Section 3.7 Elasticity of Demand

A demand function, $q = D(x)$, relates the quantity q of units purchased at the price x, in dollars per unit. The demand function is usually a decreasing function. Why?

We often need to know how a small increase in price will affect demand and total revenue. A small increase in price may result in a relatively small drop in demand, but ______________________. In this case, a price increase may be wise.

However, if a small price increase results in a significant drop in demand and______________________, then the price increase may not be wise. To measure the sensitivity of demand to a small increase in price, economists calculate the ***elasticity of demand***.

Example 1:
Klix Video games has found that the demand for one of its popular video games is given by $q = D(x) = 120 - 20x$, where q is the number of games rented per day at x dollars per rental.

a. If the price per rental is currently \$2, find the demand and total revenue.
b. If the price is raised to \$2.20, find the demand and total revenue.
c. Interpret these results.
d. If the price per rental is currently \$5, find the demand and total revenue.
e. If the price is increased to \$5.50, find the demand and total revenue.
f. Interpret these results.

The **elasticity of demand** E is given as a function of price x by

$$E(x) = \frac{-x \cdot D'(x)}{D(x)}$$

For a particular value of the price x:

1. Demand is *inelastic* if $E(x) < 1$.

A small increase in price creates an increase in revenue. If demand is inelastic, then revenue is increasing.

2. Demand has *unit elasticity* if $E(x) = 1$.

The demand has unit elasticity when revenue is at a maximum.

3. Demand is *elastic* if $E(x) > 1$.

A small increase in price creates a decrease in revenue. If demand is elastic, then revenue is decreasing.

Example 2: Klix Video games has found that the demand for one of its popular video games is given by $q = D(x) = 120 - 20x$, where q is the number of games rented per day at x dollars per rental.

a. Find the elasticity of demand as a function of x.
b. Find the elasticity at $x = 2$ and at $x = 5$. Interpret the meaning of these values of the elasticity.
c. Find the value of x for which $E(x) = 1$. What is the significance of this price?

Example 3: Wayne's Honey Farm sells honey-based moisturizing cream for \$12 per jar. At this price, it sells 14,500 jars per month. To see if the price is too high or too low, Wayne's Honey Farm raises the price to \$18 per jar. At this price, it sells 10,750 jars per month. The demand function is $q = D(x) = -625x + 22{,}000$ where x is the price in dollars.

a. Find the elasticity of demand, $E(x)$.
b. Find $E(12)$ and $E(18)$ and interpret the meaning of each result.
c. Find the price at which demand has unit elasticity and interpret its meaning.

Section 3.7 Elasticity of Demand

A demand function, $q = D(x)$, relates the quantity q of units purchased at the price x, in dollars per unit. The demand function is usually a decreasing function. Why?

as price increases, demand decreases

We often need to know how a small increase in price will affect demand and total revenue. A small increase in price may result in a relatively small drop in demand, but higher revenue. In this case, a price increase may be wise.

However, if a small price increase results in a significant drop in demand and lower revenue, then the price increase may not be wise. To measure the sensitivity of demand to a small increase in price, economists calculate the ***elasticity of demand***.

Example 1:

Klix Video games has found that the demand for one of its popular video games is given by $q = D(x) = 120 - 20x$, where q is the number of games rented per day at x dollars per rental.

a. If the price per rental is currently \$2, find the demand and total revenue.
b. If the price is raised to \$2.20, find the demand and total revenue.
c. Interpret these results.
d. If the price per rental is currently \$5, find the demand and total revenue.
e. If the price is increased to \$5.50, find the demand and total revenue.
f. Interpret these results.

a) if $x = 2$

$D(2) = 120 - 20(2) = 80$

$R(2) = \text{price} * \#\text{sold} = 2(80) = \$160$

b) when $x = 2.20$

$D(2.20) = 120 - 20(2.20) = 76$

$R(2.20) = (2.20)(76) = \$167.20$

c) small increase in price results in higher revenue

d) when $x = 5$

$D(5) = 120 - 20(5) = 20$

$R(5) = 5(20) = 100$

e) when $x = 5.50$

$D(5.50) = 120 - 20(5.50) = 10$

$R(5.50) = (5.50)(10) = \$55$

f) small increase in price results in lower revenue

The **elasticity of demand** E is given as a function of price x by

$$E(x) = \frac{-x \cdot D'(x)}{D(x)}$$

For a particular value of the price x:

1. Demand is *inelastic* if $E(x) < 1$.

A small increase in price creates an increase in revenue. If demand is inelastic, then revenue is increasing.

2. Demand has *unit elasticity* if $E(x) = 1$.

The demand has unit elasticity when revenue is at a maximum.

3. Demand is *elastic* if $E(x) > 1$.

A small increase in price creates a decrease in revenue. If demand is elastic, then revenue is decreasing.

Example 2: Klix Video games has found that the demand for one of its popular video games is given by $q = D(x) = 120 - 20x$, where q is the number of games rented per day at x dollars per rental.

a. Find the elasticity of demand as a function of x.
b. Find the elasticity at $x = 2$ and at $x = 5$. Interpret the meaning of these values of the elasticity.
c. Find the value of x for which $E(x) = 1$. What is the significance of this price?

a) $E(x) = \frac{-x\,D'(x)}{D(x)} = \frac{-x\,(-20)}{120-20x} = \frac{20x}{120-20x}$

b) $E(2) = \frac{20(2)}{120-20(2)} = \frac{40}{80} = \frac{1}{2}$

At \$2.00 per game rental, the demand is inelastic and a small increase in price causes revenue to increase

$E(5) = \frac{20(5)}{120-20(5)} = \frac{100}{20} = 5$

At \$5 per game rental, the demand is elastic and a small increase in price causes revenue to drop

c) $E(x) = 1$

$\frac{20x}{120-20x} = 1$

$20x = 120 - 20x$

$40x = 120$

$x = 3$

when the price of the rental is \$3 the revenue is maximized.

Example 3: Wayne's Honey Farm sells honey-based moisturizing cream for \$12 per jar. At this price, it sells 14,500 jars per month. To see if the price is too high or too low, Wayne's Honey Farm raises the price to \$18 per jar. At this price, it sells 10,750 jars per month. The demand function is $q = D(x) = -625x + 22{,}000$ where x is the price in dollars.

a. Find the elasticity of demand, $E(x)$.
b. Find $E(12)$ and $E(18)$ and interpret the meaning of each result.
c. Find the price at which demand has unit elasticity and interpret its meaning.

a) $E(x) = \dfrac{-x\,D'(x)}{D(x)} = \dfrac{-x(-625)}{-625x + 2200} = \dfrac{625x}{-625x + 2200}$

b) $E(12) = \dfrac{625(12)}{-625(12) + 2200} = .517$

At \$12 a jar, the demand is inelastic and a small increase in price causes revenue to increase.

$E(18) = \dfrac{625(18)}{-625(18) + 2200} = 1.046$

At \$18 a jar, the demand is elastic and a small increase in price causes revenue to decrease.

c) $E(x) = 1$

$\dfrac{625x}{-625x + 2200} = 1$

$625x = -625x + 2200$

$1250x = 22000$

$x = \dfrac{22000}{1250} = \$17.60$

At \$17.60 per jar, the revenue is maximized