Section 4.1 Antiderivatives and Integration

Antidifferentiation is the process of differentiation performed in reverse. Given a function f, we find another function F such that $\frac{d}{dx}F(x) = f(x)$. The function F is an antiderivative of f.

Example 1: If we found $f'(x) = 2x$, then what was $f(x)$?

Theorem

The **antiderivative** of $f(x)$ is the set of functions $F(x) + C$ such that $\frac{d}{dx}[F(x) + C] = f(x)$. The constant C is called the **constant of integration**.

If F is an antiderivative of f, we write $\int f(x)\, dx = F(x) + C.$

This equation is read as "the antiderivative of $f(x)$, with respect to x, if $F(x) + C$" or as "the integral of $f(x)$, with respect to x, is $F(x) + C$." The expression on the left side is called an **indefinite integral.** The symbol $\int$ is the integral sign, and $f(x)$ is the integrand. The symbol dx can be regarded as indicating that x is the variable of integration, similar to d/dx indicating that the expression that follows it is to be differentiation with respect to x.

Example 2: Determine these indefinite integrals. Find the antiderivative of each integrand. How can you check your answer?

a) $\int 8\, dx$

b) $\int 3x^2 dx$

c) $\int e^x dx$

d) $\int \frac{1}{x} dx, x \neq 0$

Rules for Antiderivatives

1. Constant Rule $\int k\,dx = kx + C$

2. Power Rule (where n≠1) $\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C$

3. Natural Logarithm Rule $\int \frac{1}{x}dx = \ln|x| + C$ and for $x > 0$, $\int \frac{1}{x}dx = \ln x + C$

4. Exponential Rule (base e and $a \neq 0$) $\int e^{ax}\,dx = \frac{1}{a}e^{ax} + C$

The Power Rule for Antiderivatives can be viewed as a two-step process:

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C$$

1
2

1. Increase the exponent by 1.
2. Divide the term by the new power.

Example 3: Determine these indefinite integrals using the power rule for antiderivatives.

a). $\int x^8\,dx$

b). $\int x^2 dx$

c) $\int \sqrt{x}\,dx$

d) $\int \frac{1}{x^3}dx, x \neq 0$

Example 4: Determine these indefinite integrals using the exponential rule for antiderivatives.

a). $\int e^{4x}\,dx$

b). $\int e^{2x} dx$

c) $\int e^{-x}\,dx$

Properties of Antiderivatives

1. A constant multiplier can be factored to the front of the indefinite integral.

$$\int c \cdot f(x)\, dx = c \cdot \int f(x)\, dx$$

2. The antiderivative of a sum or different is the sum or difference of the antiderivatives.

$$\int f(x) \pm g(x)\ dx = \int f(x)\, dx \pm \int g(x)\, dx$$

Example 5: Find the following indefinite integrals. Assume $x > 0$.

a) $\int (x^4 - x + 5)\, dx$

b) $\int (3x^5 + 7x^2)\, dx$

c) $\int (x-3)^2 dx$

d) $\int \frac{3x+2x^4}{x}\, dx$

e) $\int \frac{4}{x} dx$

Example 6: Initial Condition: Use the information given to find C.

$F(x) = \int (2x+3)\, dx$ and $F(1) = -2$

5. $\int x^{1/4}\,dx$

7. $\int (x^2 + x - 1)\,dx$

9. $\int (2t^2 + 5t - 3)\,dt$

11. $\int \frac{1}{x^3}\,dx$

13. $\int \sqrt[6]{x}\,dx$

15. $\int \sqrt{x^5}\,dx$

17. $\int \frac{dx}{x^4}$

19. $\int \frac{10}{x}\,dx$

21. $\int \left(\frac{3}{x} + \frac{5}{x^2}\right) dx$

23. $\int \frac{-7}{\sqrt[3]{x^2}}\,dx$

25. $\int e^{3x}\,dx$

27. $\int 2e^{2x}\,dx$

29. $\int 6e^{x/2}\,dx$

31. $\int 100e^{0.02x}\,dx$

Section 4.1 Antiderivatives and Integration

Antidifferentiation is the process of differentiation performed in reverse. Given a function f, we find another function F such that $\frac{d}{dx}F(x) = f(x)$. The function F is an antiderivative of f.

Example 1: If we found $f'(x) = 2x$, then what was $f(x)$?

$f'(x) = 2x$ then $f(x) = x^2$

$x^2 + 5$

$x^2 - 17$

In general

$f(x) = x^2 + C$

Theorem

The **antiderivative** of $f(x)$ is the set of functions $F(x) + C$ such that $\frac{d}{dx}[F(x) + C] = f(x)$. The constant C is called the **constant of integration**.

If F is an antiderivative of f, we write $\int f(x)\,dx = F(x) + C$.

integral

variable

constant

This equation is read as "the antiderivative of $f(x)$, with respect to x, if $F(x) + C$" or as "the integral of $f(x)$, with respect to x, is $F(x) + C$." The expression on the left side is called an **indefinite integral.** The symbol $\int$ is the integral sign, and $f(x)$ is the integrand. The symbol dx can be regarded as indicating that x is the variable of integration, similar to d/dx indicating that the expression that follows it is to be differentiation with respect to x.

Example 2: Determine these indefinite integrals. Find the antiderivative of each integrand. How can you check your answer?

a) $\int 8\,dx$

$= 8x + C$

b) $\int 3x^2 dx$

$= x^3 + C$

c) $\int e^x dx$

$= e^x + C$

d) $\int \frac{1}{x} dx, x \neq 0$

$= \ln|x| + C$

why absolute value?

can't take the log of a negative number.

Rules for Antiderivatives

1. Constant Rule $\quad \int k\,dx = kx + C$

2. Power Rule (where n≠1) $\quad \int x^n\,dx = \frac{1}{n+1}x^{n+1} + C$

3. Natural Logarithm Rule $\quad \int \frac{1}{x}dx = \ln|x| + C$ and for $x > 0$, $\int \frac{1}{x}dx = \ln x + C$

4. Exponential Rule (base e and $a \neq 0$) $\quad \int e^{ax}\,dx = \frac{1}{a}e^{ax} + C$

The Power Rule for Antiderivatives can be viewed as a two-step process:

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C$$

1. Increase the exponent by 1.
2. Divide the term by the new power.

Example 3: Determine these indefinite integrals using the power rule for antiderivatives.

a). $\int x^8\,dx =$

$\frac{1}{8+1} x^{8+1} + C = \boxed{\frac{1}{9}x^9 + C}$

b). $\int x^2 dx =$

$\frac{1}{2+1} x^{2+1} + C = \boxed{\frac{1}{3}x^3 + C}$

c) $\int \sqrt{x}\,dx = \int x^{1/2} dx =$

$\frac{1}{\frac{1}{2}+1} x^{\frac{1}{2}+1} + C = \boxed{\frac{2}{3}x^{3/2} + C}$

d) $\int \frac{1}{x^3}dx, x \neq 0 = \int x^{-3} dx$

$\frac{1}{-3+1} x^{-3+1} + C$

$\boxed{\frac{1}{-2}x^{-2} + C}$

Example 4: Determine these indefinite integrals using the exponential rule for antiderivatives.

a). $\int e^{4x}\,dx =$ $\quad a = 4$

$\boxed{\frac{1}{4}e^{4x} + C}$

b). $\int e^{2x} dx =$ $\quad a = 2$

$\boxed{\frac{1}{2}e^{2x} + C}$

c) $\int e^{-x}\,dx =$ $\quad a = -1$

$\boxed{-e^{-x} + C}$

Properties of Antiderivatives

1. A constant multiplier can be factored to the front of the indefinite integral.

$$\int c \cdot f(x)\, dx = c \cdot \int f(x)\, dx$$

2. The antiderivative of a sum or different is the sum or difference of the antiderivatives.

$$\int f(x) \pm g(x)\ dx = \int f(x)\, dx \pm \int g(x)\, dx$$

Example 5: Find the following indefinite integrals. Assume $x > 0$.

a) $\int (x^4 - x + 5)\, dx$

$\frac{1}{5}x^5 - \frac{1}{2}x^2 + 5x + C$

b) $\int (3x^5 + 7x^2)\, dx$

$3(\frac{1}{6})x^6 + 7(\frac{1}{3})x^3 + C$

$\frac{1}{2}x^6 + \frac{7}{3}x^3 + C$

c) $\int (x-3)^2 dx = \int x^2 - 6x + 9\, dx$

$\frac{1}{3}x^3 - 6(\frac{1}{2})x^2 + 9x + C$

$\frac{1}{3}x^3 - 3x^2 + 9x + C$

d) $\int \frac{3x+2x^4}{x} dx = \int \frac{3x}{x} + \frac{2x^4}{x}\, dx$

$= \int 3 + 2x^3\, dx$

$= 3x + 2\frac{1}{4}x^4 + C$

$= 3x + \frac{1}{2}x^4 + C$

e) $\int \frac{4}{x} dx$

$4 \int \frac{1}{x}\, dx$

$4 \ln x + C$

Example 6: Initial Condition: Use the information given to find C.

$F(x) = \int (2x+3)\, dx$ and $F(1) = -2$

$F(x) = \int (2x+3)\, dx = x^2 + 3x + C$

$F(1) = 1^2 + 3(1) + C = -2$

$4 + C = -2$

$C = -6$

$\boxed{F(x) = x^2 + 3x - 6}$

WS in class

5. $\int x^{1/4}\,dx = \frac{4}{5}X^{5/4}+C$

7. $\int (x^2+x-1)\,dx = \frac{1}{3}X^3+\frac{1}{2}X^2-X+C$

9. $\int (2t^2+5t-3)\,dt = \frac{2}{3}t^3+\frac{5}{2}t^2-3t+C$

11. $\int \frac{1}{x^3}\,dx = \int X^{-3}dX = -\frac{1}{2}X^{-2}+C$

13. $\int \sqrt[6]{x}\,dx = \int X^{1/6}dX = \frac{6}{7}X^{7/6}+C$

15. $\int \sqrt{x^5}\,dx = \int X^{5/2}dX = \frac{2}{7}X^{7/2}+C$

17. $\int \frac{dx}{x^4} = \int X^{-4}dX = \frac{1}{-3}X^{-3}+C$

19. $\int \frac{10}{x}\,dx = 10\ln|X|+C$

21. $\int \left(\frac{3}{x}+\frac{5}{x^2}\right)dx = 3\ln|X|-5X^{-1}+C$

23. $\int \frac{-7}{\sqrt[3]{x^2}}\,dx = \int -7X^{-2/3}dX = -21X^{1/3}+C$

25. $\int e^{3x}\,dx = \frac{1}{3}e^{3X}+C$

27. $\int 2e^{2x}\,dx = e^{2X}+C$

29. $\int 6e^{x/2}\,dx = 12e^{\frac{X}{2}}+C$

31. $\int 100e^{0.02x}\,dx = 5000e^{0.02X}+C$