Section 2.2 Derivatives of Exponential (Base-e) Functions

The derivative of the function $f(x) = e^x$ is the function itself, $f'(x) = e^x$.

Example 1: Find the derivative of the following.

a. $y = e^x$

b. $y = 3e^x$

c. $y = x^2 e^x$

d. $y = \frac{e^x}{x^3}$

Example 2: Find the first derivative of the following with the Chain Rule.

a. $y = 6e^{8x}$

b. $y = 4 - 2e^{x^2}$

Example 3: Find the second derivative. $y = e^{-5x^2}$

Example 4: Franco's Fishing Emporium invested \$50,000 in an account that earns 1.25% annual interest, compounded continuously. The value of the account after *t* years is given by $A(t) = 50{,}000e^{0.0125t}$. Find $A(5)$ and $A'(5)$, and interpret the meaning of each of these values.

After ________years, the value of Franco's Fishing Emporium's account is _____________, and at that instant, the value is growing at the rate of ____________________ per year.

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The derivative of the function $f(x) = e^x$ is the function itself, $f'(x) = e^x$.

Example 1: Find the derivative of the following.

a. $y = e^x$ $\quad y' = e^x$

b. $y = 3e^x$ $\quad y' = 3e^x$

c. $y = x^2e^x$ product

$y' = x^2(e^x)' + e^x(x^2)'$

$y' = x^2 \cdot e^x + e^x \cdot 2x$

d. $y = \frac{e^x}{x^3}$ quotient

$y' = \dfrac{x^3(e^x)' - e^x(x^3)'}{(x^3)^2} = \dfrac{x^3 e^x - e^x \cdot 3x^2}{x^6}$

Example 2: Find the first derivative of the following with the Chain Rule.

a. $y = 6e^{8x}$

$y' = 6e^{8x} \cdot 8 = 48e^{8x}$

b. $y = 4 - 2e^{x^2}$

$y' = -2e^{x^2} \cdot 2x = -4xe^{x^2}$

Example 3: Find the second derivative. $\quad y = e^{-5x^2}$

$y' = e^{-5x^2} \cdot -10x = -10xe^{-5x^2}$

$y'' = (-10x)(e^{-5x^2})' + e^{-5x^2} \cdot (-10x)'$

$= -10xe^{-5x^2} \cdot -10x + e^{-5x^2} \cdot -10 = 100x^2e^{-5x^2} - 10e^{-5x^2}$

Example 4: Franco's Fishing Emporium invested \$50,000 in an account that earns 1.25% annual interest, compounded continuously. The value of the account after t years is given by $A(t) = 50{,}000e^{0.0125t}$. Find $A(5)$ and $A'(5)$, and interpret the meaning of each of these values.

$A(5) = 50{,}000\,e^{0.0125 \cdot 5} = \$53{,}224.73$

$A'(t) = 50{,}000\,e^{0.0125t} \cdot (0.0125)$

$A'(5) = 50{,}000\,e^{0.0125 \cdot 5} \cdot (0.0125) = \$665.31$

After 5 years, the value of Franco's Fishing Emporium's account is \$53,224.73, and at that instant, the value is growing at the rate of \$665.73 per year.