Section 2.3 Derivatives of Natural Logarithmic Functions

For any positive number *x*, and $y = \ln x$, then $y' = \frac{1}{x}$

Example 1: Differentiate the following. Assume x is positive.

a. $y = x^3 + 4e^x + 5x + 3\ln x$

b. $y = 3x^2 \ln x$

c. $y = \frac{\ln x}{x^3}$

Example 2: Differentiate. Assume x is positive.

a. $y = \ln(x^2 - 3)$

b. $y = (\ln x)^4$

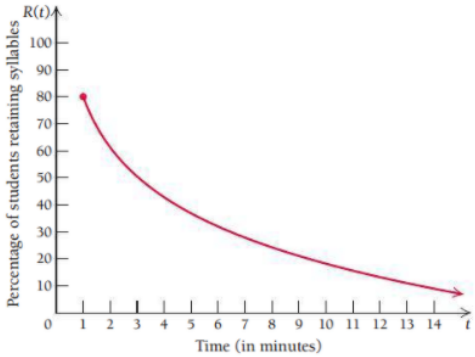
Example 3: Find an equation for the tangent line to the graph of $f(x) = 3 + \ln x$ at the point (1, 3).

Example 4: Find an equation for the tangent line to the graph of $f(x) = e^{3x}$ at the point (0, 1).

Example 5: Find y''' when $y = \sqrt[3]{x} + e^{2x} + \ln x$

Example 6: In a psychological experiment, students were shown a set of nonsense syllables and asked to recall them every minute thereafter. The percentage *R(t)* who retained the syllables after *t* minutes was found to be given by the logarithmic learning model

$R(t) = 80 - 27 \ln t$, for $1 \le t \le 15$



a. What percentage of students retained the syllables after 1 minute?
b. Find *R'(2)* and explain what it represents.

Example 7: As part of a study, students in a psychology class took a final exam and then took equivalent forms of the exam at monthly intervals thereafter. After *t* months, the average score $S(t)$, as a percentage, was found to be given by

$$S(t) = 78 - 15 \ln(t + 1), \ 0 \le t \le 80.$$

a. What was the average score when the students initially took the test?
b. What was the average score after 4 months?
c. Find $S'(t)$.
d. Find $S'(4)$ and interpret the meaning.

Section 2.3 Derivatives of Natural Logarithmic Functions

For any positive number x, and $y = \ln x$, then $y' = \frac{1}{x}$

Example 1: Differentiate the following. Assume x is positive.

a. $y = x^3 + 4e^x + 5x + 3\ln x$

$y' = 3x^2 + 4e^x + 5 + 3\left(\frac{1}{x}\right)$

b. $y = 3x^2 \ln x$

$y' = 3x^2 \cdot \frac{1}{x} + \ln x \cdot 6x = 3x + 6x \ln x$

c. $y = \frac{\ln x}{x^3}$

$y' = \frac{x^3 \cdot \frac{1}{x} - \ln x \cdot 3x^2}{(x^3)^2} = \frac{x^2 - 3x^2 \ln x}{x^6}$

Example 2: Differentiate. Assume x is positive.

a. $y = \ln(x^2 - 3)$

$y' = \frac{1}{x^2-3} \cdot 2x$

$y' = \frac{2x}{x^2-3}$

b. $y = (\ln x)^4$

$y' = 4(\ln x)^3 \cdot \frac{1}{x}$

Example 3: Find an equation for the tangent line to the graph of $f(x) = 3 + \ln x$ at the point (1, 3).

$f' = \frac{1}{x}$

$m = f' = \frac{1}{1} = 1$

$y - y_1 = m(x - x_1)$

$y - 3 = 1(x - 1)$

$y - 3 = x - 1$

$y = x + 2$

Example 4: Find an equation for the tangent line to the graph of $f(x) = e^{3x}$ at the point (0, 1).

$f' = e^{3x} \cdot 3$

$m = f' = e^{3 \cdot 0} \cdot 3 = 1 \cdot 3 = 3$

$y - y_1 = m(x - x_1)$

$y - 1 = 3(x - 0)$

$y - 1 = 3x$

$y = 3x + 1$

Example 5: Find y''' when $y = \sqrt[3]{x} + e^{2x} + \ln x$ $= x^{\frac{1}{3}} + e^{2x} + \ln x$

$y' = \frac{1}{3}x^{-2/3} + e^{2x} \cdot 2 + \frac{1}{x} = \frac{1}{3}x^{-2/3} + 2e^{2x} + x^{-1}$

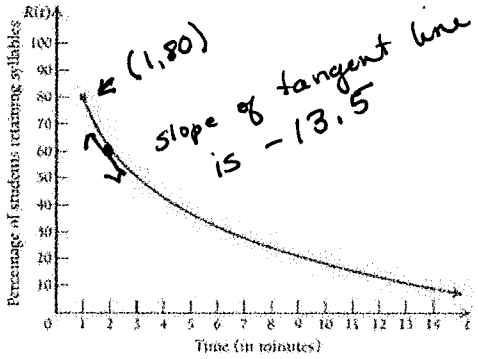
$y'' = -\frac{2}{9}x^{-5/3} + 2e^{2x} \cdot 2 - x^{-2} = -\frac{2}{9}x^{-5/3} + 4e^{2x} - x^{-2}$

$y''' = \frac{10}{29}x^{-8/3} + 4e^{2x} \cdot 2 + 2x^{-3} = \frac{10}{29}x^{-8/3} + 8e^{2x} + 2x^{-3}$

Example 6: In a psychological experiment, students were shown a set of nonsense syllables and asked to recall them every minute thereafter. The percentage *R(t)* who retained the syllables after *t* minutes was found to be given by the logarithmic learning model

$R(t) = 80 - 27\ln t$, for $1 \le t \le 15$

$R'(t) = -27 \cdot \frac{1}{t}$



a. What percentage of students retained the syllables after 1 minute?
b. Find *R'(2)* and explain what it represents.

a. $R(1) = 80 - 27\ln 1 = 80\%$

b. $R'(2) = -27 \cdot \frac{1}{2} = -13.5$

After 2 minutes, the percentage who retained the syllables is decreasing at the rate of 13.5% per minute

Example 7: As part of a study, students in a psychology class took a final exam and then took equivalent forms of the exam at monthly intervals thereafter. After *t* months, the average score $S(t)$, as a percentage, was found to be given by

$S(t) = 78 - 15\ln(t+1)$, $0 \le t \le 80$.

a. What was the average score when the students initially took the test?
b. What was the average score after 4 months?
c. Find $S'(t)$.
d. Find $S'(4)$ and interpret the meaning.

a. $S(0) = 78 - 15\ln(0+1) = 78\%$

b. $S(4) = 78 - 15\ln(4+1) = 53.86\%$

c. $S'(t) = -15 \cdot \frac{1}{t+1}$

d. $S'(4) = -15 \cdot \frac{1}{4+1} = -15 \cdot \frac{1}{5} = -3$

After 4 months, the average scores are decreasing at a rate of 3% per month