Section 3.1 First Derivatives and Curve Sketching

A ________________ of a function is any number in the domain for which the tangent line is horizontal (____________________) or for which the derivative does not exist.

A function is ________________________ when __________________

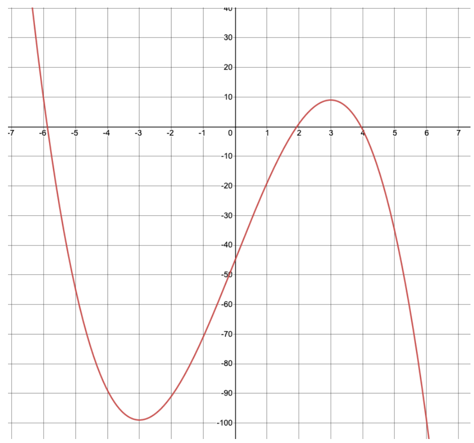
A function is ________________________ when __________________

A function has a ___________________________ when f'(x) changes from _______________

A function has a ___________________________ when f'(x) changes from _______________

Example 1:
Consider the graph of
$f(x) = -x^3 + 27x - 45.$
Discuss intervals where f(x) is increasing or decreasing.
Does f(x) have a relative max or min?

Use desmos.com to graph



a) Find the critical values for *f(x)*.

b) Make a Number Line for *f'(x)*

c) Give the open intervals where the function is
Increasing
Decreasing

d) Find the points in (x, y) form where the function has relative extrema.
Relative Max
Relative Min

Example 2: $f(x) = 5x^4 + 20x^3$

a) Find the critical values for *f(x)*.

b) Make a Number Line for *f'(x)*

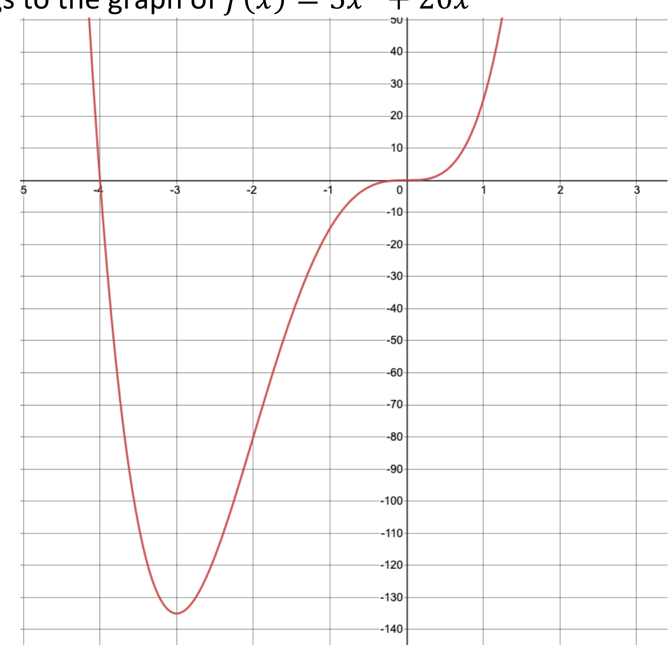
c) Give the open intervals where the functions is
Increasing

Decreasing

d) Find the points in (x, y) form where the function has relative extrema.
Relative Max

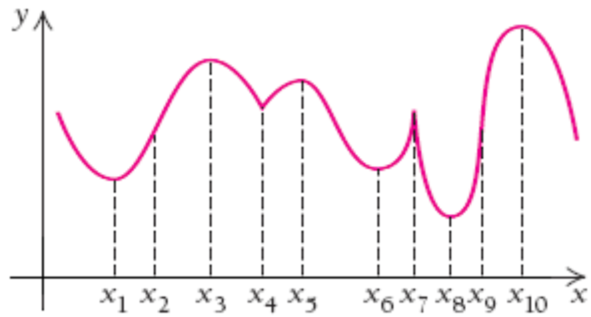
Relative Min

e) Compare your findings to the graph of $f(x) = 5x^4 + 20x^3$



Example 3:

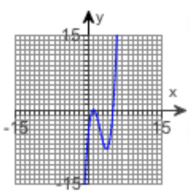
Consider the graph of *f(x)*. Explain the idea of a critical value. Then determine which x-values are critical values, and state why.



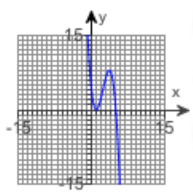
Example 4:
Draw a graph to match this description. The function *f(x)* has a positive derivative over $(-\infty, 1)$ and $(1, 5)$, a negative derivative over $(5, \infty)$, and a derivative equal to 0 at x=1.

Which of the following graphs matches the description?

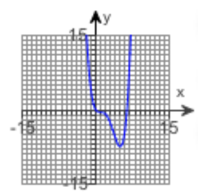
○ **A.**



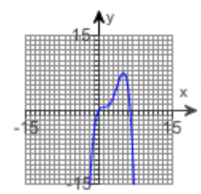
○ **B.**



○ **C.**



○ **D.**



Example 5:
Find any relative extrema of the function $f(x) = x^3 - 3x^2$. Identify intervals over which the function is increasing and over which it is decreasing. Then sketch a graph of the function.

a) Find the critical values for *f(x)*.

b) Make a Number Line for *f′(x)*

c) Increasing

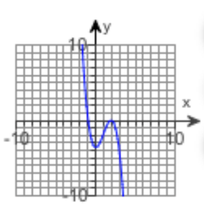
Decreasing

d) Relative Max

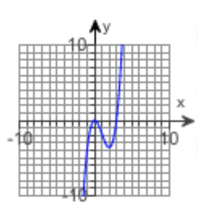
Relative Min

Choose the correct graph below.

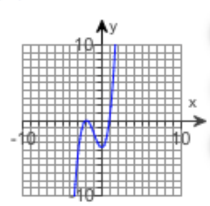
○ **A.**



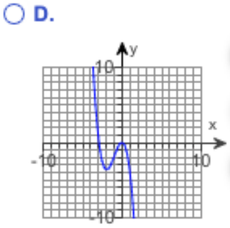
○ **B.**



○ **C.**



○ **D.**



Section 3.1 First Derivatives and Curve Sketching

A critical value of a function is any number in the domain for which the tangent line is horizontal ($f'(x) = 0$) or for which the derivative does not exist.

A function is increasing ↗ when $f'(x) > 0$ positive

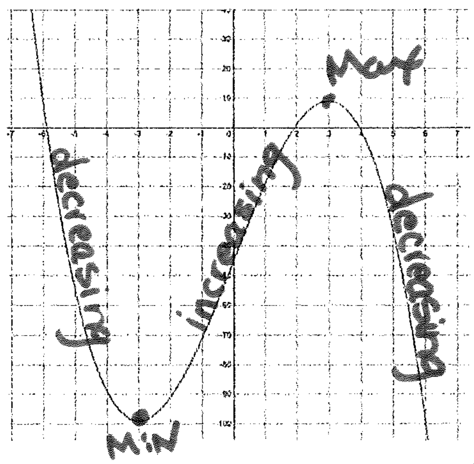
A function is decreasing ↘ when $f'(x) < 0$ negative

A function has a relative maximum when f'(x) changes from + −

A function has a relative minimum when f'(x) changes from − +

Example 1:
Consider the graph of
$f(x) = -x^3 + 27x - 45$.
Discuss intervals where f(x) is increasing or decreasing.
Does f(x) have a relative max or min?

Use desmos.com to graph



a) Find the critical values for *f(x)*. means take derivative =0 and solve

$f'(x) = -3x^2 + 27 = 0$

$-3x^2 = -27$

$x^2 = 9$

so $x = \pm 3$ are critical values

b) Make a Number Line for *f'(x)* use critical values

$$\begin{array}{ccccccc} & - & & + & & - & \\ -4 & -3 & & 0 & 3 & & 4 \\ & CV & & & CV & & \end{array}$$

$f'(-4) = -3(-4)^2 + 27 = -$

$f'(0) = -3(0)^2 + 27 = +$

$f'(4) = -3(4)^2 + 27 = -$

c) Give the open intervals where the functions is

Increasing $(-3, 3)$

Decreasing $(-\infty, -3) \cup (3, \infty)$

d) Find the points in (x, y) form where the function has relative extrema.

Relative Max $(3, 9)$

Relative Min $(3, -99)$

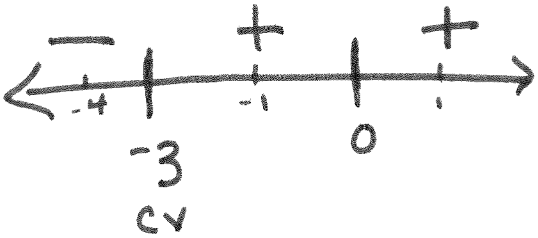
use original f(x) to find y values

Example 2: $f(x) = 5x^4 + 20x^3$

a) Find the critical values for $f(x)$.

$$f'(x) = 20x^3 + 60x^2 = 0$$
$$20x^2(x+3) = 0$$
$$20x^2 = 0 \qquad x+3 = 0$$
$$x^2 = 0 \qquad x = -3$$
$$x = 0$$

b) Make a Number Line for $f'(x)$



$f'(-4) = -$

$f'(-1) = +$

$f'(1) = +$

c) Give the open intervals where the functions is

Increasing $(-3, 0)(0, \infty)$

Decreasing $(-\infty, -3)$

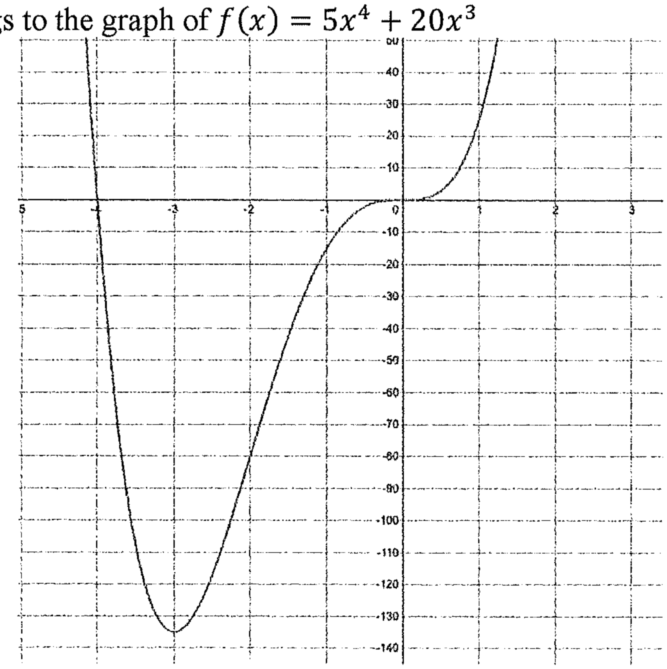
d) Find the points in (x, y) form where the function has relative extrema.

Relative Max None

Relative Min $(-3, -135)$

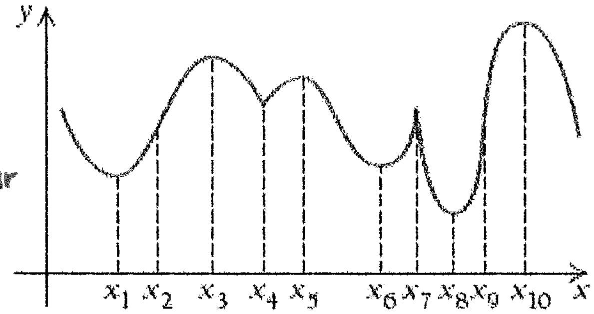
$f(-3) = 5(-3)^4 + 20(-3)^3 = -135$

e) Compare your findings to the graph of $f(x) = 5x^4 + 20x^3$



Example 3:

Consider the graph of *f(x)*. Explain the idea of a critical value. Then determine which x-values are critical values, and state why.



CV are x-values where extrema may occur

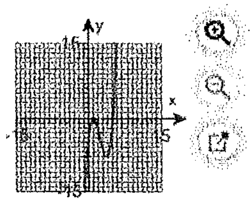
$x_1, x_3, x_4, x_5, x_6, x_7, x_8$ and x_{10} because $f'(x) = 0$ or DNE

Example 4:

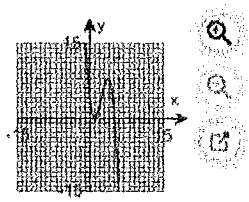
Draw a graph to match this description. The function *f(x)* has a positive derivative over $(-\infty, 1)$ and $(1, 5)$, a negative derivative over $(5, \infty)$, and a derivative equal to 0 at x=1.

Which of the following graphs matches the description?

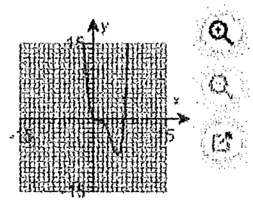
A.



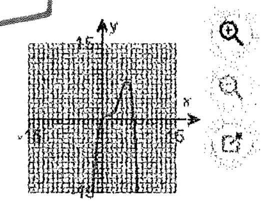
B.



C.



D.



Example 5:

Find any relative extrema of the function $f(x) = x^3 - 3x^2$. Identify intervals over which the function is increasing and over which it is decreasing. Then sketch a graph of the function.

a) Find the critical values for *f(x)*.

$f'(x) = 3x^2 - 6x = 0$

$3x(x-2) = 0$

$x = 0 \quad x = 2$

b) Make a Number Line for *f′(x)*

$+ \quad - \quad +$

0 CV, 2 CV

c) Increasing $(-\infty, 0)\ (2, \infty)$

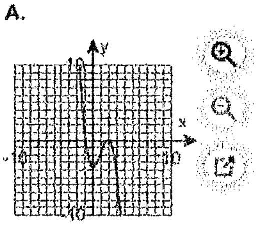
Decreasing $(0, 2)$

d) Relative Max $(0, 0)$

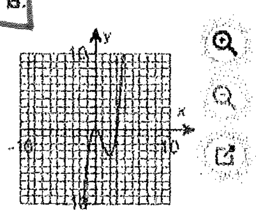
Relative Min $(2, -4)$

Choose the correct graph below.

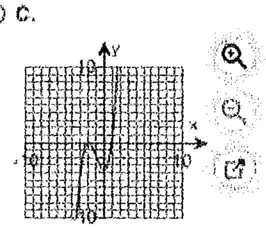
A.



B.



C.



D.