## Section 3.2 Second Derivative and Curve Sketching

A graph of a function is ________________________ if ______________________________

A graph of a function is ________________________ if ______________________________

An ______________________ point happens where the ______________________ changes.

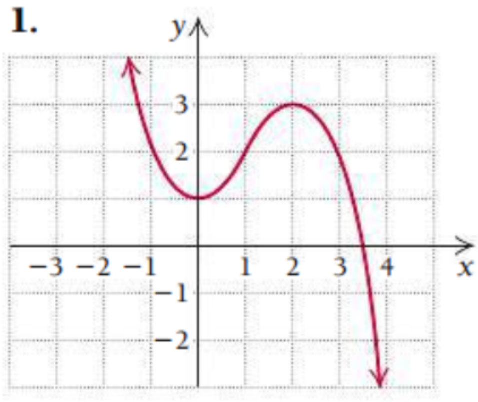
It must occur when $f''(x) = 0$ or when $f''(x)$ does not exist.

| Shape of Graph | $f'(x) > 0$ | $f'(x) < 0$ |
| --- | --- | --- |
| $f''(x) > 0$ | | |
| $f''(x) < 0$ | | |

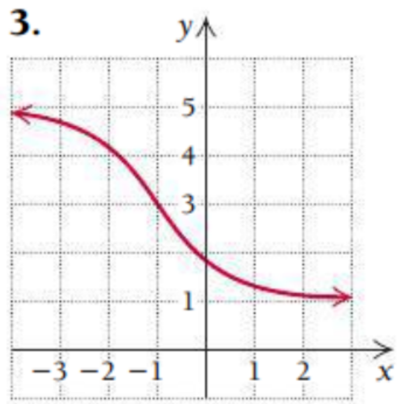
**Example 1**

Identify the points of infection and the intervals where the function is concave up or concave down.

**1.**




**3.**



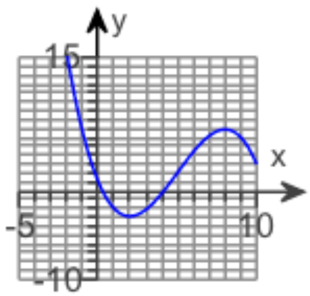

**The Second Derivative Test for Relative Extrema**

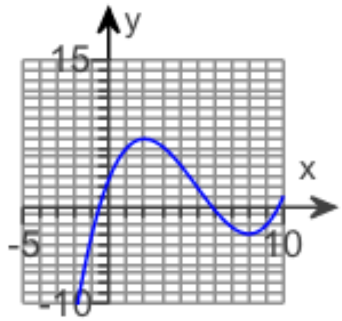
Suppose that c is a critical value for the function (means that $f'(c) = 0$).

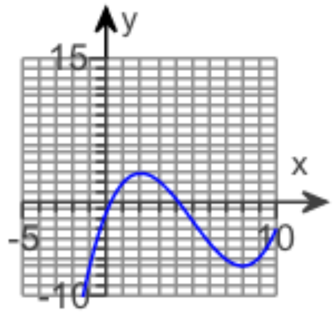
Then $f(c)$ is a relative minimum if ___________________________

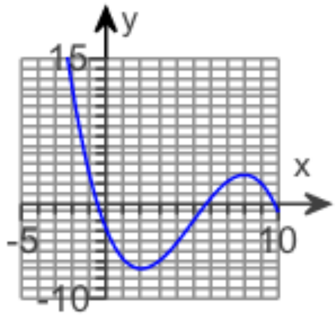
and $f(c)$ is a relative maximum if ____________________________

**Example 2:**
Sketch the graph that possesses the characteristics listed. f is concave down at the point (1, 6), concave up at the point (9, -2), and has an inflection point at (5,2).







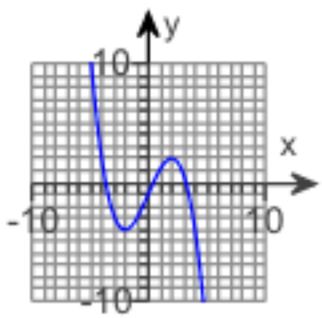


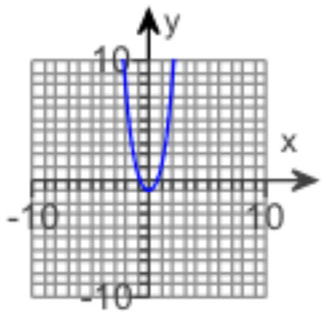

**Example 3:**
Sketch the graph that possesses the characteristics listed.

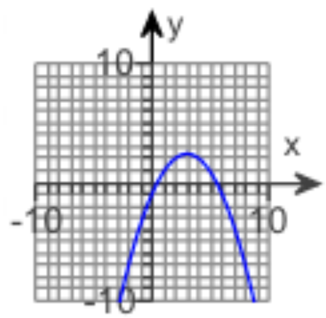
$f'(2) = 0, \quad f''(2) < 0, \quad f(2) = 2;$

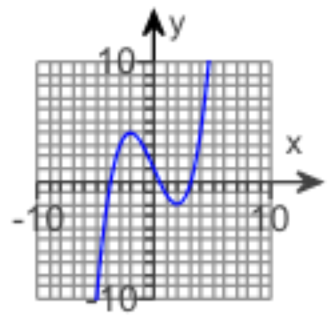
$f'(-2) = 0, \quad f''(-2) > 0, \quad f(-2) = -4;$

$f''(0) = 0, \quad f(0) = -1;$










**Example 4:**
The percentage of the US civilian labor force aged 45-54 can be modeled by the function given by $f(x) = -0.0115x^2 + 0.125x + 81.7$, where *x* is the number of years after 1994. Find the year in which the percentage of the labor force aged 45-54 was highest.

The first and second derivatives enhance our ability to sketch curves. Use the first derivative to find critical values, increasing, decreasing, and relative extrema. Use the second derivative to find possible inflection points, concave up, concave down, and inflection points. Put all the information together for a complete graph. Check with a graphing calculator or using desmos.com.

**Example 5:**

$$f(x) = x^3 + 3x^2 - 9x - 13$$

| Intervals where f(x) is | Points in (x,y) form |
|---|---|
| Increasing | Relative Max |
| Decreasing | Relative Min |
| Concave up | Inflection point |
| Concave down | Don't forget to sketch |

**Example 6:**

$$f(x) = x^4 - 4x^3 + 10$$

| Intervals where f(x) is | Points in (x,y) form |
|---|---|
| Increasing | Relative Max |
| Decreasing | Relative Min |
| Concave up | Inflection point |
| Concave down | Don’t forget to sketch |

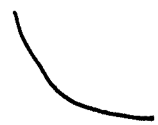
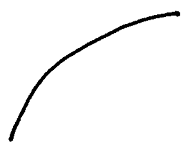
# Section 3.2 Second Derivative and Curve Sketching

A graph of a function is Concave up ↺ if $f''(x) > 0$

A graph of a function is Concave down if $f''(x) < 0$

An inflection point happens where the concavity changes.

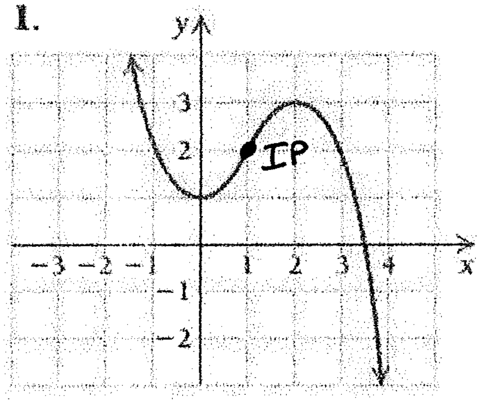
It must occur when $f''(x) = 0$ or when $f''(x)$ does not exist.

| Shape of Graph | $f'(x) > 0$ increasing | $f'(x) < 0$ decreasing |
|---|---|---|
| $f''(x) > 0$ Concave up |  |  |
| $f''(x) < 0$ concave down |  |  |

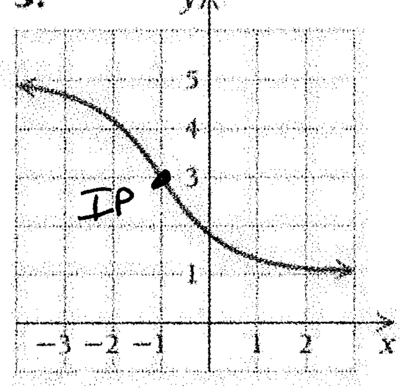
**Example 1**

Identify the points of infection and the intervals where the function is concave up or concave down.

1.




3.




Concave up $(-\infty, 1)$

concave down $(1, \infty)$

Inflection Point $(x, y)$ at $(1, 2)$

Concave up $(-1, \infty)$

concave down $(-\infty, -1)$

inflection point at $(-1, 3)$

**The Second Derivative Test for Relative Extrema**

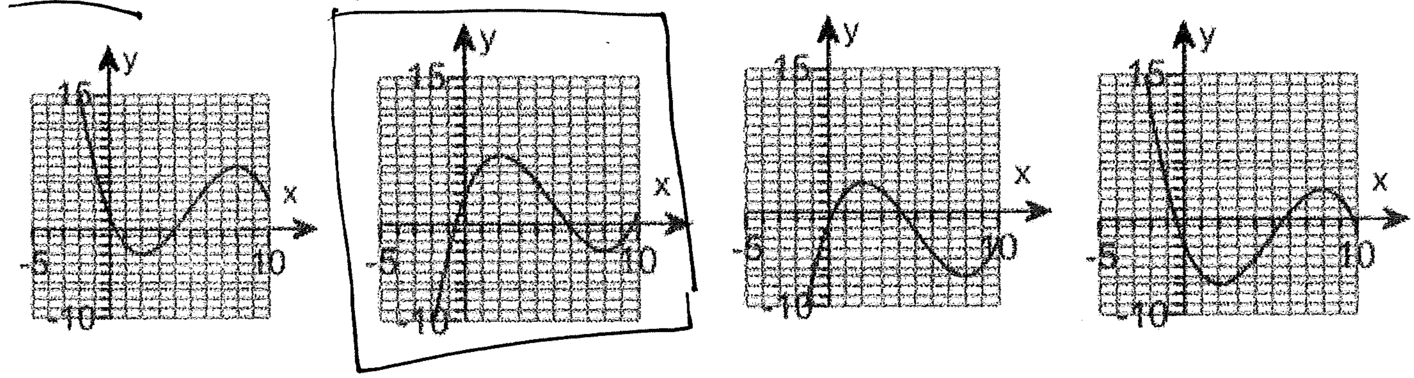
Suppose that c is a critical value for the function (means that $f'(c) = 0$).

Then $f(c)$ is a relative minimum if $f''(x) > 0$

and $f(c)$ is a relative maximum if $f''(x) < 0$

**Example 2:**

Sketch the graph that possesses the characteristics listed. f is concave down at the point (1, 6), concave up at the point (9, -2), and has an inflection point at (5,2).



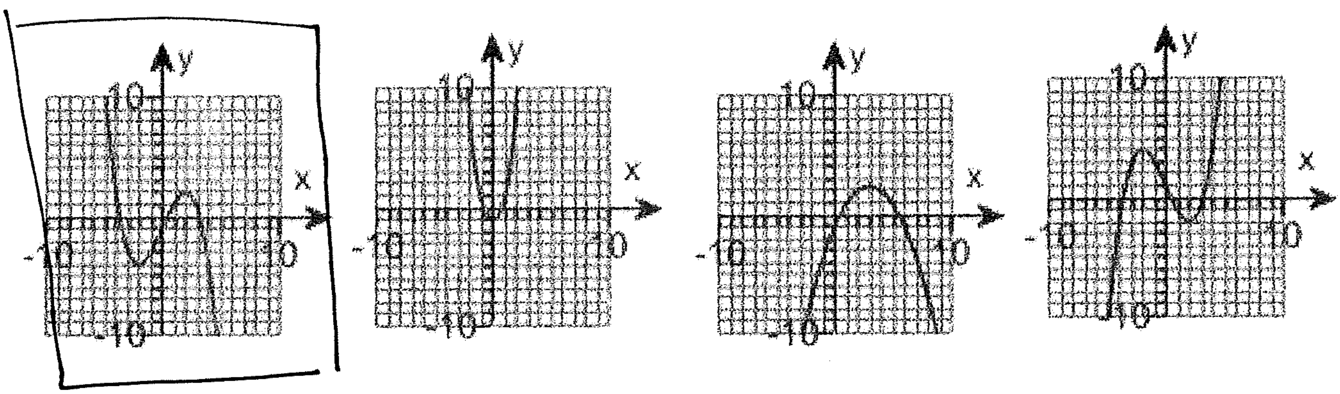
**Example 3:**

Sketch the graph that possesses the characteristics listed.

$f'(2) = 0,\quad f''(2) < 0,\quad f(2) = 2;$ point (2,2) concave down, rel. max

$f'(-2) = 0,\quad f''(-2) > 0,\quad f(-2) = -4;$ point (-2,-4) concave up, rel. min

$f''(0) = 0,\quad f(0) = -1;$ point (0,-1) inflection point



**Example 4:**

The percentage of the US civilian labor force aged 45-54 can be modeled by the function given by $f(x) = -0.0115x^2 + 0.125x + 81.7$, where $x$ is the number of years after 1994. Find the year in which the percentage of the labor force aged 45-54 was highest.

$f'(x) = -0.0230x + 0.125 = 0$

$f''(x) = -0.023$

concave down

$-0.023x = -0.125$

$x = \dfrac{-0.125}{-0.023}$

$x = 5.43$ ← max

year $= 1994 + 5.43 \approx 1999$

The first and second derivatives enhance our ability to sketch curves. Use the first derivative to find critical values, increasing, decreasing, and relative extrema. Use the second derivative to find possible inflection points, concave up, concave down, and inflection points. Put all the information together for a complete graph. Check with a graphing calculator or using desmos.com.

**Example 5:**

$$f(x) = x^3 + 3x^2 - 9x - 13$$

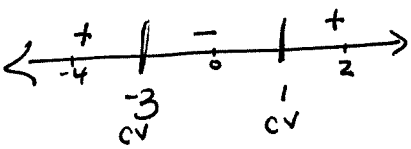
$f'(x) = 3x^2 + 6x - 9$

$3(x^2 + 2x - 3) = 0$

$3(x+3)(x-1) = 0$

$x + 3 = 0 \quad x - 1 = 0$

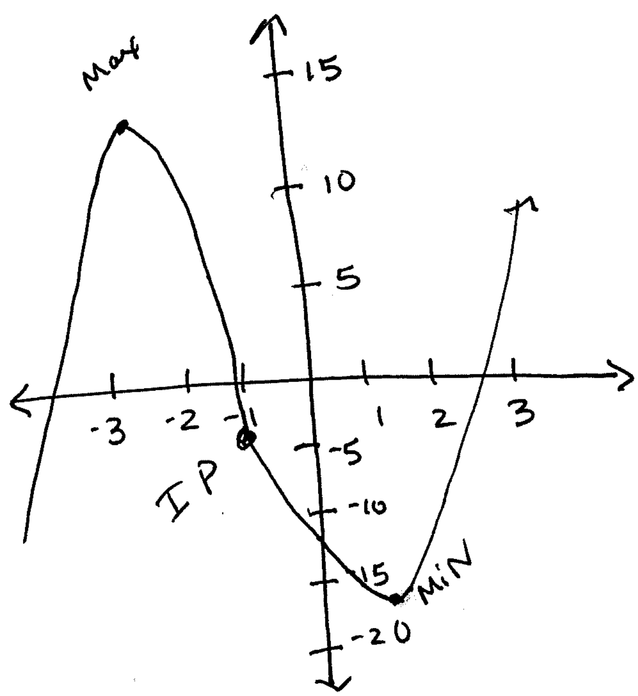
$x = -3 \quad x = 1$




$f'(-4) = +15$

$f'(0) = -9$

$f'(2) = +15$




$f''(x) = 6x + 6$

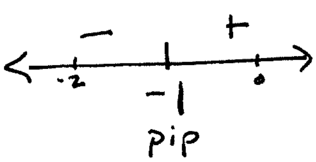
$6x + 6 = 0$

$6x = -6$

$x = -1$

$f''(-2) = -6$

$f''(0) = 6$




| Intervals where f(x) is | | Points in (x,y) form | |
|---|---|---|---|
| Increasing | $(-\infty, -3) \cup (1, \infty)$ | Relative Max | $(-3, 14)$ |
| Decreasing | $(-3, 1)$ | Relative Min | $(1, -18)$ |
| Concave up | $(-1, \infty)$ | Inflection point | $(-1, -2)$ |
| Concave down | $(-\infty, -1)$ | Don't forget to sketch | |

$f(-3) = 14$

$f(1) = -18$

$f(-1) = -2$

**Example 6:**

$$f(x) = x^4 - 4x^3 + 10$$

$f'(x) = 4x^3 - 12x^2$

$4x^3 - 12x^2 = 0$

$4x^2(x-3) = 0$

$4x^2 = 0 \qquad x - 3 = 0$

$x^2 = 0 \qquad x = 3$

$x = 0$

$-\quad -\quad +$

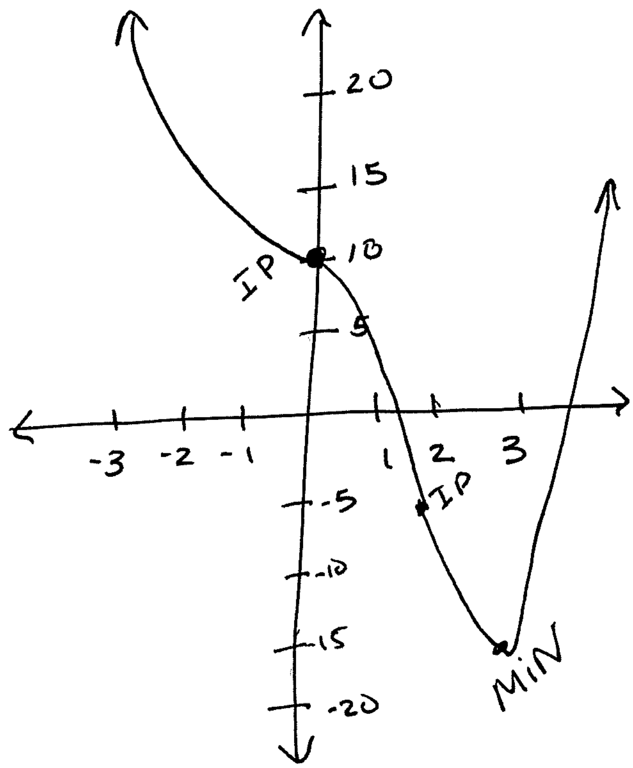
$-1 \quad 0 \quad 1 \quad 3 \quad 4$

0 cv   3 cv

$f'(-1) = -16$

$f'(1) = -8$

$f'(4) = +64$




$f''(x) = 12x^2 - 24x$

$12x^2 - 24x = 0$

$12x(x-2) = 0$

$12x = 0 \qquad x - 2 = 0$

$x = 0 \qquad x = 2$

$+\quad -\quad +$

$-1 \quad 0 \quad 1 \quad 2 \quad 3$

0 pip   2 pip

$f''(-1) = +36$

$f''(1) = -12$

$f''(3) = +36$

| Intervals where f(x) is | | Points in (x,y) form | |
|---|---|---|---|
| Increasing | $(3, \infty)$ | Relative Max | None |
| Decreasing | $(-\infty, 3)$ | Relative Min | $(3, -17)$ |
| Concave up | $(-\infty, 0) \cup (2, \infty)$ | Inflection point | $(0, 10)$ and $(2, -6)$ |
| Concave down | $(0, 2)$ | Don't forget to sketch | |

$f(3) = -17$

$f(0) = 10$

$f(2) = -6$