x - 5)^2$

Example 4: Differentiate and simplify. $y = \frac{4x^2}{(7-5x)^3}$

Example 5: Find the equation of the tangent line to the curve $y = \sqrt{x^2 + 3x}$ at the point (1, 2).

Example 6: Differentiate $f(x) = (3 - 5x)^{250}$

A. $f'(x) = -1250(3 - 5x)^{250}$
B. $f'(x) = 250(3 - 5x)^{249}$
C. $f'(x) = 1250(3 - 5x)^{249}$
D. $f'(x) = -1250(3 - 5x)^{249}$

Example 7: Differentiate $f(x) = 2x(2x + 5)^3$

A. $f'(x) = 2(2x + 5)^2(8x + 5)$
B. $f'(x) = 2(2x + 5)^3(5x + 5)$
C. $f'(x) = 2(8x + 5)^2$
D. $f'(x) = 2(2x + 5)^2$

Example 8: Given a total-revenue function $R(x) = 1200\sqrt{x^2 - 0.2x}$ and a total-cost function $C(x) = 2200(x^2 + 3)^{\frac{1}{3}} + 500$, both in thousands of dollars, find the rate at which total profit is changing when x items have been produced and sold.

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The Chain Rule for Derivatives $\frac{d}{dx}[f(g(x)] = f'(g(x)) \cdot g'(x)$
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Keep the inside, take the derivative of the outside, multiply by the derivative of the inside.

Example 2: Find the derivatives of the following.

a. $y = (2x^2 + 5x)^{10}$

$y' = 10(2x^2+5x)^9 \cdot (4x+5)$

b. $y = (1 + x^3)^3$

$y' = 3(1+x^3)^2 \cdot (3x^2)$

c. $y = (5 - 2x)^{\frac{3}{2}}$

$y' = \frac{3}{2}(5-2x)^{\frac{1}{2}} \cdot (-2)$

d. $y = \sqrt{1-x} = (1-x)^{1/2}$

$y' = \frac{1}{2}(1-x)^{-1/2}(-1)$

e. $y = \frac{1}{(3x-16)^2} = (3x-16)^{-2}$

$y' = -2(3x-16)^{-3}(3)$

product rule

Example 3: Differentiate and simplify. $y = (\underset{f}{x^2 - 7x})^4(\underset{g}{3x - 5})^2$

$$y' = f \cdot g' + g \cdot f'$$

$$= (x^2-7x)^4\left((3x-5)^2\right)' + (3x-5)^2\left((x^2-7x)^4\right)'$$

$$= (x^2-7x)^4 \cdot 2(3x-5)^1 \cdot 3 + (3x-5)^2 \cdot 4(x^2-7x)^3 \cdot (2x-7) \checkmark$$

$$= 6(3x-5)(x^2-7x)^4 + 4(2x-7)(3x-5)^2(x^2-7x)^3 \checkmark$$

quotient rule

Example 4: Differentiate and simplify. $y = \frac{4x^2}{(7-5x)^3}$

$$y' = \frac{DN' - ND'}{D^2}$$

$$= \frac{(7-5x)^3 \cdot 8x - 4x^2 \cdot 3(7-5x)^2 \cdot (-5)}{\left((7-5x)^3\right)^2} \checkmark$$

$$= \frac{8x(7-5x)^3 + 60x^2(7-5x)^2}{(7-5x)^6} \checkmark$$

Example 5: Find the equation of the tangent line to the curve $y = \sqrt{x^2 + 3x}$ at the point (1, 2).

$$y = (x^2+3x)^{1/2}$$

$$y' = \tfrac{1}{2}(x^2+3x)^{-1/2}(2x+3)$$

$$m = y' = \tfrac{1}{2}(1^2+3(1))^{-1/2}(2 \cdot 1+3)$$

$$m = 5/4$$

equation

$$y - y_1 = m(x - x_1)$$

$$y - 2 = \tfrac{5}{4}(x-1)$$

$$y - 2 = \tfrac{5}{4}x - \tfrac{5}{4}$$

$$y = \tfrac{5}{4}x + \tfrac{3}{4}$$

Example 6: Differentiate $f(x) = (3 - 5x)^{250}$

A. $f'(x) = -1250(3 - 5x)^{250}$
B. $f'(x) = 250(3 - 5x)^{249}$
C. $f'(x) = 1250(3 - 5x)^{249}$
D. $f'(x) = -1250(3 - 5x)^{249}$

$$f'(x) = 250 \cdot (3-5x)^{249} \cdot (-5)$$
$$= -1250(3-5x)^{249}$$

Example 7: Differentiate $f(x) = 2x(2x + 5)^3$

A. $f'(x) = 2(2x + 5)^2(8x + 5)$
B. $f'(x) = 2(2x + 5)^3(5x + 5)$
C. $f'(x) = 2(8x + 5)^2$
D. $f'(x) = 2(2x + 5)^2$

product rule

$$f'(x) = f \cdot g' + g \cdot f'$$
$$= 2x \cdot 3(2x+5)^2(2) + (2x+5)^3 \cdot 2$$
$$= 12x(2x+5)^2 + 2(2x+5)^3$$
$$= 2(2x+5)^2[6x + (2x+5)]$$
$$= 2(2x+5)^2(8x+5)$$

Example 8: Given a total-revenue function $R(x) = 1200\sqrt{x^2 - 0.2x}$ and a total-cost function $C(x) = 2200(x^2 + 3)^{\frac{1}{3}} + 500$, both in thousands of dollars, find the rate at which total profit is changing when x items have been produced and sold.

Total-Profit = total-revenue — total-cost

$$P(x) = R(x) - C(x)$$
$$= 1200\sqrt{x^2 - 0.2x} - (2200(x^2+3)^{1/3} + 500)$$
$$P(x) = 1200(x^2 - 0.2x)^{1/2} - 2200(x^2+3)^{1/3} - 500$$

$$P' = 1200 \cdot \tfrac{1}{2}(x^2 - 0.2x)^{-1/2}(2x - 0.2) - 2200 \cdot \tfrac{1}{3}(x^2+3)^{-2/3} \cdot (2x)$$