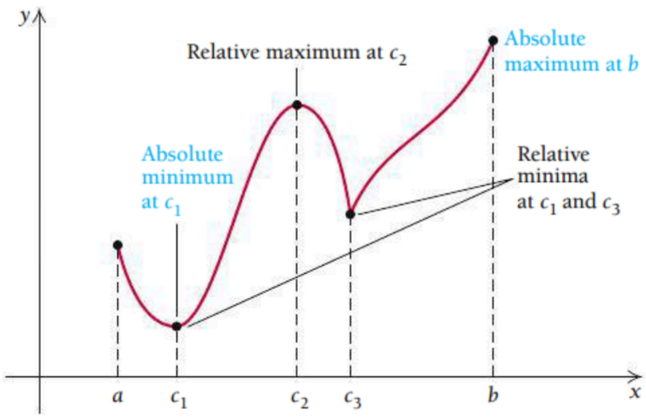
Section 3.4 Absolute Extrema

Absolute Maximum

Absolute Minimum



Extreme Value Theorem
If a function f is continuous on a closed interval [a, b], then f has both a maximum and minimum value on the interval [a, b].

How to find Absolute Extrema on a closed interval [a, b]

1. Find critical values -where f'(x)=0 or f' undefined
2. Evaluate f(x) at all the critical values **and** at the endpoints a and b
3. The largest value is max, the smallest value is min

Example 1: Find the absolute extrema of $f(x) = 4x - x^2$ over the interval [1, 4]

Example 2: Find the absolute extrema of $f(x) = x^3 - 3x + 2$ on [-2, $\frac{3}{2}$]

Example 3: Find the absolute extrema of $f(x) = 2x^3 - 15x^2 + 36x$ on [1, 5]

Example 4: Find the absolute extrema of $f(x) = \sqrt{9 - x^2}$ on [-1,2]

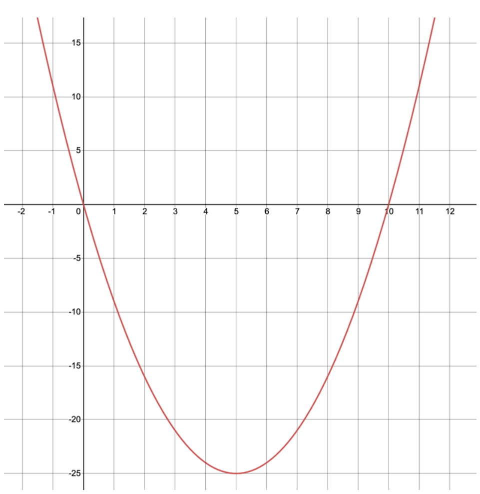
Example 5: Find the absolute extrema of $f(x) = x + \frac{9}{x}$ on [3, 10]

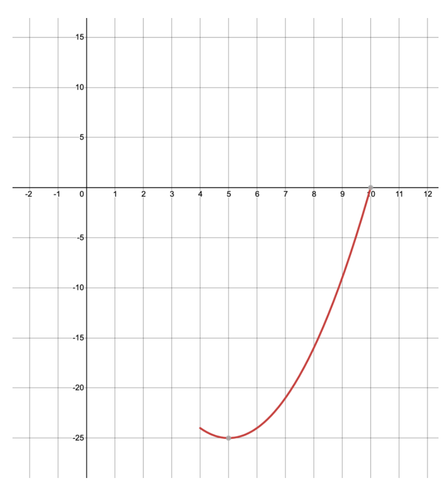
What if the interval isn't closed?

Example 6: Find the absolute extrema of $f(x) = x^2 - 10x$ over each interval.

a. $(-\infty, \infty)$

b. [4, 10]

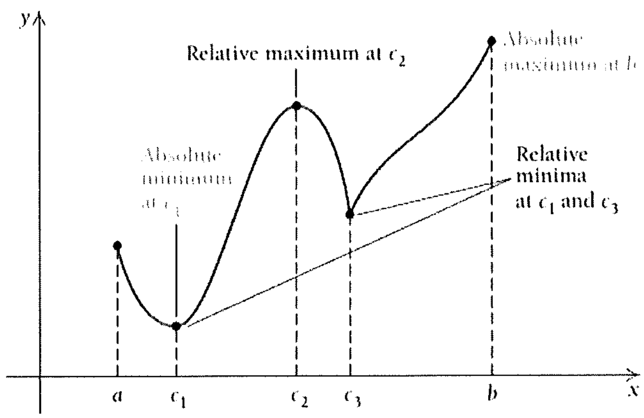




Section 3.4 Absolute Extrema

Absolute Maximum THE largest maximum, highest

Absolute Minimum THE smallest minimum, lowest



Absolute Max at $x = b$

Absolute Min at $x = c_1$

Extreme Value Theorem

If a function f is continuous on a closed interval [a, b], then f has both a abs. maximum and abs. minimum value on the interval [a, b].

How to find Absolute Extrema on a closed interval [a, b]

1. Find critical values -where f′(x)=0 or f′ undefined
2. Evaluate f(x) at all the critical values **and** at the endpoints a and b   plug into original
3. The largest value is max, the smallest value is min

Example 1: Find the absolute extrema of $f(x) = 4x - x^2$ over the interval [1, 4]

$f'(x) = 4 - 2x$
$0 = 4 - 2x$
$2x = 4$
$x = 2$
critical value

evaluate f(x)
$f(2) = 4$
$f(1) = 3$
$f(4) = 0$

The absolute maximum value of f is 4 when $x = 2$

The absolute minimum value of f is 0 when $x = 4$

Example 2: Find the absolute extrema of $f(x) = x^3 - 3x + 2$ on $[-2, \frac{3}{2}]$

$f'(x) = 3x^2 - 3$
$0 = 3x^2 - 3$
$3 = 3x^2$
$1 = x^2$
$\pm 1 = x$

$f(1) = 0$
$f(-1) = 4$
$f(-2) = 0$
$f(3/2) = 7/8$

The abs max value is 4 when $x = -1$

The abs min value is 0 when $x = 1$ and $x = -2$

Example 3: Find the absolute extrema of $f(x) = 2x^3 - 15x^2 + 36x$ on [1, 5]

$f'(x) = 6x^2 - 30x + 36$
$0 = 6(x^2 - 5x + 6)$
$0 = 6(x - 2)(x - 3)$
$x = 2 \quad x = 3$

$f(2) = 28$
$f(3) = 27$
$f(1) = 23$ Abs Min
$f(5) = 55$ Abs Max

$$f(x) = (9-x^2)^{1/2}$$

Example 4: Find the absolute extrema of $f(x) = \sqrt{9-x^2}$ on [-1,2]

$$f'(x) = \frac{1}{2}(9-x^2)^{-1/2}(-2x)$$

$$f'(x) = \frac{-2x}{2(9-x^2)^{1/2}}$$

$$f'(x) = \frac{-x}{\sqrt{9-x^2}}$$

numerator = 0
$-x = 0$
$x = 0$
denominator = 0
$9 - x^2 = 0$
$9 = x^2$
don't use $\rightarrow$ $\pm 3 = x$
not in [-1, 2]

$f(0) = 3$ Abs Max
$f(-1) = \sqrt{8}$
$f(2) = \sqrt{5}$ Abs Min

Example 5: Find the absolute extrema of $f(x) = x + \frac{9}{x}$ on [3, 10]

$$f(x) = x + 9x^{-1}$$

$$f'(x) = 1 - 9x^{-2}$$

$$f'(x) = 1 - \frac{9}{x^2}$$

$$f'(x) = \frac{x^2}{x^2} - \frac{9}{x^2}$$

$$f'(x) = \frac{x^2-9}{x^2}$$

numerator = 0
$x^2 - 9 = 0$
$x = \pm 3$

denominator = 0
$x^2 = 0$
$x = 0$

$f(3) = 6$ Abs.Min
$f(10) = 10.9$ Abs. Max

What if the interval isn't closed?

Example 6: Find the absolute extrema of $f(x) = x^2 - 10x$ over each interval.

a. $(-\infty, \infty)$

$f'(x) = 2x - 10$
$0 = 2x - 10$
$10 = 2x$
$5 = x$

b. [4, 10]

$f(5) = -25$ Abs. Min
$f(4) = -24$
$f(10) = 0$ Abs Max