Section 2.6 Derivatives of a^x and $log_a x$

If $y = a^x$, then $y' = a^x (\ln a)$

Example 1 Differentiate the following.

a. $y = 2^x$

b. $y = (1.4)^x$

c. $f(x) = e^x$

d. $y = 10 \cdot 2.3^x$

Example 2 Differentiate the following.

a. $y = 3^{2x+1}$

b. $f(x) = 7^{x^2}$

If $y = log_a x$, then $y' = \frac{1}{x}\left(\frac{1}{\ln a}\right) = \frac{1}{x \ln a}$

Example 3 Differentiate the following.

a. $y = log_8 x$

b. $y = \log x$

c. $y = 3\, log_4 x$

Example 4 Differentiate the following.

a. $f(x) = log_3(x^2 + 1)$

b. $y = x^3 log_5 x$

Example 5

In 2012, 34.2% of all glass containers were recycled. A beverage company used 400,000 lb of glass containers per year. After recycling, the amount of glass, in pounds, still in use after *t* years is given by $N(t) = 400{,}000(0.341)^t$

a. Find *N(4)* and explain its meaning.

b. Find *N'(4)* and explain its meaning.

Section 2.6 Derivatives of a^x and $log_a x$

If $y = a^x$, then $y' = a^x (\ln a)$

Example 1 Differentiate the following.

a. $y = 2^x$

$y' = 2^x \ln 2$

b. $y = (1.4)^x$

$y' = 1.4^x (\ln 1.4)$

c. $f(x) = e^x$

$y' = e^x \ln e = e^x (1) = e^x$

d. $y = 10 \cdot 2.3^x$

$y' = (10)(2.3^x) \ln 2.3$

Example 2 Differentiate the following.

a. $y = 3^{2x+1}$

$y' = 3^{2x+1} (\ln 3)(2)$

b. $f(x) = 7^{x^2}$

$y' = 7^{x^2} (\ln 7)(2x)$

If $y = log_a x$, then $y' = \frac{1}{x}\left(\frac{1}{\ln a}\right) = \frac{1}{x \ln a}$

Example 3 Differentiate the following.

a. $y = log_8 x$

$y' = \left(\frac{1}{x}\right)\left(\frac{1}{\ln 8}\right)$

b. $y = \log x$ = common log $= \log_{10} x$

$y' = \left(\frac{1}{x}\right)\left(\frac{1}{\ln 10}\right)$

c. $y = 3\, log_4 x$

$y' = (3)\left(\frac{1}{x}\right)\left(\frac{1}{\ln 4}\right)$

Example 4 Differentiate the following.

a. $f(x) = log_3(x^2 + 1)$

$f'(x) = \frac{1}{x^2+1} \cdot \frac{1}{\ln 3} \cdot 2x$

b. $y = x^3 log_5 x$ product

$y' = x^3 \cdot \frac{1}{x} \cdot \frac{1}{\ln 5} + (\log_5 x)(3x^2)$

Example 5

In 2012, 34.2% of all glass containers were recycled. A beverage company used 400,000 lb of glass containers per year. After recycling, the amount of glass, in pounds, still in use after t years is given by $N(t) = 400{,}000(0.341)^t$

$N'(t) = 400{,}000\,(0.341)^t (\ln .341)$

a. Find *N(4)* and explain its meaning.

$N(4) = 5408.51$

After 4 years, there are still 5408 lbs of glass in use.

b. Find *N'(4)* and explain its meaning.

$N'(4) = -5818.87$

After 4 years, the amount of glass still in use is decreasing by 5818 lbs/year.