Section 1.8 Higher Order Derivatives

Notation for higher order derivatives

y'	y''	y'''	y^{iv}
$f'(x)$	$f''(x)$	$f^{(3)}(x)$	$f^{(4)}(x)$
$\frac{dy}{dx}$	$\frac{d^2y}{dx^2}$	$\frac{d^3y}{dx^3}$	$\frac{d^4y}{dx^4}$

<u>Example 1:</u> Find the fourth derivative of $y = x^5 - 3x^4 + x$.

<u>Example 2:</u> For the following, find the third derivative.

a. $f(x) = -2x^5 - 2$

b. $f(x) = \frac{1}{x^{12}}$

c. $f(x) = \sqrt{x}$

d. $f(x) = \frac{9}{x^6}$

Example 3: For the following, find $\frac{d^2y}{dx^2}$

a. $y = (x^2 + 5)^6$

b. $y = \frac{2x+1}{3x-5}$

Velocity and Acceleration

A function's derivative represents an instantaneous rate of change. When the function relates distance traveled to time, the instantaneous rate of change is called speed or velocity.

The velocity of an object that is $s(t)$units from a starting point at time t is given by
$Velocity = v(t) = s'(t)$.

The rate at which velocity changes is called acceleration.
$Acceleration = a(t) = v'(t) = s''(t)$

Example 4: If a stone is dropped from a cliff, the distance it falls in *t* seconds is approximately $s(t) = 4.9t^2$ where $s(t)$ is in meters.

a. Find how far the stone has traveled 5 seconds after being dropped.

b. Find how fast it is traveling 5 seconds after being dropped.

c. Find its acceleration after it has been falling for 5 seconds.

Example 5: The number of Apple iPads sold per year, in millions, t years after 2010, can be approximated by

$$N(t) = 0.106t^4 - 1.452t^3 + 5.659t^2 + 1.946t + 20.714$$

Find $N(7)$, $N'(7)$, and $N''(7)$, and explain what these values represent.

a. $N(7)$

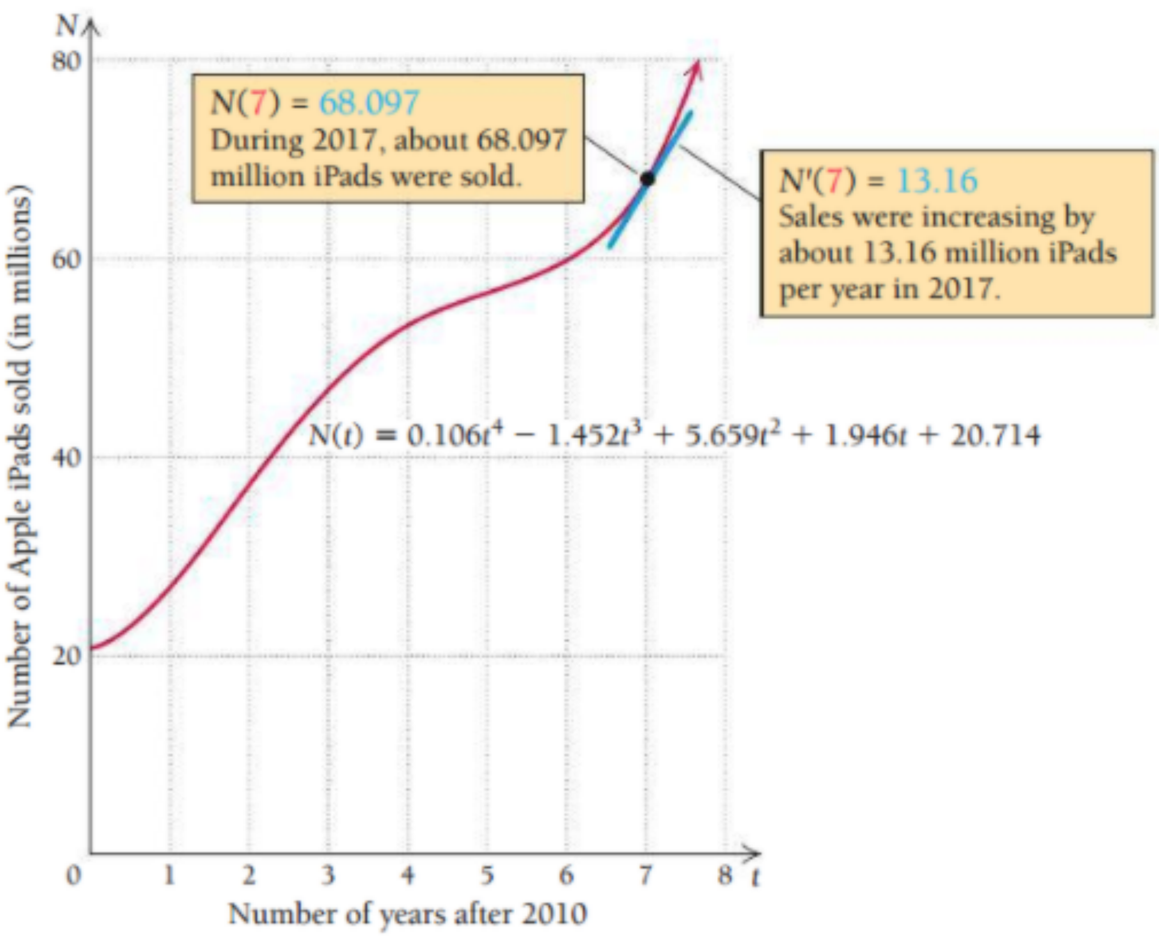
In __________, Apple sold approximately _____________ million iPads

b. $N'(7)$

In __________, Apple's yearly sales of iPads were _____________ at the rate of about _____________ million units per year.

c. $N''(7)$

In __________, the rate of change is, itself, changing at a rate of _____________ million units/year^2.
Since the change in the rate of change is ______________, sales are _____________________



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$\frac{dy}{dx}$	$\frac{d^2y}{dx^2}$	$\frac{d^3y}{dx^3}$	$\frac{d^4y}{dx^4}$

<u>**Example 1:**</u> Find the fourth derivative of $y = x^5 - 3x^4 + x$.

$y' = 5x^4 - 12x^3 + 1$

$y'' = 20x^3 - 36x^2$

$y''' = 60x^2 - 72x$

$\boxed{y^{iv} = 120x - 72}$

<u>**Example 2:**</u> For the following, find the third derivative.

a. $f(x) = -2x^5 - 2$

$f'(x) = -10x^4$

$f''(x) = -40x^3$

$\boxed{f'''(x) = -120x^2}$

b. $f(x) = \frac{1}{x^{12}} = x^{-12}$

$f'(x) = -12x^{-13}$

$f''(x) = 156x^{-14}$

$\boxed{f'''(x) = -2184x^{-15}}$

c. $f(x) = \sqrt{x} = x^{1/2}$

$f'(x) = \frac{1}{2}x^{-1/2}$

$f''(x) = -\frac{1}{4}x^{-3/2}$

$\boxed{f'''(x) = \frac{3}{8}x^{-5/2}}$

d. $f(x) = \frac{9}{x^6} = 9x^{-6}$

$f'(x) = -54x^{-7}$

$f''(x) = 378x^{-8}$

$\boxed{f'''(x) = -3024x^{-9}}$

Example 3: For the following, find $\frac{d^2y}{dx^2}$

a. $y = (x^2 + 5)^6$

$\frac{dy}{dx} = 6(x^2+5)^5(2x) = 12x(x^2+5)^5$

$\frac{d^2y}{dx^2} = fg' + gf' = (12x)(5(x^2+5)^4(2x)) + (x^2+5)^5(12)$

b. $y = \frac{2x+1}{3x-5}$

$\frac{dy}{dx} = \frac{gf' - fg'}{g^2} = \frac{(3x-5)(2)-(2x+1)(3)}{(3x-5)^2} = \frac{6x-10-6x-3}{(3x-5)^2} = \frac{-13}{(3x-5)^2} = -13(3x-5)^{-2}$

$\frac{d^2y}{dx^2} = 26(3x-5)^{-3}(3)$

Velocity and Acceleration

A function's derivative represents an instantaneous rate of change. When the function relates distance traveled to time, the instantaneous rate of change is called speed or velocity.

The velocity of an object that is $s(t)$units from a starting point at time t is given by
$Velocity = v(t) = s'(t)$.

The rate at which velocity changes is called acceleration.
$Acceleration = a(t) = v'(t) = s''(t)$

Example 4: If a stone is dropped from a cliff, the distance it falls in *t* seconds is approximately $s(t) = 4.9t^2$ where $s(t)$ is in meters.

a. Find how far the stone has traveled 5 seconds after being dropped.

$s(5) = 4.9(5)^2 = 122.5$ meters

b. Find how fast it is traveling 5 seconds after being dropped.

$v(t) = s'(t) = 9.8t$

$v(5) = 9.8(5) = 49$ meters/sec

c. Find its acceleration after it has been falling for 5 seconds.

$a(t) = s''(t) = 9.8$

$a(5) = 9.8$ m/sec^2

Example 5: The number of Apple iPads sold per year, in millions, t years after 2010, can be approximated by

$$N(t) = 0.106t^4 - 1.452t^3 + 5.659t^2 + 1.946t + 20.714$$

Find $N(7)$, $N'(7)$, and $N''(7)$, and explain what these values represent.

a. $N(7) = 68.097$

In 2017, Apple sold approximately 68.097 million iPads

b. $N'(7) = 13.16$ $\qquad N'(t) = 0.424t^3 - 4.356t^2 + 11.318t + 1.946$

In 2017, Apple's yearly sales of iPads were increasing at the rate of about 13.16 million units per year.

c. $N''(7) = 12.662$ $\qquad N''(t) = 1.272t^2 - 8.712t + 11.318$

In 2017, the rate of change is, itself, changing at a rate of 12.662 million units/year^2.
Since the change in the rate of change is positive, sales are accelerating