Section 3.5 Optimization: Business, Economics, and General Applications

Example 1:
Raggs, Ltd. a clothing firm, determines that in order to sell x suits, the price per unit, in dollars, must be $p(x) = 180 - 0.5x$. It also determines that the total cost of producing x suits is given by $C(x) = 2000 + 0.25x^2$.

a. Find the total revenue, $R(x)$
b. Find the total profit, $P(x)$
c. How many suits must the company produce and sell in order to maximize profit?
d. What is the maximum profit?
e. What price per suit must be charged in order to maximize profit?

Example 2:

Cruzing Tunes determines that in order to sell x units of a new car audio receiver, the price per unit, in dollars, must be $p(x) = 1000 - x$.

It also determines that the total cost of producing x units is given by $C(x) = 3000 + 20x$.

a. Find the total revenue, $R(x)$

b. Find the total profit, $P(x)$

c. How many units must be made and sold in order to maximize profit?

d. What is the maximum profit?

e. What price per unit yields this maximum profit?

Example 3:

By keeping records, a theater determines that at a ticket price of $26, it averages 1000 people in attendance. For every drop in price of $1, it gains 50 customers. Each customer spends an average of $4 on concessions. What ticket price should the theater charge in order to maximize total revenue?

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Example 1:
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a. Find the total revenue, $R(x)$
b. Find the total profit, $P(x)$
c. How many suits must the company produce and sell in order to maximize profit?
d. What is the maximum profit?
e. What price per suit must be charged in order to maximize profit?

a) $R(x) = \text{price} * \#\text{ sold}$

$R(x) = (180 - 0.5x)x$

$R(x) = 180x - 0.5x^2$

b) $P(x) = \text{Revenue} - \text{Cost}$

$P(x) = R(x) - C(x)$

$P(x) = (180x - 0.5x^2) - (2000 + .25x^2)$

$P(x) = -.75x^2 + 180x - 2000$

c) $P'(x) = -1.50x + 180$

$0 = -1.50x + 180$

$1.50x = 180$

$x = \frac{180}{1.50} = 120$ suits

d) $P(120) = -.75(120)^2 + 180(120) - 2000$

$= \$8,800$

e) $\text{price} = p(x) = 180 - 0.5x$

$p(120) = 180 - 0.5(120)$

$= \$120$

Example 2:

Cruzing Tunes determines that in order to sell x units of a new car audio receiver, the price per unit, in dollars, must be $p(x) = 1000 - x$.
It also determines that the total cost of producing x units is given by $C(x) = 3000 + 20x$.

a. Find the total revenue, $R(x)$
b. Find the total profit, $P(x)$
c. How many units must be made and sold in order to maximize profit?
d. What is the maximum profit?
e. What price per unit yields this maximum profit?

a) $R(x) = \text{price} * \#\text{sold}$
$R(x) = (1000 - x)(x)$
$R(x) = 1000x - x^2$

b) $P(x) = \text{Revenue} - \text{Cost}$
$P(x) = R(x) - C(x)$
$P(x) = (1000x - x^2) - (3000 + 20x)$
$P(x) = -x^2 + 980x - 3000$

c) $P'(x) = -2x + 980$
$0 = -2x + 980$
$2x = 980$
$x = 490$ units

d) $P(490) = -490^2 + 980(490) - 3000$
$= \$237,100$

e) $\overset{\text{price}}{p}(490) = 1000 - 490$
$= \$510$

Example 3:

By keeping records, a theater determines that at a ticket price of \$26, it averages 1000 people in attendance. For every drop in price of \$1, it gains 50 customers. Each customer spends an average of \$4 on concessions. What ticket price should the theater charge in order to maximize total revenue?

let x = number of dollars by which ticket price decreased

$$\text{Revenue} = \text{revenue from tickets} + \text{revenue from concessions}$$
$$= (\text{ticket price})(\#\text{ people}) + (\$4)(\#\text{ people})$$
$$= (26 - x)(1000 + 50x) + (4)(1000 + 50x)$$
$$= 26000 + 1300x - 1000x - 50x^2 + 4000 + 200x$$
$$R(x) = -50x^2 + 500x + 30000$$

$$R'(x) = -100x + 500$$
$$0 = -100x + 500$$
$$-500 = -100x$$
$$5 = x$$

ticket price? $26 - 5 = \$21$